

Welfare costs due to economic fluctuations according to different model setups

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Abstract. *The topic related to the costs in terms of welfare generated by economic fluctuations was highly debated in the macroeconomic research. Lucas opened the debate in this regard in a seminal paper in 1987 by asking what will be the gain in terms of consumption units by eliminating the economic fluctuations. Some weak assumptions raised several criticisms on the Lucas (1987) work. Reis (2009) proposed a robust approach by assuming the data generating process of consumption is defined through a persistent auto-regressive approach without drift. The current paper aims at calculating the welfare cost generating by fluctuations in Romanian economy. As compared with Alupoaiei (2013), here was investigated the relationship between welfare costs and different scenarios regarding the level of consumption volatility and the risk aversion parameter. In that sense were constructed surfaces to emphasize these connections.*

Keywords: Auto-regressive process, consumption, costs, risk aversion, random walk, volatility, welfare.

JEL Classification: E32, F41.

1. Introduction

Given some previous attempts to assess the welfare costs associated with the reduction of inflation, Lucas (1987) proposed in a seminal paper an innovative approach designed to measure the welfare gain from eliminating economic fluctuations. In fact, Lucas (1987) addressed the standard problem faced by a representative agent: how to smooth the consumption across different periods as to maximize the utility, given some exogenous forces that generate fluctuations. The problem of Lucas (1987) is formulated in the sense of finding some appropriate stabilization policies. On the other hand, Cochrane (1989) investigated if the agents tend to avoid the pure optimal policy in consumption owing to the small costs associated with the related deviation. More exactly, Cochrane (1989) showed that if the agents are near-rational instead of pure rational, the cost stemming from this deviation are very small. Therefore, many important economists were concerned on the link among the decisions in consumption, the related welfare and the stochastic environment. Apart from the general objective, it's important to ensure robustness to final conclusions. For example, apart from the innovative idea, the Lucas (1987) work was criticized for the weak assumptions that conducted to very low welfare costs determined by the normal economic fluctuations. In that sense, Reis (2009) proposed an approach to overcome the limits of the models developed in this field. One of the main contributions of the Reis (2009) work was to consider a persistent stochastic process as data generating process for consumption dynamic. He showed that a higher persistence of consumption innovations will lead to higher welfare costs.

The current paper comes to assess the welfare costs due to fluctuations in Romanian economy by using the approach of Reis (2009). On the other hand, the present paper intends to complete the work of Alupoai (2013) who addressed the problem of welfare cost for Romanian economy. As compared with Alupoai (2013), the contribution of this paper is twofold. Increasing the sample size for consumption evolution allows for better estimates regarding the parameters of the assumed law of motion. This aspect is particularly important for the current work in terms of econometric inference, given the short available sample alongside with a structural break followed by only a few registrations. A second contribution of this paper consists in constructing two surfaces in order to allow for the variation of consumption volatility and the risk aversion parameter. This feature is fairly important, given the uncertainty related with the real level of these two parameters. More than that, the two surfaces allow for a deeper analysis of the welfare costs, highlighting interesting facts as it is the non-linearity.

2. Methodology

Lucas (2003) describes how to form the welfare measure designed to capture the gains caused by achieving a target. In the current context, the problem in hand is about consumption fluctuations and welfare measure is defined as:

$$E \left[\sum_{t=0}^{\infty} e^{-\beta t} u(C_t(1 + \lambda)) \right] = \sum_{t=0}^{\infty} e^{-\beta t} u(\bar{C}_t) \quad [1]$$

where E denotes the rational expectation operator, \bar{C}_t represents the level of consumption unaffected by fluctuations, $u(C_t)$ is the standard utility function, while β is the subjective discount factor. In fact, according to Lucas (2003), the welfare measure represents the solution to an optimisation problem that is designed to equalise the two utility functions. Therefore, λ could be viewed as being the loss measure generated by fluctuations in consumption flows. In order to solve the above optimisation problem, it is necessary to set up a stochastic process used to model $[\sum_{t=0}^{\infty} e^{-\beta t} u(C_t(1 + \lambda))]$, as well to find a counterfactual level of consumption \bar{C} . In respect with the two mentioned necessities, Reis (2009) adopted the following assumptions: *i)* the existence of a representative agent, *ii)* time separable and iso-elastic preferences and *iii)* persistent stochastic process for consumption evolution. While the first two hypothesis are the same as in Lucas (1987), the third one differs fundamentally. Thus, as compared with Lucas (1987) that assumed non-correlated innovations in consumption dynamic, Reis (2009) stressed the importance of very correlated shocks and even permanent, in order obtain more plausible figures for the loss measure. In that sense, Reis (2009) resorted two alternative approaches: the rational expectation permanent income model of Hall (1978) and a very persistent first-order autoregressive model.

Therefore Reis (2009) uses an economic model, respectively a statistical one to define the data generating process behind consumption dynamics. In this regard, Reis (2009) defines the counterfactual \bar{C} as being that level of consumption which corresponds to a constant rate of growth: $\bar{C}_t = C_0 e^{gt}$. Therefore, no fluctuation feature is exploited within the previous fashion. From a probabilistic view point, Reis (2009) called the log-normal feature for consumption distribution. Even this is a strong assumption, is supported by the empirical evidences. By exploiting the properties of the log-normal distribution, the following function form it is obtained for [1]:

$$\ln(1 + \lambda) + (1 - e^{-\beta})E \left[\sum_{t=0}^{\infty} e^{-\beta t} u(c_t) \right] = (1 - e^{-\beta})E \left[\sum_{t=0}^{\infty} e^{-\beta t} u(c_t) + \frac{\sigma^2}{2} \right] \quad [2]$$

The utility function it is assumed to be of CRRA type $\frac{(c_{t+i})^{1-\gamma}}{1-\gamma}$, which means the above relation it is obtained under $\gamma = 1$. By using the approximation $\ln(1 + \lambda) \cong \lambda$, from [2] it will result that depending on the values registered by risk aversion parameter γ , the loss measure is defined through the following two relations:

$$\lambda = \begin{cases} \frac{1}{2}(1 - e^{-\rho}) \sum_{t=0}^{\infty} e^{-\rho t} \text{Var}(c_t) & \text{dacă } \gamma = 1 \\ (\gamma - 1)^{-1} \ln \left[(1 - e^{-\rho}) \sum_{t=0}^{\infty} e^{-\rho t} e^{\frac{1}{2}\gamma(\gamma-1)\text{Var}(c_t)} \right] & \text{dacă } \gamma \neq 1 \end{cases} \quad [3]$$

In [3], the ρ is the effective discount factor defined by $(\beta + (\gamma - 1)g)$. Therefore, the degree of persistence in process' innovations are crucial for the amount of loss generated by fluctuations in consumption flows.

One of the main criticism related to Lucas (1987) paper is the assumed process for the data generating process is i.i.d. In fact, several authors consider the previous feature as being the main reason behind the low figures in term of welfare loss due to uncertainty that were reported by Lucas (1987). On the other hand, Hall (1978) assumed the underlying data generating process is described by permanent effects of innovations. It is important to note the work of Hall (1978) launched one of the most interesting and long-standing debates in the economy, by linking the consequences of rational expectations to the stochastic properties of aggregate consumption. For these considerations and for robustness reasons, Reis (2009) has chosen a comprehensive stochastic process for the data generating process:

$$\hat{c}_t = \eta \hat{c}_{t-1} + \varepsilon_t \quad [4]$$

According with the first order auto-regressive process [4], with $\varepsilon \sim N(0, \sigma)$, if $\eta = 0$, then log of consumption evolve as in Lucas (1987). Therefore, for $\eta = 0$ the welfare loss due to utility are reduces. But Reis (2009) showed instead that if $\eta = 1$, the size of welfare loss caused by fluctuations could be even ten times higher as compared with the case when $\eta = 0$. Another important remark mad by Reis (2009) is related to the variance of [4]. Thus, if the log of consumption is define throughout [4], its conditional variance is defined by $Var(c_t) = \frac{\sigma^2(1-\eta^{2t})}{1-\eta^2}$. By calling the approximation $e^\rho - 1 \cong \rho$ and using a first order Taylor expansion around $\sigma^2 = 0$, if $\gamma = 1$, then welfare loss will be:

$$\lambda = \frac{\frac{1}{2}\sigma^2}{e^\rho - \eta^2} \quad [5]$$

By the contrary, when the risk aversion parameter is different from one ($\gamma \neq 1$), the related formula for the welfare costs is:

$$\lambda = \frac{1}{\gamma - 1} \ln \left[(1 - e^{-\rho}) \sum_{t=0}^{\infty} e^{-\rho t} e^{\frac{1}{2}\gamma(\gamma-1)\frac{\sigma^2(1-\eta^{2t})}{1-\eta^2}} \right] \quad [6]$$

Again, by resorting a first order Taylor expansion around $\sigma^2 = 0$, the formula for welfare costs defined in [6] will be reduced to the following expression:

$$\lambda \cong \frac{\frac{1}{2}\gamma(1-\eta^2)}{\rho + 1 - \eta^2} \left(\frac{\sigma^2}{1-\eta^2} \right) \quad [7]$$

From [7] results that welfare costs generated by fluctuations in aggregate consumption are increasing in γ , σ and η . In other words, the higher risk aversion, the higher costs faced by our representative agent owing to fluctuations in consumption and therefore more dislike for volatility. Therefore, the higher the volatility and persistence related to innovations that affect consumption, the agents want event more to eliminate the fluctuations. On the other hand, the subjective discount factors could represent a measure of uncertainty the agents assign it to future economic prospects. Referring to subjective discount factor, a higher β is inversely related to the level of assigned uncertainty to economic outlook.

Reis (2009) maintained the same form of the utility function used by Lucas in his work from 1987. Therefore, parameters γ and ρ should be interpreted in respect with the signification underlined by the utility function meaning, even there is the case of the statistical model, as well the economic model. Given that utility function used by Lucas (1987) and Reis (2009) is nothing more than the first order condition of a standard consumption problem, γ and ρ denote thus the so called taste parameters. In that sense, they describe the behaviour of a representative agent in regard with the shocks that affect him. In other train of thoughts, an important departure from the Lucas (1987) model is that Reis (2009) uses conditional moments in his approach and not unconditional ones. This feature it is emphasized by the derived formula for the costs λ . In fact, Lucas (1987) called the unconditional variance $\left(\frac{\sigma^2}{1-\eta^2}\right)$ to calculate the related λ for his approach. A important consequence is caused by the feature previously mentioned about Lucas (1987) model, namely when the persistence of the process increases, the costs due to fluctuations will decrease owing to the unconditional variance set-up. This phenomenon can be explained as: given a constancy in terms of innovations' volatility, the increase of agent's predictability will determine a smoothing in his consumption process. Obviously the set-up used by Reis will increase the costs associated with fluctuations in consumption as compared with the Lucas (1987) figures.

The second approach resorted by Reis (2009) contains more economic substance, by comparison with the first one. In that sense, the economic approach starts from the standard problem faced by a representative consumer:

$$\max_{\{C_t\}} E \left[\sum_{t=0}^{\infty} e^{-\beta t} u(C_t) \right] \quad [8]$$

$$K_{t+1} + C_t = R_t K_t \quad [9]$$

where K_t denotes the level of endowed capital, while R_t is the return expected from a standard investment. Reis (2009) assumed the capital R_t is log-normally distributed. From the familiar Euler equation for consumption flows during sequential periods, the following law of motion is obtained for consumption dynamics:

$$c_t = c_{t-1} + g - \frac{1}{2}\sigma^2 + \varepsilon_t \quad [10]$$

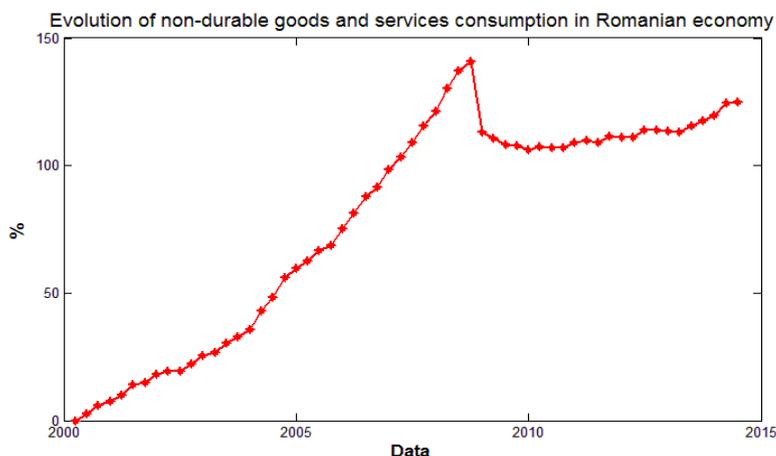
According to above formula, basically it is assumed the martingale hypothesis of Hall (1978) for consumption dynamics, while its growth rate evolves as: $r - \rho + 0.5\gamma(\gamma + 1)\sigma^2 - \gamma\sigma^2$. Proceeding in a similar way as in the previous case, namely by using the new law of motion for consumption across periods and plugging it in the formula for the associated cost, then it results the following relation:

$$\lambda \cong \frac{\frac{1}{2}\gamma\sigma^2}{\rho - \frac{1}{2}\gamma(\gamma - 1)\sigma^2} \quad [11]$$

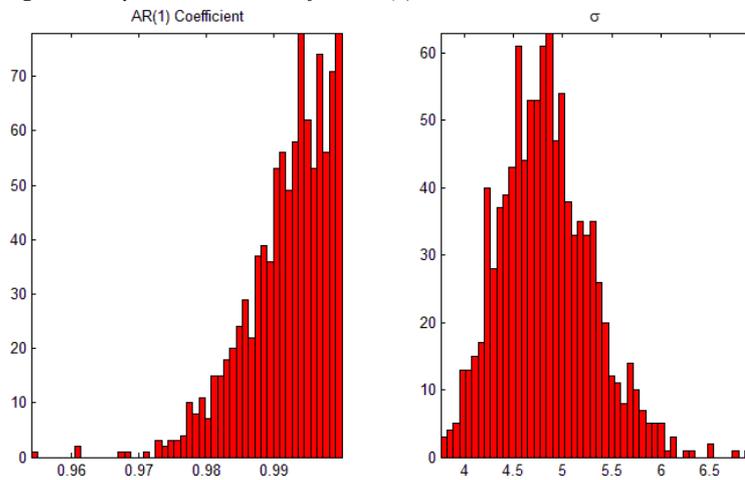
3. Data and results

To implement the approach of Reis (2009), the following specifications are necessary to be done. The data used for estimation and calibrations purposes consist in time series on Romanian household consumption level with quarterly frequency, spanning the period between 2000Q2 – 2014Q3. Use of quarterly data is augmented by lack of yearly statistics on consumption data that could result in biased estimates when standard inference techniques are used. The data denotes the expenditures related to non-durable goods and services expressed in national currency, being provided by EUROSTAT. In order to obtain per capita expressed data, the population was assumed to be constant within a year. By looking at historical evidence on the evolution of private consumption on non-durable goods and services in figure 1, the following mentions can be briefly done. From an economic view, it is easily to observe there are two distinct regimes alongside the recession. The first phase it is described by high growth rates of consumption level, while in the second one can be observed a calm evolution. This observation emphasizes an interesting stylized fact, namely that before the recession occurrence Romanian consumers were very enthusiastic. Instead, after the significant shrink of economic activity during the recession, the behavior of economic agents became a precautionary one. These facts also determine important econometric implications. More exactly, the evolution of consumption level until the occurrence of economic recession is likely to be the subject of a random walk behavior. The situation it is tremendous different after 2010 in terms of stochastic properties.

Figure 1. *Empirical evidence*



In order to calibrate the Reis (2009) model, the first step consisted in determining the parameters which describes the law of motion for the level of consumption. In that sense, here was used the Bayesian econometrics, given that underlining series could be the subject of a unit root. In such a case, standard econometric techniques are suspected to underestimate the parameters related to real data generating process. As in Reis (2009), consumption was modeled with an AR(1) stochastic process without drift. Estimation for the AR(1) model was ruled out with a Gibbs on the base of running 50,000 simulations.

Figure 2. Bayesian estimation of the AR(1) model

Estimates related to the assumed law of motion for C_t are reported in figure 2. As the histogram show, the stochastic process that governs dynamic of Romanian C_t behaves similarly as a random walk process, but still not. As Reis (2009) emphasizes, between the costs obtained with a random walk process, respectively with a persistent AR(1) process could exists important differences as magnitude. For this purpose and given the context described by no such long time series along with a sudden change in dynamic, in order to ensure robustness to final decision, here were constructed grids to calibrate each of the two models. The use of grids is motivated by varying the figures related to σ^2 , respectively to γ . The decision to vary these two parameters come from the uncertainty on the size of σ^2 of consumption and of the risk aversion parameter γ . In our case, the differences related to the level of σ^2 could stem on the first ground if it is used a conditional or unconditional set of information as well could depend on the frequency of underlying data. Not at least, different methods could conduct to different figures for σ^2 and γ . On the hand, η differs anyway between the statistical and economic. Of course grids for σ^2 and γ are constructed around most plausible levels recommended by the estimates as well other similar papers such is the work of Alupoaiei (2013). The other parameters are calibrated in a similar way.

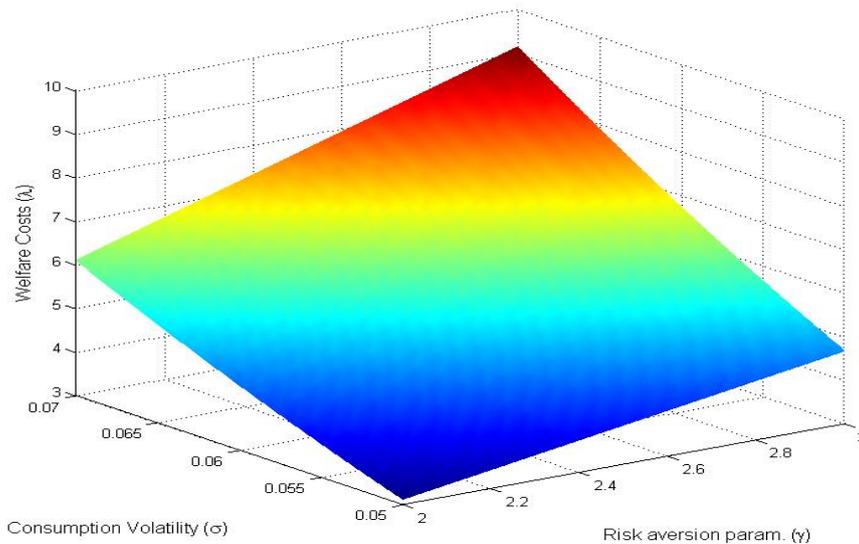
Table 1. Calibration of the Reis (2009) models

η	σ	γ	β
0.98/1	5 – 7 %	2 – 3	0.96

Bayesian estimate show a figure for η of around 0.98, while the standard inference methods provide a level of approximately 0.97. As it is well known the standard OLS method underestimate the auto-regressive coefficients in the case of unit roots and even local unit roots, here were preferred the figure obtain with the Bayesian approach. In the case of Reis' (2009) economic model, η is automatically set to 1. The rate of time preference is calibrated on the base of subjective discount factor β , according to rational expectation relation. Even in general ρ is calibrated from in such a way, there also exist and other alternatives as is in Reis (2009). β was calibrated according with Alupoaiei

(2013) on the base of an Euler equation approach as well on the estimation of a general equilibrium approach. In that manner, the level of β and implicitly ρ was directly linked with η . An important specification is that β was estimated by calling data with quarterly frequency and only then was calibrated at a yearly frequency. The chosen of a yearly frequency is mainly motivated by the sake of comparison. More exactly, is more intuitive to express the welfare costs at the level of a year in order to compare with different policies projected in the government's budget. In such a way, for example it is interesting to compare the asymptotic calibrated welfare costs with level of social benefits provided by the government for the poorer people to help them to smooth consumption. On the other hand, given the limits of using yearly data on Romanian consumption, the grid was constructed as σ vary between 5 – 7 %, even the most plausible figure is likely to be around 6.6 %. The risk aversion parameter was calibrated in the same manner as β . Given that between general and partial equilibrium could exists important differences between mechanics, the risk aversion parameter γ varies between 2 and 3. Even that, in my opinion proper values for γ should be between 2.5 and 3, but mainly closely to 3.

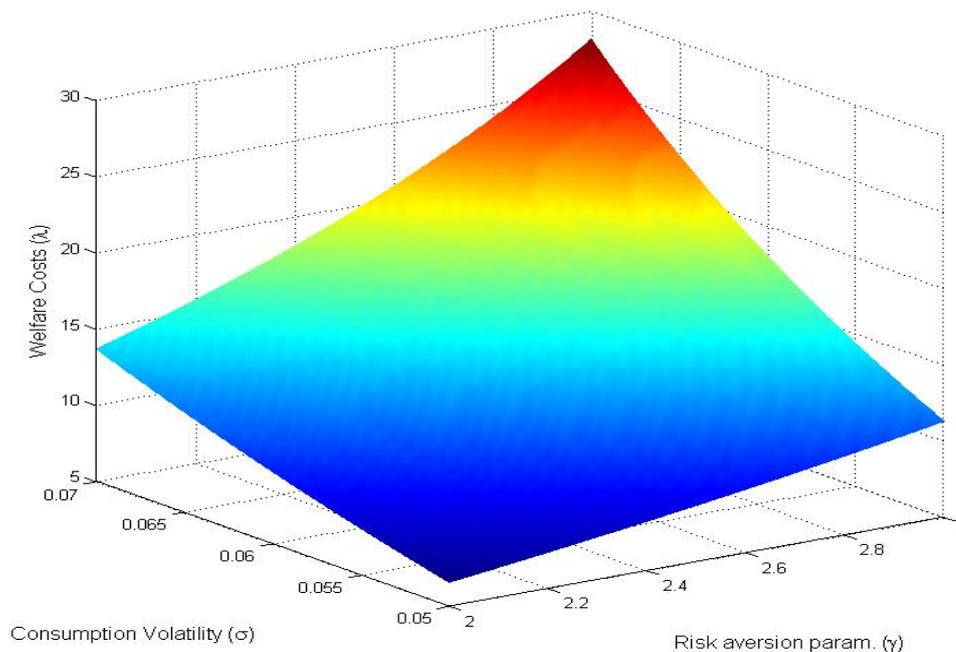
Figure 3. Surface on the calibrated statistical model



In figure 3 it is reported the calibrated surface related to Reis' (2009) statistical model. As we already know, the welfare costs increase with the degree of risk aversions, as well with size of consumption volatility. The calibration exercise shows that according with the statistical model, consumers from Romanian economy would be willing to give an amount of up to approximately 9 % of their annual consumption to remove the economic fluctuations in their consumption. In another train of thoughts, the amount of around 9 % of their annual consumption denotes the welfare costs generated by economic fluctuations. More exactly, from the calibration exercise results that maximum amount of welfare costs is 9.14 %, while the minimum level is 3.11 %. On average, Romanian

consumers are willing to give up a share of approximately 5.65 from their annual consumption.

Figure 4. *Surface on the calibrated economic model*



In figure 4 is reported the surface calibrated for the case of economic model, by using the same parameters. In this case, the auto-regressive coefficient is 1. By comparing the two figures it can be observed there is more non-linearity for the random walk economic model. On the other hand, the maximum welfare costs provided by the second approach are around 28.14% as compared with only 9.14% in the first case. Instead, the average welfare cost is only two times higher than in the first model, being approximately 13.73%. In the same time, the minimum welfare cost obtained with the random walk economic model is 6.52%. Therefore, comparing the basic stats between the two models we conclude the major difference between the two models come in a very uncertain context, described by high figures for the volatility of consumption, as well the degree of risk aversion.

4. Conclusion

Obtained results show that consumers from Romanian economy are willing to give up to 9.14% of their annual consumption according with the first approach, respectively up to approximately 28.14% as the second approach underlines. Given the second approach is based on the random walk economic model of Hall (1978) and the related martingale hypothesis it is difficult to be accepted for Romanian economy, as Alupoai (2013)

notes, the maximum welfare loss of 9.14% is more plausible. In fact, the plausibility of the first approach is motivated by the higher likelihood as the real data generating process to be a persistent auto-regressive process. The resulted figures are pretty higher than those ones reported in Reis (2009). This fact could be put on the back of higher persistence and fluctuation in the process innovations, as well on the higher uncertainty assigned to future economic outlook. Given that expenditures on Romanian government with social benefits were more than 20% of the aggregate consumption, the reduction of economic volatility could lead to a significant increase in the general welfare.

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