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Mixed strategy Nash Equilibrium and Quantal Response Equilibrium. An experimental comparison using RPS games(1)

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Abstract. *In this paper I test the hypothesis according to which the Quantal Response Equilibrium (QRE) solution concept significantly outperforms the Mixed Strategy Nash Equilibrium (MSNE) in experimental situations. The testing ground for the hypothesis is an experiment with variations of the RPS game applied in June 2013 to students from various universities in Bucharest. The results of the experiment show that the QRE solution does not perform substantially better than the Nash Equilibrium for two of the three games studied and that it does represent a slight improvement in only one of the games. This result lies in stark contrast to the bulk of the literature developed thusfar on the QRE solution*

Keywords: experiment, Mixed Strategy Nash Equilibrium, Quantal Response Eqilibrium, strategies, solution concept.

JEL Classification: C72, C73, C79, C91, D83.
REL Classification: 7L, 9J, 9Z.

1. Introduction

The solution introduced in (1950) by John Nash, i.e. the Nash Equilibrium, is without a doubt the most used solution concept in game theory[(3)]. A vast strand of the meta-theoretical literature in this field has been focused solely on producing refinements to the Nash Equilibrium solution for the past 60 years. But still, the Nash Equilibrium solution has often come under heavy criticism for often being inadequate when it comes to predicting observed human behavior. The Quantal Response Equilibrium[(4)], proposed by McKelvey and Palfrey (1995) seeks to address this concern by providing an alternate solution concept, one which is not a refinement of the Nash Equilibrium but rather a statistical counterpart. In a nutshell, the QRE replaces the best response utility functions of individuals, used in classical game theory, with quantal response functions that attach a wide array of probabilities to playing pure strategies according to an error parameter[(5)].

The purpose of this paper is to experimentally test the presumed superiority of the Quantal Response Equilbrium over the Mixed Strategy Nash Equilibrium[(6)] in 3×3 games, and, more specifically for several variations of the RPS game. The paper is structured as follows: in the second section I present an outline of the Quantal Response Equilibrium in which I detail some of its general characteristics and the logit QRE operationalization which has been commonplace in the economic literature. In the third section I describe the RPS game in its original form. I then describe the experimental design and present the general results obtained which are relevant to my paper, i.e. the mixed strategies used by players in each of the three games played. In the fourth section I analyze the predictive capacity of the Mixed Strategy Nash Equilibrium solution for each of these games. In the fifth section I perform an identical task for the QRE solution and compare it with the Nash Equilibrium. In the last part I present the conclusions and limitations of the paper.

2. Theoretical background: the MSNE and QRE solutions

The Nash Equilibrium solution is characteristic of a profile which has the property that no player has any reason to unilaterally change his strategy. In other words, a Nash Equilibrium is the best response of each player to the strategy used by the other player. Formally, if we have two players i and j with agendas $S_i = \{s_1, s_2\}, S_j = \{s_3, s_4\}$, $\{s_1, s_3\}$ is a Nash Equilibrium *iff* $u_i(s_1, s_3) \geq u_i(s_2, s_3)$ and at the same time $u_j(s_1, s_3) \geq u_j(s_1, s_4)$. Since a game can have more than a single Nash equilibrium[(7)], its usage is not able to guarantee that we will obtain a single profile as a result (and consequently make definite predictions). Still, the Nash Equilibrium solution is "without doubt the single game theoretic solution concept that is most frequently applied in economics" (Aumann, 2000, p. 67). The Mixed Strategy Nash Equilibrium (MSNE) is an extension of the concept of Nash Equilibrium from pure strategies to mixed strategies. A mixed strategy is a combination of pure strategies, in which players assign probabilities to choosing each alternative within a round. Formally, we can represent a mixed strategy[(8)] as $\sigma_i(s_i)$ with the agenda being represented by the set of probability distributions defined on S_i, namely $\Delta_i = \Delta(S_i)$ (McCarty, Meirowitz, 2007, p. 90). The MSNE solution therefore characterizes the profile of mixed strategies where each player provides the best response to the mixed strategy used by the other player[(9)].

The main reason for the introduction of the Quantal Response Equilibrium[(10)] in the mainstream solution set of game theory is the widespread failure of classical solution concepts, and in particular the Mixed Strategy Nash Equilibrium, to accurately predict human behavior in empirical and experimental situations, especially in games where the classical solutions yield somewhat counterintuitive results, predicting that the game will end with low-payoff outcomes for all players. There are various examples which support this finding. One of them is the centipede game, studied by McKelvey and Palfrey (1992). In the general form of this game, two players compete in an *n*-round game, with the number of rounds being common knowledge between the players. On every round of the game one of the players has the option to either take the money (*t*), in which case the game ends, or pass (*p*), in which case the game continues. During every round in which a player passes, the payoff is multiplied by a number *k*, with the highest payoff corresponding to the player who chooses an alternative. For instance, in McKelvey and Palfrey's experiment, the game typically lasted for 4 rounds[(11)], with the standard payoffs being (0.4, 0.1) in the first round, where player 1 had to make the choice, (0.2, 0.8) in the second one, where player 2 had to make the choice, (1.6, 0.4) in the third round where it was once again player 1's turn to make the choice, and (0.8, 3.2) in the fourth round when the roles were once again reversed. Although it was clear that both players received higher payoffs if the game lasts longer, the Nash Equilibrium solution predicts that the first player will choose to take the money in the first round. The experimental results proved to thoroughly refute this hypothesis however, with only 7% of the 4-round games and 1% of the 6-round games ending on the first round.

Another such example is the ultimatum game, introduced by Guth et al. (1982), in the case of which Binmore et al. (1995) experimentally show that the subgame perfect Nash Equilibrium fails to correctly predict the vast majority of player strategies. Still another frequently encountered example, described at length by McKelvey and Palfrey (1995) and used by Goeree et al. (2005) to explain the concept of Regular QRE is that of the matching pennies game, in its original and modified versions, as introduced by Ochs (1995). The author considers three versions of the matching pennies game with two players. In the first one the payoffs are identical between the players, with the first one gaining +1 (and the second one obtaining 0) if the strategies selected coincide and the second one obtaining +1 (with the first one obtaining 0) if the profile resulted is composed of distinct strategies. The two modified versions of the game yield +9 (in the first version) and +4 (in the second one) instead of +1 for the profile in which both player 1 and player 2 choose s_1, with all the other payoffs (included 0 for the second player in this profile) remaining unchanged. As Ochs (1995, p.205) explains, although the MSNE prediction in this case would yield different probabilities of playing pure strategies for player 2, in the case of player 1 he would have to play each strategy with the same 0.5 probability since that ensures that his opponent is indifferent amongst his/her alternatives. The results obtained by Ochs, on an experimental sample consisting of 48 subjects for all three sessions (1995, p.208), thoroughly disproved the hypothesis that player 1 would play according to the predictions of the MSNE in the second and third game, with the choice of s_1 in both games significantly exceeding 0.5[(12)].

Thus, the unravelling of these examples and others which similarly showed that the MSNE or other classical solution concepts are poor predictors of actual human behavior, represented the context in which the QRE solution arises as a powerful contender in game theory during the 1990s.

QRE is based on the idea that “players are ‘better responders’ rather than best responders” (Goeree et al., 2005, p. 364). The solution concept is introduced for normal-form games[(13)] by McKelvey and Palfrey (1995) who sought to replace “the perfectly rational expectations equilibrium embodied in Nash Equilibrium with an imperfect, or noisy, rational expectations equilibrium” (McKelvey, Palfrey, 1995, p. 7). Thus, the QRE can be defined as “an extension of standard random utility models of discrete (‘quantal’) choice to strategic settings, or as a generalization of Nash Equilibrium that allows noisy optimizing behavior while maintaining the internal consistency of rational expectations” (Haile et al., 2008, p.180).

The basic idea behind the QRE[(14)] is to define an equilibrium solution while allowing individuals to play the game in a sub-optimal fashion, due to the fact that they are characterized by various cognitive constraints and limitations. Thus, bounded rationality[(15)], as the core element which underlies the QRE[(16)], is then used to replace the best response functions of individuals which characterizes a Nash Equilibrium solution with a *statistical reaction function* (McKelvey, Palfrey, 1995, p. 10) or *quantal response function* (Goeree et al., 2005, p. 351).

One interesting aspect of the QRE solution is that in of itself it only specifies the procedure for generating predictions regarding strategy choices but it does not specify any exact mathematical tool employed in this regard. The most frequent way to operationalize it is to use the *Logit QRE*, which uses the following response functions: $P_{ij} = \frac{e^{\lambda \pi_{ij}}}{\sum_{k=1}^{J_i} e^{\lambda \pi_{ik}}}$, where P_{ij} is the quantal response function of player *i* which corresponds to strategy *j*, λ is an error parameter and π_{ij} is the expected payoff obtained by player *i* when playing strategy *j*[17].

As we can see from their Logit expression above, quantal response functions vary according to an error parameter λ, which can take values from 0, where the choices made are completely random, or as McKelvey and Palfrey phrase it, "actions consist of all error" (1995, p.11) to $+\infty$,where no error exists and the choices made overlap or asymptotically approach the Nash Equilibrium solution. The intersection of each of these functions generate the Quantal Response Equilibrium, which is a fixed point[18].

3. The RPS game

"Rock-Paper-Scissors" [19] is a game well known to individuals due to its frequent usage in common day situations to solve minor disputes between parties where no one is perceived to be more entitled to a disputed good than the other[20], [21]. In its standard form, the game is played between two players, let us call them p_1 and p_2. Each player has three available strategies on the agenda: Rock (*r*), Paper (*p*) and Scissors (*s*), thus $S_i = \{r, p, s\}$. Since both players have the same agenda, there are $S_i \times S_j$ profiles possible, namely $\{(r,r),(r,p),(r,s),(p,r),(p,p),(p,s),(s,r),(s,p),(s,s)\}$. The payoffs for each player are determined by the following pattern: a) Rock crushes Scissors, i.e. $r > s$, b) Scissors cuts Paper, i.e. $s > p$, c) Paper covers Rock, i.e. $p > r$ and d) in case an alternative is played against its twin the game ends in a draw.

The general game is depicted in its strategic form in Figure 1. In its standard form, for each win the players gain +1, for each loss -1 and for each draw 0. Thus, for player 1 we have $u_1(r,s) = u_1(p,r) = u_1(s,p) = +1$, $u_1(r,r) = u_1(p,p) = u_1(s,s) = 0$ and $u_1(r,p) = u_1(p,s) = u_1(s,r) = -1$. Since the game is symmetrical player 2 has an identical payoff scheme. The first thing we notice is that there is no Pure Strategy Nash Equilibrium since in every profile at least one player has the incentive to unilaterally change his strategy. To see this let us take for instance the profile (r, p). Player 2 gains +1 while Player 1 receives a payoff of -1. Naturally, Player 1 has an incentive to unilaterally change his strategy to s. In the resulting profile, i.e. (s, p) Player 2 now has an incentive to change his strategy to r. But the resulting profile, (s, r) is now once again sub-optimal for Player 1 who changes his strategy to p and so forth. In the case that players reach a tie the reasoning is similar. Suppose the profile resulting from their choices is $u_1(p, p)$. Both players receive 0 and they would like to receive +1. Therefore both of them would be stimulated to unilaterally adopt the strategy s in the following round. But this would result in a tie once again which would lead both players to adopt strategy p in the subsequent round and so forth(22).

Figure 1. *RPS strategic-form game*

p_1 \ p_2	Rock (*r*)	Paper (*p*)	Scissors (*s*)
Rock (*r*)	$u_2(r,r)$ $u_1(r,r)$	$u_2(r,p)$ $u_1(r,p)$	$u_2(r,s)$ $u_1(r,s)$
Paper (*p*)	$u_2(p,r)$ $u_1(p,r)$	$u_2(p,p)$ $u_1(p,p)$	$u_2(p,s)$ $u_1(p,s)$
Scissors (*s*)	$u_2(s,r)$ $u_1(s,r)$	$u_2(s,p)$ $u_1(s,p)$	$u_2(s,s)$ $u_1(s,s)$

Source: Author.

Although there are no pure strategies Nash equilibria, each two-player zero-sum game has at least one equilibria in mixed strategies, which in this case is the profile generated by strategies $\sigma_1(s_1) = \sigma_2(s_2) = \frac{1}{3}r + \frac{1}{3}p + \frac{1}{3}s$. The main idea is "to 'mix' the possible ways of playing with certain probabilities" so that "he can protect himself against loss" (von Neumann, Morgenstern, 1944, p.144) since the other player becomes indifferent amongst his own alternatives (the mix of strategies is built in such a way in which the expected payoff of the opponent is equivalent for all alternatives and equal to 0). The proof of this solution for the standard RPS game is straightforward and I will not detail it here, since the mathematical procedure is also described in Section 4 for the modified versions(23).

4. The experiment

4.1. Experimental design

The experiment consisted of three repeated Rock-Paper-Scissors games, with a modified payoff structure for each game. There were 22 participants in total (all of them responded to a call for participants), with 17 of them studying at the National School of Political and Administrative Studies (10 of them enrolled at that time in Political Science programs at a BA level, 2 of them studying International Relations at a BA level, 3 of them studying Project Evaluation at an MA level, 2 of them studying Government and Society at an MA level and one studying Project Management at an MA level), 3 studying Law either at the University of Bucharest or Nicolae Titulescu University (2 at a BA level and one at an MA level), 1 studying Political Sciences at the University of Bucharest (BA level) and 1 studying Applied Electronics at the Polytechnic University of Bucharest (BA level). Each pair of players (except one) was formed between players who were not previously acquainted in order to mitigate any potential side-effects caused by the establishment of prior social relations between the participants(24).

The three games played by the participants are described in detail in Section 5 of this paper. At the beginning of the experiment, each pair played one round of the regular RPS game in order to determine which one of them would be Player 1 and Player 2(25). The first game which was played for 24 rounds between the players was an RPS game (subsequently called RPS-1; see Figure 2) where strategy *p* yielded a higher payoff (+2) than the other two (which yielded +1) when used as a winning strategy. The game was symmetric, in the sense that the payoff structure satisfied the anonimity property. The second game (subsequently called RPS-2; see Figure 4), which was also played for 24 rounds was identical to RPS-1 except for the fact that strategy *p* yielded a much higher payoff (+8) than before. The other payoffs remained constant. The third game, which was played only for 21 rounds was an RPS game (subsequently called RPS-3; see Figure 6) where winning with strategy *r* yielded a higher payoff for player 1 (+2 instead of the regular +1) and winning with strategy *s* yielded a higher payoff for player 2 (+2 instead of the regular +1). All other payoffs were kept the same. The number of rounds for each game was not communicated to the participants and was varied in order to mitigate any potential end-game effects.

In order to create the conditions for instrumentally rational behavior directed towards winning the game, the players were remunerated according to their performance, with the winner of each game receiving the sum of 6 lei for a victory, with the sum being divided equally between both players in case of ties. If one player won all three games a 2 lei bonus was added to his total payoff, in order to stimulate him to adopt a competitive attitude throughout the entirety of the game, thus a player could win at most 20 lei. If the player did not win any games or lost two games and drew the other he received a 5 lei sum for participation. The length of each game was of approximately 10 minutes. The intention was to balance the length so that players would have time to make calculations between rounds but still keep each game as short as possible, so that the monetary incentives would not be overturned by temporal costs associated with participation to the experiment.

At the beginning of the experiment, each player was handed a sheet of paper with the Instructions for Participants alongside the payoff tables for each game and the Experiment form. The rules, the payoff structures and other logistical details were also explained verbally by the operator of the experiment prior to each of the three games. After the explanations were concluded, the players were given a few minutes in order to reflect on the array of strategies available. Regarding the mechanism of play, participants were required to play physically, i.e. by a show of hands depicting each alternative(26), (27), showing their choices simultaneously after the elapse of the countdown kept by the operator of the experiment. After the choices were revealed each player had to write the initial of his own choice of the respective round on the Experiment form and then consult the payoff table in order to see what the payoff corresponding to the respective profile was. Especially since all of the three games played had only mixed strategy equilibria, the players were allowed to call for a suspension of the game between rounds in order to evaluate the history of the game(28), (29).

After the experiment was conducted, the participants received a questionnaire(30) via e-mail in which they were asked to describe their strategies for each game and especially if the strategies employed were built rather on the history of the game, on the history of the previous round or on a pre-determined probabilistic pattern. The participants were also asked to describe their previous level of knowledge regarding game theoretical concepts such as Nash Equilibrium, Mixed Strategy Nash Equilibrium and Quantal Response Equilibrium and, if familiar, if they intended to play strategies leading to these sort of solutions.

4.2. An overview of the results

From the perspective of this study, the main element of interest generated by the experiment was the combination of pure strategies used by the players during each game, namely the proportion with which each strategy was played. These results are summarized in Table 1.

Each cell of the table represents the probability with which a player played a specific strategy during a game. The double line divides the Player 1-type and the Player 2-type of participants in the experiment. Although the sample size was relatively small, I contend that the results can be validly used to construct a preliminary image of the issues discussed in the following sections.

Table 1. *Percentages of pure strategies played in the three RPS games*

	Game 1			Game 2			Game 3		
Players	*r*	*p*	*s*	*r*	*p*	*s*	*r*	*p*	*s*
1	0.25	0.08	0.66	0	0.16	0.83	0.66	0	0.33
2	0.29	0.42	0.29	0.37	0.37	0.25	0.42	0.38	0.19
3	0.33	0.45	0.2	0.25	0.45	0.29	0.23	0.52	0.23
4	0.33	0.41	0.25	0.25	0.04	0.7	0.14	0.19	0.66
5	0.12	0.62	0.25	0.2	0.45	0.33	0.42	0	0.57
6	0.08	0.54	0.37	0.12	0.58	0.29	0.42	0.28	0.28
7	0.29	0.33	0.37	0.45	0.16	0.37	0.47	0.04	0.47
8	0.25	0.33	0.41	0.33	0.16	0.5	0.28	0.33	0.38
9	0.25	0.37	0.37	0.12	0.5	0.37	0.71	0	0.28
10	0.16	0.54	0.29	0.04	0.45	0.5	0.38	0	0.61
11	0.37	0.41	0.2	0.41	0.12	0.45	0.47	0	0.52
12	0.16	0.62	0.2	0.12	0.62	0.25	0.19	0.57	0.23
13	0.29	0.29	0.37	0.41	0.45	0.12	0.28	0.52	0.19
14	0.2	0.16	0.62	0.12	0.25	0.62	0.23	0.23	0.52
15	0.25	0.41	0.33	0.08	0.62	0.29	0.04	0.66	0.28
16	0.41	0.33	0.25	0.25	0.29	0.45	0.28	0.28	0.42
17	0.37	0.08	0.54	0.16	0.33	0.5	0.23	0.33	0.42
18	0.2	0.37	0.41	0	0.79	0.2	0.28	0.52	0.19
19	0.25	0.37	0.37	0.29	0.41	0.29	0.28	0.23	0.47
20	0.29	0.25	0.45	0.33	0.16	0.5	0.28	0.23	0.47
21	0.2	0.37	0.41	0.12	0.66	0.2	0.09	0.38	0.52
22	0.29	0.33	0.37	0.33	0.37	0.29	0.33	0.33	0.33

Source: Author.

5. Mixed Strategy Nash Equilibrium performance in predicting strategy choices

5.1. MSNE predictive power in RPS-1

The first game played by the participants in the experiment, termed RPS-1 was a symmetrical RPS game with the strategy *p* yielding a higher payoff than *r* or *s* when winning, i.e. +2 instead of +1. By analogy, losing with *r* against *p* yielded a negative payoff of -2 instead of -1. The game is depicted in Figure 2. The game has a unique Mixed Strategy Nash Equilibrium, which will be computed subsequently. Let us consider that $u_i(s_i)$ represents the expected utility gained by *i* when playing strategy s_i and that $\sigma_i(s_i)$ is the mixed strategy played by player *i*.

Let us further consider that $u_i(s_i) = \sum u_i(s_i, s_j)P(s_j)$, where $u_i(s_i, s_j)$ represents the utility gained by player *i* when gaining the payoff of the profile (s_i, s_j) and $P(s_j)$ represents the probability that player *j* will play strategy s_j. Thus, for any known system of payoffs we can solve for $P(s_j)$ in the Mixed Strategy Nash Equilibrium by solving the system of equations generated by the three expected payoffs of *i*.

Figure 2. *RPS-1 game*

p_2 p_1	Rock (*r*)	Paper (*p*)	Scissors (*s*)
Rock (*r*)	0 0	+2 –2	–1 +1
Paper (*p*)	–2 +2	0 0	+1 –1
Scissors (*s*)	+1 –1	–1 +1	0 0

Source: Author.

In RPS-1, let us first solve for the probability parameters of Player 2. Notice that since the sum of all probabilites must be equal the following equivalency holds: $P_2(s) = 1 - P_2(r) - P_2(p)$. Since the game is symmetrical, the exact mixed strategy is prescribed for Player 1 as well in the MSNE. Thus for Player 1 we have the following system of equations:

$$u_1(r) = u(r,r)P_2(r) + u(r,p)P_2(p) + u(r,s)(1 - P_2(r) - P_2(p)) \quad (1.1)$$

$$u_1(p) = u(p,r)P_2(r) + u(p,p)P_2(p) + u(p,s)(1 - P_2(r) - P_2(p)) \quad (2.1)$$

$$u_1(s) = u(s,r)P_2(r) + u(s,p)P_2(p) + u(s,s)(1 - P_2(r) - P_2(p)) \quad (3.1)$$

From (1.1), (2.1) and (3.1) we further obtain:

$$u_1(r) = -2P_2(p) + 1 - P_2(r) - P_2(p) = 1 - P_2(r) - 3P_2(p) \quad (1.2)$$

$$u_1(p) = 2P_2(r) - 1 + P_2(r) + P_2(p) = -1 + 3P_2(r) + P_2(p) \quad (2.2)$$

$$u_1(s) = -P_2(r) + P_2(p) \quad (3.2)$$

But by definition of MSNE we have $u_1(r) = u_1(p) = u_1(s)$, therefore we can replace (3.2) in (1.2) and (2.2) Thus we have:

$$1-P_2(r)-3P_2(p)=-P_2(r)+P_2(p)\Rightarrow 1-P_2(r)-3P_2(p)+P_2(r)-P_2(p)=0 \quad (1.3)$$

$$-1+3P_2(r)+P_2(p)=-P_2(r)+P_2(p)\Rightarrow -1+3P_2(r)+P_2(p)+P_2(r)-P_2(p)=0 \quad (2.3)$$

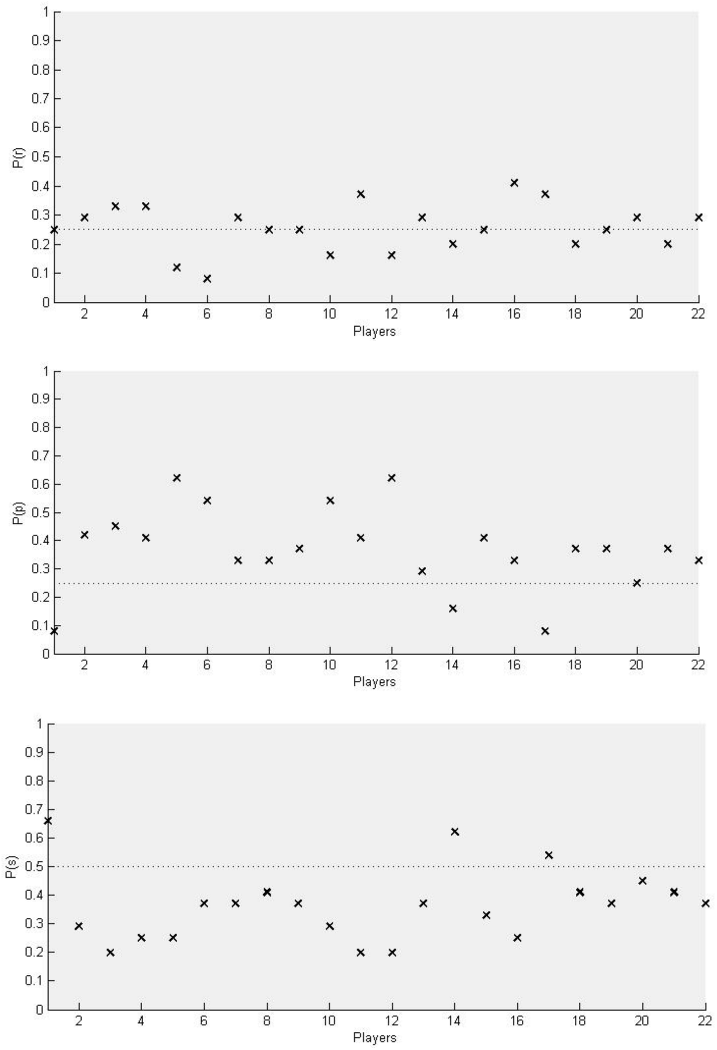
From (1.3) we then obtain $1-4P_2(p)=0\Rightarrow P_2(p)=\frac{1}{4}=0.25$. Also, for (2.3) we have $-1+4P_2(r)=0\Rightarrow P_2(r)=\frac{1}{4}=0.25$. Further, since we know that $P_2(s)=1-P_2(r)-P_2(p)$, we have $P_2(s)=1-0.25-0.25=0.5$. The Mixed Strategy Nash Equilibrium for RPS-1 is therefore achieved when $\sigma_1(s_i)=\sigma_2(s_i)=\{0.25r,0.25p,0.5s\}$.

The MSNE is not immediately intuitive. As we can see, the strategy which yields the highest payoff (*p*) is not the strategy which should be played most often and in fact it is on par with what appears to be the worst possible strategy since losing with *r* yields a negative payoff of -2 while winning with *r* yields a positive payoff of only 1. The MSNE strategy therefore seeks to exploit the temptation of the opponent to frequently use the higher paying strategy, while in the meantime ensuring that the negative payoff associated with a defeat is not significantly large.

In Figure 3 the experimental results are plotted against the MSNE for RPS-1. The first plot corresponds to $P(r)$, the second one to $P(p)$ and the third one to $P(s)$. The dotted lines represent the percentage prescribed by MSNE, with the $\times$'s representing the percentages of each strategy as played in the experiments. By looking at the plots it is easy to see that the Nash Equilibrium predictions perform very poorly. Although in some of the cases $P(r)$ is indeed played as prescribed by the MSNE, i.e. with 0.25 probability, the predictions for $P(p)$ and $P(s)$ fall way outside of their intended targets. Indeed, the bulk of participants fell prey to the temptation to use $P(p)$ more often than prescribed due to its higher payoff (approximately 81% of players), with $P(s)$ being under-used by 86% of players. All in all, none of the participants played the NE mixed strategy.

Further, considering that due to the relatively small number of rounds combined with a lack of knowledge regarding the position of the final turn, it would have been possible for participants to mix the strategies so as to converge towards the Nash Equilibrium but their failure to do so was caused by the end of the game, let us assume that some small margin of disturbance in the percentages required for a mixed strategy to generate a Nash Equilibrium is allowed. With a 0.05 vicinity (which corresponds to an error of +/- 1 in the number of strategies played), we only find one participant who plays consistently with the MSNE.

Figure 3. *MSNE and experimental results in RPS-1*



Source: Author.

5.2. MSNE predictive power in RPS-2

The second game played by the participants in the experiment, termed RPS-2 was a symmetrical RPS game with the strategy *p* yielding an even higher payoff than in RPS-1, i.e. +8 instead of +2. By analogy, losing with *r* against *p* yielded a negative payoff of -8 instead of -2. The game is depicted in Figure 4.

Figure 4. *RPS-2 game*

p_1 \ p_2	Rock (*r*)	Paper (*p*)	Scissors (*s*)
Rock (*r*)	0, 0	–8, +8	+1, –1
Paper (*p*)	+8, –8	0, 0	–1, +1
Scissors (*s*)	–1, +1	+1, –1	0, 0

Source: Author.

In order to compute the MSNE we will proceed in an identical manner as in section 5.1. The game is once again symmetrical, therefore solving the probability parameters for Player 2 will automatically yield the parameters for player one. Once again we consider that $P_2(s) = 1 - P_2(r) - P_2(p)$. We obtain:

$$u_1(r) = u(r,r)P_2(r) + u(r,p)P_2(p) + u(r,s)(1 - P_2(r) - P_2(p)) \quad (4.1)$$

$$u_1(p) = u(p,r)P_2(r) + u(p,p)P_2(p) + u(p,s)(1 - P_2(r) - P_2(p)) \quad (5.1)$$

$$u_1(s) = u(s,r)P_2(r) + u(s,p)P_2(p) + u(s,s)(1 - P_2(r) - P_2(p)) \quad (6.1)$$

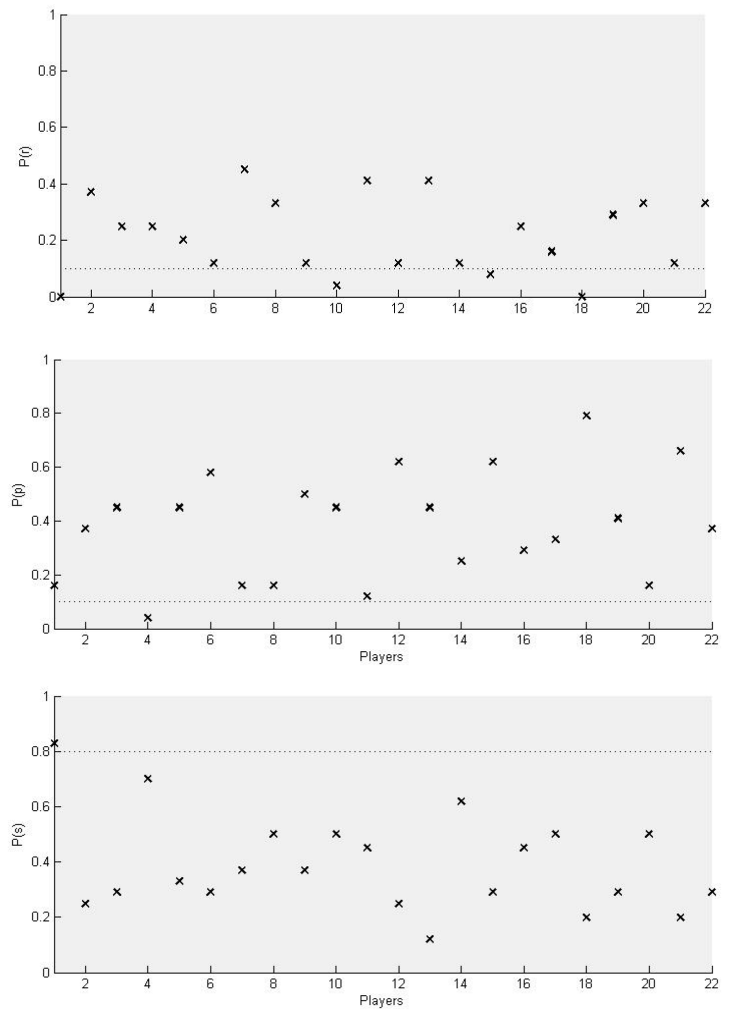
From (4.1), (5.1) and (6.1) we further obtain:

$$u_1(r) = -8P_2(p) + 1 - P_2(r) - P_2(p) = 1 - P_2(r) - 9P_2(p) \quad (4.2)$$

$$u_1(p) = 8P_2(r) - 1 + P_2(r) + P_2(p) = -1 + 9P_2(r) + P_2(p) \quad (5.2)$$

$$u_1(s) = -P_2(r) + P_2(p) \quad (6.2)$$

Figure 5. *MSNE and experimental results in RPS-2*



Source: Author.

But by definition of MSNE we have $u_1(r) = u_1(p) = u_1(s)$, therefore we can replace (6.2) in (4.2) and (5.2) Thus we have:

$$1 - P_2(r) - 9P_2(p) = -P_2(r) + P_2(p) \Rightarrow 1 - P_2(r) - 9P_2(p) + P_2(r) - P_2(p) = 0 \quad (4.3)$$

$$-1 + 9P_2(r) + P_2(p) = -P_2(r) + P_2(p) \Rightarrow -1 + 9P_2(r) + P_2(p) + P_2(r) - P_2(p) = 0 \quad (5.3)$$

From (4.3) we then obtain $1-10P_2(p)=0 \Rightarrow P_2(p)=\frac{1}{10}=0.1$. Also, for (5.3) we have $-1+10P_2(r)=0 \Rightarrow P_2(r)=\frac{1}{10}=0.1$. Further, since we know that $P_2(s)=1-P_2(r)-P_2(p)$, we have $P_2(s)=1-0.1-0.1=0.8$. The Mixed Strategy Nash Equilibrium for RPS-2 is therefore achieved when $\sigma_1(s_i)=\sigma_2(s_i)=\{0.1r, 0.1p, 0.8s\}$.

The MSNE is once again somewhat counterintuitive at first sight, since the strategy which is projected to be played no less than 80% of time in equilibrium is once again not the one yielding the higher payoff, but the one which is able to defeat it. Observing the pattern, we can therefore arrive at a very interesting, and perhaps paradoxical result, namely that as a strategy becomes *prima facie* more attractive in an RPS game, it is rational for individuals to avoid using it. The relation between the payoff (not *expected payoff*) of a strategy and the probability assigned to its usage in symmetrical RPS games with a single strategy modified from the standard game is one of inverse proportionality. The MSNE is compared with the experimental results in Figure 5. Once again, we notice that there are a few mixed strategies which assign to *P(r)* the probability prescribed by the MSNE (although half of them still depart with more than 10% from it).

The predictions for the other two strategies however are decidedly inconsistent with individual behavior. Only 6 of the 22 players are in a 0.1 vicinity of the prescribed *P(p)* and the situation for *P(s)* is the worst of all with 21 out of 22 players underplaying this strategy. There are only 2 strategies out of 22 (0.09) which are in a 0.1 vicinity to the MSNE prediction for *s*, with players assigning probabilities as low as 0.2 (two cases) or even 0.12 (one case) to *s* when its prescribed probability according to the MSNE is 0.8.

5.3. MSNE predictive power in RPS-3

The third game played by the participants in the experiment, termed RPS-3 was an asymmetrical RPS game with the strategy *r* yielding a payoff of +2 only for Player 1 when winning and strategy *s* yielding a payoff of +2 only for Player 2 when winning. All the other payoffs are symmetrical between the players and equal to +1. The game is depicted in Figure 6.

We will once again proceed with finding the Nash Equilibrium, however, since the game is asymmetric from the perspective of payoffs, $\sigma_1(s_i) \neq \sigma_2(s_i)$, we will need to identify the solution for each of the players. I will start with player 2.

I begin once again with the standard system of equations:

$$u_1(r) = u(r,r)P_2(r) + u(r,p)P_2(p) + u(r,s)(1 - P_2(r) - P_2(p)) \quad (7.1)$$

$$u_1(p) = u(p,r)P_2(r) + u(p,p)P_2(p) + u(p,s)(1 - P_2(r) - P_2(p)) \quad (8.1)$$

$$u_1(s) = u(s,r)P_2(r) + u(s,p)P_2(p) + u(s,s)(1 - P_2(r) - P_2(p)) \quad (9.1)$$

Figure 6. *RPS-3 game*

p_1 \ p_2	Rock (*r*)	Paper (*p*)	Scissors (*s*)
Rock (*r*)	0, 0	–1, +1	+2, –2
Paper (*p*)	+1, –1	0, 0	–2, +2
Scissors (*s*)	–1, +1	+1, –1	0, 0

Source: Author.

From (7.1), (8.1) and (9.1) we further obtain:

$$u_1(r) = -P_2(p) + 2 - 2P_2(r) - 2P_2(p) = 2 - 2P_2(r) - 3P_2(p) \quad (7.2)$$

$$u_1(p) = P_2(r) - 2 + 2P_2(r) + 2P_2(p) = -2 + 3P_2(r) + 2P_2(p) \quad (8.2)$$

$$u_1(s) = -P_2(r) + P_2(p) \quad (9.2)$$

But by definition of MSNE we have $u_1(r) = u_1(p) = u_1(s)$, therefore we can replace (9.2) in (7.2) and (8.2) Thus we have:

$$2 - 2P_2(r) - 3P_2(p) = -P_2(r) + P_2(p) \Rightarrow 2 - 2P_2(r) - 3P_2(p) + P_2(r) - P_2(p) = 0 \quad (7.3)$$

$$-2 + 3P_2(r) + 2P_2(p) = -P_2(r) + P_2(p) \Rightarrow -2 + 3P_2(r) + 2P_2(p) + P_2(r) - P_2(p) = 0 \quad (8.3)$$

From (7.3) we then obtain $2 - P_2(r) - 4P_2(p) = 0 \Rightarrow P_2(r) = 2 - 4P_2(p)$. By substituting $P_2(r)$ in (8.3) we obtain $-2 + 4(2 - 4P_2(p)) + P_2(p) = 0 \Rightarrow -2 + 8 - 16P_2(p) + P_2(p) = 0$ which results in $6 - 15P_2(p) = 0 \Rightarrow P_2(p) = \frac{6}{15} = 0.4$. Thus, $P_2(r) = 2 - 4 * 0.4 = 0.4$. Finally, since $P_2(s) = 1 - P_2(r) - P_2(p) \Rightarrow P_2(s) = 1 - 0.4 - 0.4 = 0.2$.

Solving for player 1, we have the following system of utility payoffs:

$$u_2(r) = u(r,r)P_1(r) + u(r,p)P_1(p) + u(r,s)(1 - P_1(r) - P_1(p)) \tag{10.1}$$

$$u_2(p) = u(p,r)P_1(r) + u(p,p)P_1(p) + u(p,s)(1 - P_1(r) - P_1(p)) \tag{11.1}$$

$$u_2(s) = u(s,r)P_1(r) + u(s,p)P_1(p) + u(s,s)(1 - P_1(r) - P_1(p)) \tag{12.1}$$

From (10.1), (11.1) and (12.1) we further obtain:

$$u_2(r) = -P_2(p) + 1 - P_2(r) - P_2(p) = 1 - P_2(r) - 2P_2(p) \tag{10.2}$$

$$u_2(p) = P_2(r) - 1 + P_2(r) + P_2(p) = -1 + 2P_2(r) + P_2(p) \tag{11.2}$$

$$u_2(s) = -2P_2(r) + 2P_2(p) \tag{12.2}$$

But by definition of MSNE we have $u_2(r) = u_2(p) = u_2(s)$, therefore we can replace (12.2) in (10.2) and (11.2). We then have:

$$-1 + 2P_2(r) + P_2(p) = -2P_2(r) + 2P_2(p) \Rightarrow 1 - P_2(r) - 2P_2(p) + 2P_2(r) - 2P_2(p) = 0 \tag{10.3}$$

$$-1 + 2P_2(r) + P_2(p) = -2P_2(r) + 2P_2(p) \Rightarrow -1 + 2P_2(r) + P_2(p) + 2P_2(r) - 2P_2(p) = 0 \tag{11.3}$$

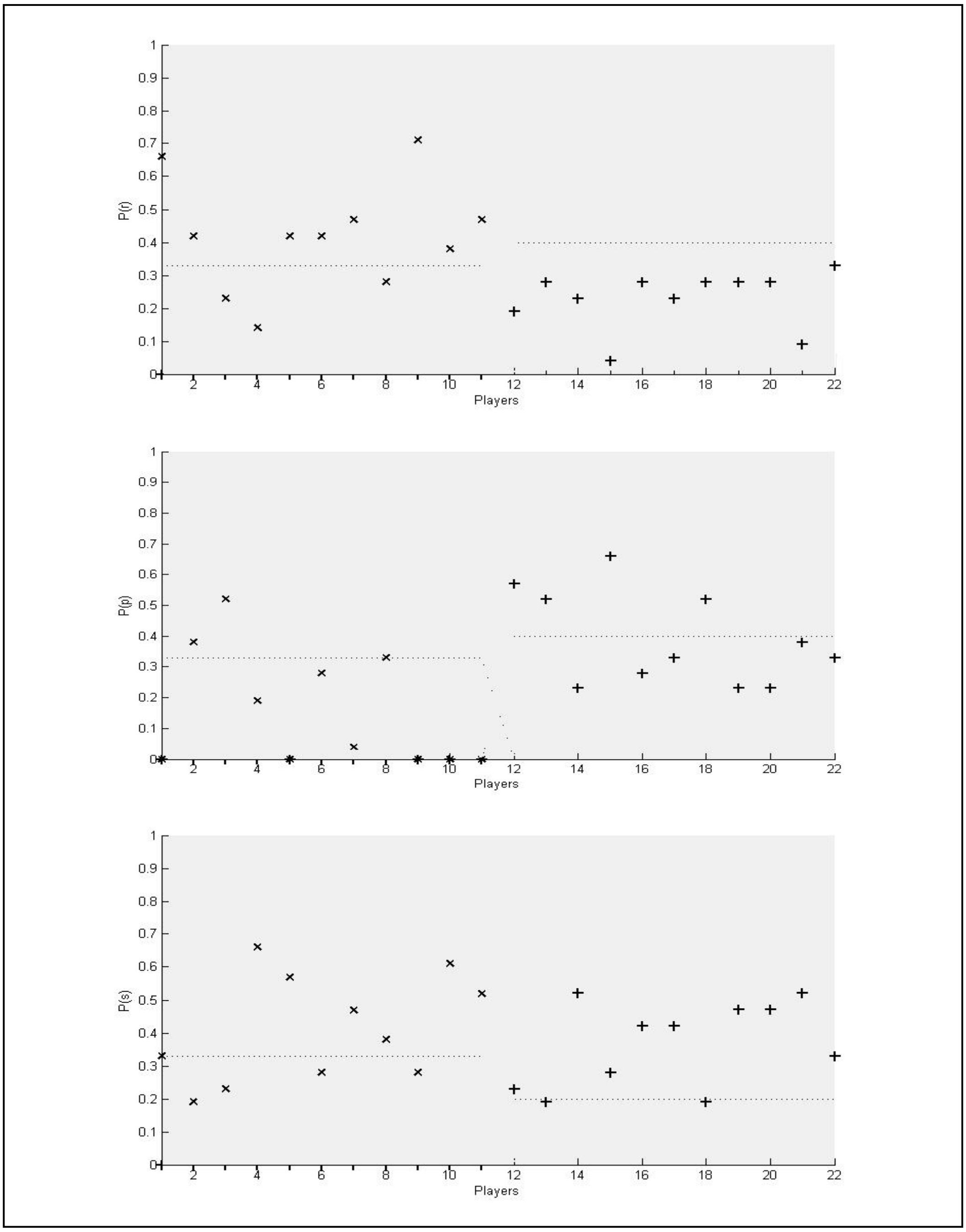
Thus we have $1 + P_2(r) - 4P_2(p) = 0$ (10.4) and $-1 + 4P_2(r) - P_2(p) = 0$ (11.4). We can then calculate that $P_2(r) = 4P_2(p) - 1$ and substitute this into (11.4). We have

$$-1 + 4(4P_2(p) - 1) - P_2(p) = 0 \Rightarrow -1 + 16P_2(p) - 4 - P_2(p) = 0 \Rightarrow$$

$$\Rightarrow -5 + 15P_2(p) = 0 \Rightarrow P_2(p) = \frac{5}{15} = 0.33.$$

Since $P_2(r) = 4P_2(p) - 1 \Rightarrow P_2(r) = 4 \cdot \frac{1}{3} - 1 = \frac{1}{3} = 0.33$ and $P_2(s) = 1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$.

Figure 7. *MSNE and experimental results in RPS-3*



Source: Author.

The MSNE of RPS-3 is therefore the mixed strategy profile where $\sigma_1(s_i) = \{0.33r, 0.33p, 0.33s\}$ and $\sigma_2(s_i) = \{0.4r, 0.4p, 0.2s\}$. Both equilibria are interesting to analyze. In the case of player 2, the strategy which yields the highest payoff is now the strategy which should be played less than each of the other two strategies. In the case of the first player, although he also has a unique strategy which yields a higher payoff than the other two when winning, the MSNE prescribes that he should play all pure strategies with equal probability. There is an intuition underlying the MSNE strategies which is exceptionally well captured by the mathematical apparatus. Although at first instance it may appear that the game is symmetrical in the sense that although the players do not have the same payoff matrix, exactly one strategy yields a higher (and identical between players) payoff for each of them, the fact that player 1 has *r* as a more attractive strategy and player 2 has *s* in the same position changes the game dynamic since the strategy which benefits player 1 the most is also the one winning against the strategy which benefits player 2 the most, but the inverse relation is *not* true. Therefore, in anticipation of player 1 choosing *r* with a high frequency, the optimal mixed strategy for player 2 is to play *s* with a lower frequency than the other alternatives.

The MSNE is compared with the experimental results in Figure 7. Each plot is divided into two halves, with the first half being represented by participants who were Player 1 during the experiment (and the corresponding Nash equilibria for Player 1) and with the second half being represented by participants who were Player 2 (and the corresponding Nash equilibria for Player 2). In this case the $\times's$ represent Player 1 and the $+'s$ Player 2. The results of RPS-3 are consistent with the results from the previous two games and support the idea that the MSNE is not an adequate predictor of individual choices in repeated games of RPS.

In the case of Player 1's no less than 5 out of the 11 (almost half) played only a 2-strategy mix, avoiding to play *P(p)*, although any strategy which does not mixes all 3 strategies is clearly exploitable by an opponent who notices this fact. Further, while in the case of Player 1, one of them plays MSNE within a 0.05 vicinity of each probability prescription, in the case of Player 2, none of the 11 participants manage to do this.

5.4. Section conclusions

In this section I questioned if the strategies adopted by individuals in RPS-type of games is consistent with the Mixed Strategy Nash Equilibrium. The evidence found from the experimental study clearly show that individuals do not play RPS in accordance with the MSNE (see Table 2 below). The claim is relevant for both scientific realist and instrumentalist perspectives. Regarding the first one I would bring the following argument: although most participants studied Political Science and more than half of the participants who filled out the post-experiment questionnaire mentioned that they had prior knowledge regarding the concept of Nash Equilibrium, when asked if they had attempted to play in accordance with the MSNE for any of the games none of them replied affirmatively. Although the sample size was relatively small and the results should be treated with caution, I have not managed to find any evidence supporting the idea that individuals intentionally seek to play in accordance with the MSNE solution in RPS games. Regarding the instrumentalist perspective I have found no evidence that individuals play the MSNE "as if" they had calculated it, with 0 out of 66 strategies being the ones prescribed by the solution concept.

Table 2. *Percentage of mixed strategies played as prescribed by the MSNE*

	MSNE	MSNE (~0.05)
RPS-1	0	4%
RPS-2	0	0
RPS-3 - Player 1	0	9%
RPS-3 - Player 2	0	0
Total	0	3%

*RPS-1, RPS-2: 22 strategies; RPS-3 – Player 1, RPS-3 – Player 2: 11 strategies; Total: 66 strategies.

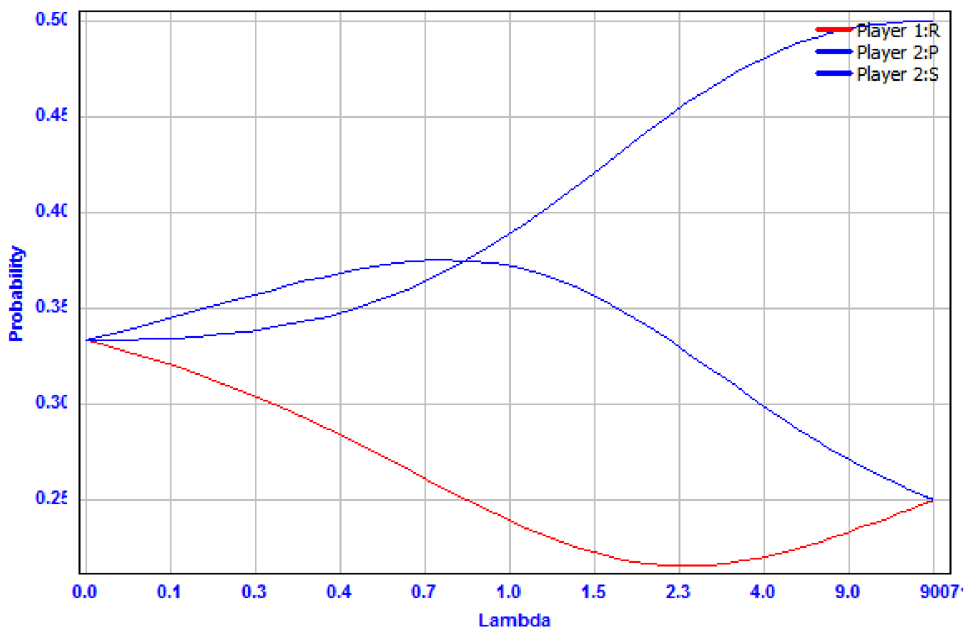
Source: Author.

Even if in order to correct for the small number of rounds played between the subjects we allow a perturbance around the probability percentages for each strategy (caused by an abrupt ending of the game) we still only find that 3% of strategies were consistent with the MSNE within 0.05 vicinity. These results are however clearly insufficient from my point of view to support the claim that individuals act either intentionally or "as if" with the purpose of reaching an MSNE.

6. Quantal Response Equilibrium performance in predicting strategy choices

6.1. QRE predictive power in RPS-1

Figure 8. *Quantal Response Functions in RPS-1*[31]



Source: Author.

Since the QRE can take an entire array of values due to the fluctuation of its λ error parameter as opposed to the Mixed Strategy Nash Equilibrium which predicts a single value for each probability, the QRE will naturally perform at least as well as the MSNE in predicting individual strategies.

However, it is not my only intention to see if the QRE significantly ouperforms the MSNE solution, but also to see if the QRE solution is able to adequately capture individual decision-making in the RPS games on its own. Figure 8 plots the quantal response functions according to the error parameter λ. As I mentioned in the previous section, as λ increases from 0 to $+\infty$, i.e. the probability of making an error is decreasing, the Quantal response function moves from random probability assignments to probability assignments which converge upon the prescriptions of the MSNE. Since the game is symmetrical, both Player 1 and Player 2 have the same quantal response functions. In Figure 8, the red convex function represents *P(r)*, the blue concave function converging to 0.25 represents *P(p)* and the blue concave function converging toward 0.5 represents *P(s)*.

Table 3. *Percentage of strategies included in the set of possible QRE for RPS-1*

	Minimum	Maximum	% of pure strategies included (in only 1 set)	% of mixed strategies included (in all 3 sets)
P(r)	0.21	0.33	54%	31.81%
P(p)	0.25	0.37	45%	
P(s)	0.33	0.5	50%	

Source: Author.

By examining Table 3 we immediately notice that only about half of the probabilities assigned by participants to each strategy in RPS-1 would be included in the set of possible QRE. Still, considering the fact that the set only expands for 13% (for $P(r)$ and $P(p)$) and 17% (for $P(s)$) of the entire probability domain, it would be reasonable to conclude that the quantal response functions have indeed some explanatory power regarding the strategies of individuals, especially since almost a third of the mixed strategies are fully included in the sets. However, in order to conclude that the QRE itself has any explanatory power, we need to see not only if the pure strategies separately belong to the QRE set, but if all three strategies correspond to the same level of λ since in a normal interpretation of the parameter it must be considered as a trait of the individual not the strategy. The λ parameter therefore must be fixed for every mixed strategies. But is this the case here? Table 4 shows the λ parameter estimates for the probability of playing each pure strategy for the 7 mixed strategies which are in the QRE set, with λ being estimated for both the actual value and a vicinity of 0.05 which tries to correct, as in the previous section, for any negative side-effects of the short number of rounds, accepting a +/- 1 error for any pure strategy.

Following this procedure we notice that for 6 of the 7 mixed strategies it is possible for the λ to be unique while for the 6^{th} strategy this is impossible since the three sets $\lambda P(r)$~0.05 and $\lambda P(p)$~0.05 do not have any common elements.

I will subseqently call all strategies within the QRE set which cannot possibly have a unique error parameter $\lambda - unfeasible$ and all strategies within the QRE set for which we cannot state with certainty that there isn't a unique λ for the mixed strategy, $\lambda - feasible$.

Table 4. *λ estimates for mixed strategies within the QRE set for RPS-1*

Nr.	$P(r)$	$\lambda P(r)$ ~0.05	$P(p)$	$\lambda P(p)$ ~0.05	$P(s)$	$\lambda P(s)$ ~0.05	$\bigcup$
1	0.29	0-0.71	0.33	0-4.72	0.37	0-1.35	0-0.71
2	0.25	0.28-3.81	0.33	0-4.72	0.41	0.81-2.45	0.81-2.45
3	0.25	0.28-3.81	0.37	0-2.18	0.37	0-1.35	0.28-1.35
4	0.29	0-0.71	0.29	0-0.05; 2.18- $+\infty$	0.37	0-1.35	0-0.05
5	0.25	0.28-3.81	0.37	0-2.18	0.37	0-1.35	0.28-1.35
6	0.29	0-0.71	0.25	4.24- $+\infty$	0.45	1.35- $+\infty$	$\varnothing$
7	0.29	0-0.71	0.33	0-4.72	0.37	0-1.35	0-0.71

Source: Author.

Proposition 1: *In any normal-form game, a mixed strategy* $\sigma_i(s_i)$ *can only determine a Quantal Response Equilibrium if it is* $\lambda-$ *feasible*.

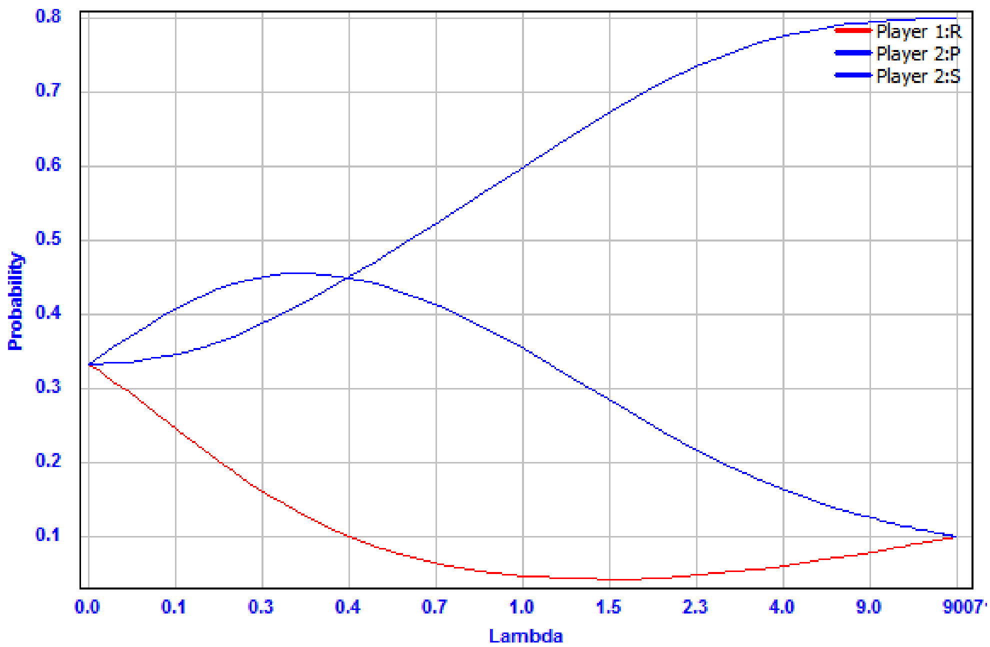
Corollary 1: *In any normal-form game, a random strategy is always* $\lambda-$ *feasible*.

Corollary 1 naturally results from Proposition 1 since the random strategy, i.e. the mixed strategy which plays every pure strategy with an equal probability has a unique λ, as $\lambda = 0$.

In RPS-1 there are 6 strategies which are in the QRE set and are $\lambda-$ *feasible*, a percentage of **27.27%** of total strategies.

6.2. QRE predictive power in RPS-2

Figure 9. *Quantal Response Functions in RPS-2*



Source: Author.

Figure 9 plots the quantal response functions for RPS-2. Like in the previous case, due to symmetry there are only 3 functions described. The convex function drawn in red corresponds to the *P(r)* strategy, the concave function which converges to 0.1 corresponds to the *P(p)* strategy and the blue concave function which converges to 0.8 represents the *P(s)* strategy.

Table 5. *Percentage of strategies included in the set of possible QRE for RPS-2*

	Minimum	Maximum	% of pure strategies included (in only 1 set)	% of mixed strategies included (in all 3 sets)
P(r)	0.04	0.33	72%	31.81%
P(p)	0.1	0.45	68%	
P(s)	0.33	0.8	54%	

Source: Author.

Although the percentage of pure strategies included within the QRE set for RPS-2 is somewhat greater than that for RPS-1, especially since in this case the Quantal response functions encompass a wider area of the co-domain (29% as opposed to 13%, 44% as opposed to 13% and 47% as opposed to 17%), the number of mixed strategies included in the QRE set is identical, namely 7 strategies or 31.81% of the total.

Just as in the case of RPS-1 we proceed to eliminate any possible mixed strategies which are $\lambda - unfeasible$.

Table 6. λ *estimates for mixed strategies within the QRE set for RPS-2*

Nr.	$P(r)$	$\lambda P(r)$ ~0.05	$P(p)$	$\lambda P(p)$ ~0.05	$P(s)$	$\lambda P(s)$ ~0.05	$\bigcup$
1	0.2	0.09-0.22	0.45	0.13-0.67	0.33	0-0.22	0.13-0.22
2	0.33	0-0.04	0.16	2.86-10.6	0.5	0.43-0.67	$\varnothing$
3	0.04	0.49-21	0.45	0.13-0.67	0.5	0.43-0.67	0.49-0.67
4	0.12	0.25-0.49; 9- $+\infty$	0.25	1.57-2.38	0.62	1-1.25	$\varnothing$
5	0.25	0.04-0.17	0.29	1.25-1.57	0.45	0.32-0.57	$\varnothing$
6	0.16	0.17-0.32	0.33	0-0.05; 1-1.25	0.5	0.43-0.67	$\varnothing$
7	0.33	0-0.4	0.16	2.86-10.6	0.5	0.43-0.67	$\varnothing$

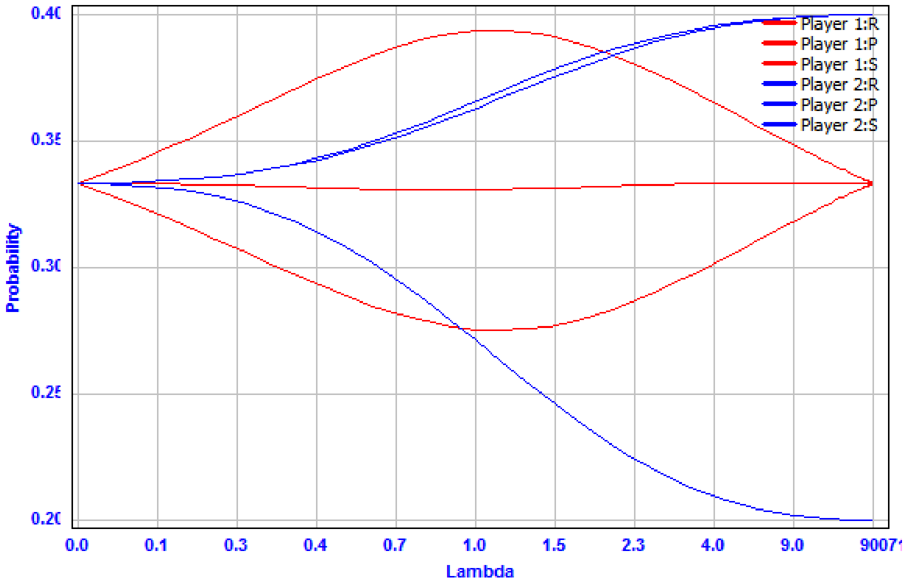
Source: Author.

By looking at the last column of Table 6 we can notice that although there are 7 mixed strategies in the QRE set, five of them could never have a unique error parameter and are eliminate as being $\lambda - unfeasible$. From the total set of 22 strategies analyzed, we can therefore conclude that only 2, a mere **9.09%** are in the QRE set and are $\lambda - feasible$.

6.3. QRE predictive power in RPS-3

Figure 10 plots the quantal response functions for RPS-3. Due to the fact that the game is asymmetrical, we have all 6 quantal response functions plotted for this game. The red functions correspond to Player 1 with the concave function representing *P(r)*, the convex function *P(p)* and the function which is almost horizontal function *P(s)*. In regard to Player 2, the two functions which converge towards 0.4 are the *P(r)* and *P(p)* strategies, with the latter placed slightly above and the downward sloping function representing *P(s)*.

Figure 10. *Quantal Response Functions in RPS-3*



Source: Author.

Table 7 describes the percentage of strategies included in the set of possible QRE. As seen from Figure 10, the functions spread out over a very small area of the co-domain, unlike the graphs in RPS-1 and especially RPS-2. This is problematic for the QRE solution because it severely limits the possibility that even pure strategies can be a part of the QRE set.

For Player 1 the functions are particularly restrictive since the fact that strategy *s* has a minimum and maximum likelihood to be played of exactly 0.33 means that the requirements for the existence of a QRE are resembling the requirements imposed by the Nash Equilibrium.

Table 7. *Percentage of strategies included in the set of possible QRE for RPS-3*

	Minimum	Maximum	% of pure strategies included (in only 1 set)	% of mixed strategies included (in all 3 sets)
P_1(r)	0.33	0.39	9%	4.54%
P_1(p)	0.27	0.33	18%	
P_1(s)	0.33	0.33	9%	
P_2(r)	0.33	0.4	9%	
P_2(p)	0.33	0.4	27%	
P_2(s)	0.2	0.33	27%	

Source: Author.

As we can see from Table 7 there is only one single mixed strategy which is a part of all three QRE sets, i.e. the mixed strategy in which all pure strategies are played with the same probability. Following corollary 1, we note that the strategy is $\lambda - feasible$ since the choice is completely random $\lambda = 0$ and therefore the union of the subsets cannot be void. To maintain uniformity I still detail the subsets in Table 8 however.

Table 8. *λ estimates for mixed strategies within the QRE set for RPS-2*

Nr.	$P(r)$	$\lambda P(r)$ ~0.05	$P(p)$	$\lambda P(p)$ ~0.05	$P(s)$	$\lambda P(s)$ ~0.05	$\bigcup$
1	0.33	0-0.31	0.33	0-0.31	0.33	0-0.16	0-0.16

Source: Author.

The consistency between individual choices and the QRE is even lower in this game than in RPS-2. In fact, in RPS-3, the Quantal Response Equilibrium manages to explain a single strategy, which incidentally is also the strategy of randomization.

6.4. Section conclusions

In spite of its critical acclaim which began ever since it was developed by McKelvey and Palfrey in (1995) and continues up until the present time, the Quantal Response Equilibrium does not appear to be a useful solution for 3×3 games in general, or at least for variations of the RPS game. Although registering some very mild success in explaining the behavior of participants in RPS-1, where it correctly framed 27.27% of the mixed strategies used by participants, in RPS-2 it only managed to explain 9.09% of the mixed strategies used and in RPS-3 only 4.54%, which is equivalent to the Nash Equilibrium prediction.

The experiment has shown that the QRE faces at least two major problems when increasing the number of players from two to three, problems which will only amplify as the number of players is further increased. The first problem regards what I have termed the issue of $\lambda - feasibility$, namely the fact that the QRE is predicated on the fact that there exists a unique λ per player per round. Although the parameter can change over time as the game progresses and different players can have different $\lambda's$ in the same round, it is impossible for the same individual to have two different error parameters during the same round. The problem is amplified with the increase in the number of strategies since in order for a mixed strategy to determine a QRE, all pure strategies played must be in the QRE set and the mixed strategy itself must be $\lambda - feasible$, i.e. to have a non-empty subset of elements after the union of all sets of λ estimates. This first problem was visible in particular in RPS-2 where 5 out of the 7 mixed strategies within the QRE set were eliminated for being $\lambda - unfeasible$. The second problem regards the possibility that the quantal response functions cover only a narrow area of the co-domain, a possibility which, *ceteris paribus*, increases with the addition of new strategies. An example in this respect is the problem with the narrowness of the functions in RPS-3, especially the function for *P(s)* of Player 1 which was identical to the Nash Equilibrium prescription.

7. Conclusions and limitations

In this paper I have comparatively tested the classical solution concept within game theory, i.e. the Mixed Strategy Nash Equilibrium, with one of its main contemporary challengers, namely the Quantal Response Equilibrium, seeking to find out if the latter significantly outperforms the former in experimental cases. In order to achieve this objective I conducted an experiment on 22 subjects (all of them students belonging to various universities in Bucharest) by using 3 modified versions of the RPS game, with 759 rounds of the game played in total.

The results of the experiments show that the QRE does not perform substantially better than the Nash Equilibrium for two of the three games studied and that it does represent a slight improvement in one of the games, where it manages to accurately predict the behavior of players in 27% of cases as opposed to the MSNE which only manages to do it in 4% of the cases. This result lies in stark contrast to the bulk of the literature developed thusfar on QRE which consistently promotes the idea that the QRE is not only a superior predictor of individual behavior than the MSNE (which is almost guaranteed by definition) but that it significantly outperforms the MSNE in every case. The results might also point to a decrease in the success of the QRE solution as the payoff structure becomes more complex, since it yielded the worst predictions in RPS-3.

Aside from the testing of this hypothesis, alongside the potentially relevant result which I have obtained in respect to the empirical difficulties of the QRE in games with more than two strategies, another important contribution could be the introduction of the QRE solution to the literature on political science and economy in Romania, since, to my knowledge, no article has been published in Romania thusfar on this subject.

However, the reader should also be aware of the limitations and drawbacks of this paper. First of all, the experiment was somewhat underdeveloped from both a human resources perspective (with only 22 participants) and from the perspective of the incentive scheme (with only 20 lei as a maximum reward). The number of turns was not communicated to the participants beforehand in order to avoid potential end-game effects, which, might be raise other types of problems since even if players were trying to play the Mixed Strategy Nash Equilibrium, it could have been possible for them to be desynchronized in mixing strategies when the game ended abruptly[32].

In spite of potential shortcomings, however, the experimental results obtained are relevant for the game theoretical literature, since the weak performance of the QRE solution depicted in this paper may be a signal to reconsider the displacement of traditional solution concepts from game theory with innovative, but less falsifiable, solution concepts imported from other scientific fields, which in many circumstances fail to significantly add to our understanding of human behavior.

Notes

(1) I would like to thank Adrian Miroiu for comments on a previous draft of this paper and for his support in the implementation phase of the experiment. I am also grateful to Andra Roescu and Mihai Ungureanu for discussions on some of the aspects of the paper, in particular for their inputs concerning the experimental design. The standard caveat applies.

(2) Alexandru Volacu is a beneficiary of the Sectoral Operational Programme for Human Resources Development 2007-2013, co-financed by the European Social Fund, under the project number POSDRU/159/1.5/S/134650 with the title "Doctoral and Postdoctoral Fellowships for young researchers in the fields of Political, Administrative and Communication Sciences and Sociology".

(3) For a brief introduction into the main concepts of game theory see Volacu (2014, pp.109-125) and for a more comprehensive one see McCarty and Meirowitz (2007).

(4) Henceforth QRE.

(5) These details are broadly explained in section 2.

(6) Henceforth MSNE.

(7) As in the case of the Stag Hunt for instance (see for instance Skyrms and Irvine: 2001).

(8) I use McCarty and Meirowitz's (2007) notation system.

(9) While it is not the case that every finite game has a Nash Equilibrium, every finite game *does* have a Mixed Strategy Nash Equilibrium

(10) McKelvey and Palfrey state that the name "is borrowed from the statistical literature on quantal choice/response models in which individual choices or responses are rational, but are based on latent variables [...] that are not observed by the econometrician" (McKelvey, Palfrey: 1995, p.7). The concept of quantal (or discrete) choice is originally introduced by McFadden (1976) to contrast with the typical quantitative choice modelling of standard economic models.

(11) Although three games were played using 6 rounds instead of 4.

(12) The MSNE prediction for the choice of s_1 by player 2 fared no better, with s_1 being chosen at a level of over 0.3 for both games although the MSNE predicted it to be played with probability of 0.1 for the second game and 0.2 for the third one.

(13) And for extensive-form games by McKelvey and Palfrey (1998).

(14) For a formal characterization see McKelvey and Palfrey (1995, pp. 8-10).

(15) Simon (1947 [1976]) introduces the concept of *bounded rationality*, which places limits on both an individual's "ability to perform" and on his "ability to make correct decisions" (Simon: 1947 [1976], p.39). His main project is to "replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist" (Simon, 1955, p. 99).

(16) To the extent that it is sometimes "referred to as a boundedly rational version of Nash Equilibrium" (Palfrey, 2007, p. 425).

(17) I use Goeree et al. (2005) notation.

(18) Existence of a QRE in any game is proven by McKelvey and Palfrey (1995, p. 10).

(19) Henceforth RPS.

(20) The widespread familiarity with the game was also confirmed by the experiment presented in this paper, where 100% of the participants stated that they knew how the standard game is played before receiving the explanations for the particular games of the experiment.

(21) Although the game is not very widely discussed in game theory, the history of its study is quite long, as it appears in von Neumann and Morgenstern's (1944) paradigmatic work and is commonly used nowadays in introductory game theoretical courses on mixed strategy equilibria.

(22) To my knowledge, Shapley (1963) proves the cycling result for 3x3 games such as RPS for the first time. He states that "rather than converging to the unique equilibrium point (at which all probabilities are equal), the sequence of mixed-strategy pairs generated by the algorithm oscillates around it, keeping a finite distance away" (Shapley: 1963, p.2). On the problem of ciclicity in RPS games see also Xu et al. (2013), who experimentally show that the cycling result observed via evolutionary game theory (and specifically using the replicator dynamics equation) is empirically superior to the MSNE of classical game theory which predicts random behavior for the classical RPS.

(23) And it can be applied analogously.

(24) Anecdotal evidence suggest that the proportion of tied rounds in standard RPS games between players who were previously acquainted is higher than normal. This is not confirmed for the one pair in which due to logistical issues the players knew each other before the game as the percentage of rounds which ended in ties during their three games was only 34% in comparison with the 37% average for the other 10 pairs. Thus, it cannot be stated that the prior acquaintance between players, *in this particular case*, had any negative side-effects of the type mentioned above.

(25) By convention, it was decided that the winner of the round would become Player 1.

(26) The standard conventions applied: a closed fist for *rock*, an open palm for *paper* and two pointing fingers for *scissors*.

(27) It was my initial intention not to have players physically play the game but only to chart their options down on a sheet of paper during each round in order to avoid. Mihai Ungureanu however pointed an interesting issue to me regarding the possibility that the actions of the players may be influenced by social factors such as the direct interaction between them and taking out these factors might distort the experimental results in some way. Since the sample size of the experiment was decidedly low, it was not possible to test for differences between the two types of mechanisms, therefore this issue remains opened for future research.

(28) Which was available to them at all stages, as the choices on each rounds were written on their Experiment forms.

(29) By providing the players with a detailed history of the game at each of its rounds we cancelled any possible noise attributed to mistakes in estimating the frequency of each strategy throughout the game history.

(30) I thank Andra Roescu for her useful suggestion in this respect.

(31) In order to construct Figures 8-10 I used the Gambit software (McKelvey et al., 2014).
(32) However, since the experimental form contained no more than 26 rows and the number of turns was 24, 24 and 21, I do not consider that this had a significant effect on the results obtained.

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