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A utility based theoretical model for the income-life expectancy curve

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Abstract. *The relationship between income and life expectancy is an established empirical fact, commonly represented by the Preston curve. Although this empirical relationship has been common knowledge for a very long time, there is no theoretical model that explicates the exact dynamic of it. The present paper fills this void by developing a utility based model which successfully estimates the Preston curve.*

Keywords: life expectancy, Preston curve, utility model, logistic function, simulation.

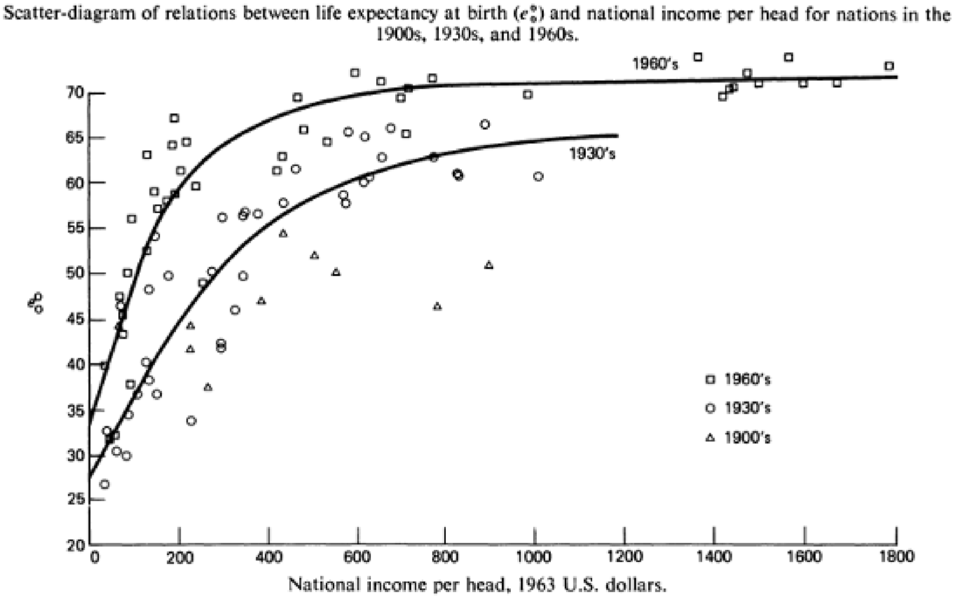
JEL Classification: I1, J1.

1. Introduction

In his influential paper, Preston (1975) derived an empirical income - life expectancy curve which is now known as the Preston curve. The literature is abundant with papers that study this empirical fact (e.g., Alho, 2006, pp. 41-51, Oeppen, 2006, pp. 55-82, Vallin and France, 2010, pp. 1-4). Nevertheless, it lacks a theoretical model that successfully explains the Preston curve. Without such a theoretical model, one cannot claim to fully explain the relationship between income and life expectancy. Moreover, a theoretical model would allow many interest groups (e.g., insurance companies, health ministries and social security institutions, etc.) to get accurate and convenient estimates of future life expectancy for particular countries. The present paper develops such a model.

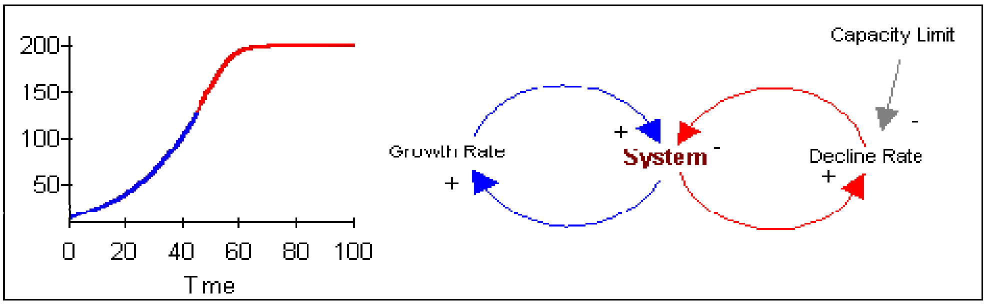
The Preston curve (Figure 1) matches life expectancies and per capita incomes of countries. Preston claimed that fundamental improvements in medical technology and health knowledge, which are the starters of new health stages, should be considered positive shocks that shift the life expectancy-income curve up. Hence, he derived two distinct curves representing different health stages (For health stages see the appendix). Each curve has an s-shape pattern and shows life expectancy as a function of per capita national income. It grows quickly at first and then decelerates its progression with income.

Figure 1. *The original Preston Curve (1975)*



S-shaped growth is the characteristic behavior of a system which consists of positive and negative feedback loops simultaneously (Kirkwood, 1998, p. 12). Movement is exponential at first while the positive feedback loop initially dominates the system, so the system variable increases at an increasing rate. However, the system approaches its limit or "carrying capacity" over time when the negative feedback loop becomes the dominant loop (Sterman, 2000, pp. 296-297). Logistic functions are chosen to model the S-shaped movements because the logistic law of growth assumes that systems grow exponentially under the constraints of an upper limit (longevity in our case), producing a typical S-shaped curve (Coelho, 2007, pp. 6-7). Preston showed that 1930s and 1960s data were fitted to logistic curves. However, he did not theoretically derive the implied logistic equations.

Figure 2. *S-shaped structure and characteristic time path*



Source: http://www.systemdynamics.org/DL-IntroSysDyn/sshp.htm

Preston argued that income is the best socio-economic indicator to explain life expectancy increases. The sketch of his argument is as follows: National income consists of all the goods and services produced in a country. These include consumption items affecting health positively, such as food, housing, medical and public health services, education, leisure, health technology and health related research. Thus, there is a positive relationship between increased per capita income and life expectancy at a decreasing rate (diminishing marginal utility of consumption). Researchers, including Preston, also acknowledge that increased income could negatively influence life expectancy through pollution, animal fats, artificial nutrition and physical inertia. Actually there is a readily available tool that can be used for theoretical modeling of this relationship: Environmental Kuznets Curve (EKC). The EKC establishes that an increasing income first leads to increasing levels of pollution, artificial nutrition and physical inertia etc. However, after a threshold level, environmental bads, which affect life expectancy negatively, start to diminish (Dinda, 2004, p. 432). Thus, there is also a negative relationship between increased per capita income and life expectancy that is characterized by an inverse u-shape. We develop a solid theoretical model of income-life expectancy relationship simply by utilizing the Environmental Kuznets Curve (EKC) and using a logistic differential equation.

The plan of the paper is as follows. The second section develops the theoretical model. The third section simulates the model and checks its estimates of life expectancy against real data.

2. Theoretical Model

According to the general form of the logistic differential equation Eq. (1), the growth rate of life expectancy can be assumed to be determined by the product of longevity $\bar{L}$,[(1)] and δ which we assume to be a time-variant parameter through income.[(2)]

$$\frac{dL}{dt} = \delta \bar{L}\, L \left(1 - \frac{L}{\bar{L}}\right) \qquad (1)$$

In the derivation, we use a utility based approach. We assume that the time – variant parameter δ depends on the utility level in the relevant period. Hence, we assume that life expectancy is affected by utility levels and, following Andreoni, Levinson (2001), we set the utility to be a function of consumption C and pollution (environmental bads) P in a specific time period.

$$\delta = U(C, P) \quad (2)$$

While higher consumption affects utility in a positive way, pollution influences it negatively.

$$U = U(C, P) = C - P \qquad \frac{\partial U}{\partial C} > 0 \quad \frac{\partial U}{\partial P} < 0 \quad (3)$$

Moreover, pollution is defined as a positive function of consumption and a negative function of environmental effort.

$$P = P(C, E) = C - C^{\alpha}E^{\beta} \quad \frac{\partial P}{\partial C} > 0 \quad \frac{\partial P}{\partial E} < 0 \quad (4)$$

Therefore, utility can be expressed as a Cobb – Douglas type quasi concave function.

$$U(C, P) = C^{\alpha}E^{\beta} \quad (5)$$

People use their income for two purposes: consumption C and environmental effort E, which have unit prices P_C and P_E respectively (all in nominal terms). P_C and P_E have initial values of P_{C0} and P_{E0} and growth rates π_C and π_E.

$$P_C C + P_E E = M \quad (6)$$

When we maximize utility, given the income constraint, it is easy to obtain the dependency of the optimum value of the utility function on income, prices and input elasticities[(3)] (Derivation of the optimized utility is given in the appendix).

$$U^*(\alpha, \beta, P_C, P_E, M) = \left(\frac{\alpha}{P_C}\right)^{\alpha}\left(\frac{\beta}{P_E}\right)^{\beta}\left(\frac{M}{\alpha+\beta}\right)^{\alpha+\beta} \quad (7)$$

For estimation purposes, we need to define δ as a function of initial values of income and prices and their growth rates where

$$M_t = M_0 e^{\rho t}$$

$$P_{Ct} = P_{C0} e^{\pi_C t}$$

and

$$P_{Et} = P_{E0} e^{\pi_E t} \quad (8)$$

$$\delta^* = \delta^*(\alpha, \beta, P_{C0}, P_{E0}, M_0, \pi_C, \pi_E, \rho, t) = \left(\frac{\alpha}{P_{C0}e^{\pi_C t}}\right)^{\alpha}\left(\frac{\beta}{P_{E0}e^{\pi_E t}}\right)^{\beta}\left(\frac{M_0 e^{\rho t}}{\alpha+\beta}\right)^{\alpha+\beta} \quad (9)$$

Having the dynamic property of the parameter δ, we let initial income, prices and their growth rates vary across countries and get the country specific equation:

$$\delta_t^i = \delta_t^i\left(\alpha, \beta, P_{C0}^{\,i}, P_{E0}^{\,i}, M_0^i, \pi_C^i, \pi_E^i, \rho^i, t\right) = \left(\frac{\alpha}{P_{C0}^{\,i}e^{\pi_C t}}\right)^{\alpha}\left(\frac{\beta}{P_{E0}^{\,i}e^{\pi_E t}}\right)^{\beta}\left(\frac{M_0^i e^{\rho t}}{\alpha+\beta}\right)^{\alpha+\beta} \quad (10)$$

That is the change in the life expectancy of a country with respect to time can be written as:

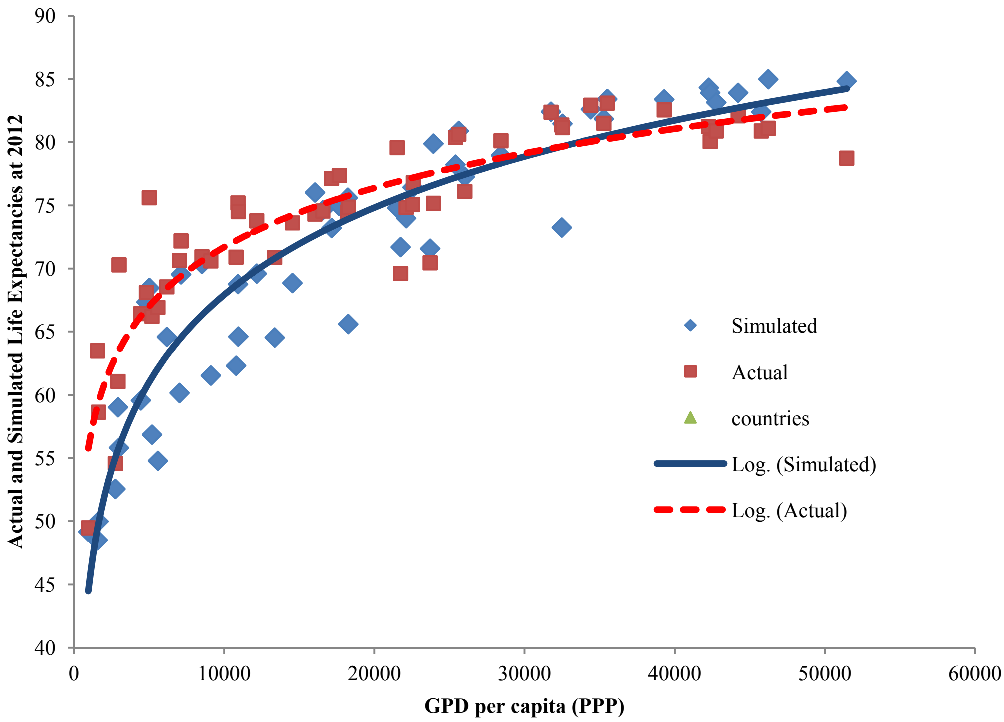
$$\frac{dL}{dt} = \left(\frac{\alpha}{P^{\,i}_{C0}e^{\pi_C t}}\right)^{\alpha} \left(\frac{\beta}{P^{\,i}_{E0}e^{\pi_E t}}\right)^{\beta} \left(\frac{M^{i}_{0}e^{\rho t}}{\alpha+\beta}\right)^{\alpha+\beta} \bar{L}\, L\left(1-\frac{L}{\bar{L}}\right) \tag{11}$$

With Eq. (11), it is possible to estimate the life expectancy of a country in period t, as long as we have estimates of input elasticities, know initial values of life expectancy, income, prices and have a projection about the growth rates of income and prices.

3. Simulations

The simple correlation between life expectancy and the logarithm of income per head is 0.885 in the 1930s and 0.880 in the 1960s. Today we are still in the health era started after the 1960s, therefore, we derive a single curve for one representative year. In Figure 3, we draw an updated Preston curve which uses IMF 2012 data for "GDP per capita, PPP"[(4)] (World Economic Outlook Database, 2015) and "life expectancy at birth" (World Data Bank, World Development Indicators) of multiple countries around the world. The curve follows a logistic pattern with an equation of $y = 6.757\ln(x) + 9.457$ ($R^2 = 0.82$) and the simple correlation between life expectancy and the logarithm of income per head for 2012 is 0.90.

Figure 3. *Actual and simulated life expectancies at 2012*



Then we simulate our model to see its fit with the actual Preston curve for 2012, by taking GDP per capita PPP, life expectancy in the initial year 1980, and the average growth figure for the period 1980-2012 for each country. (See appendix for a list of countries).

For the simulations, due to lack of proper data, we modified the budget constraint to have purchasing power instead of nominal income. In addition, following Andreoni, Levinson (2001, p. 272) we assume the relative prices of consumption and environmental effort to be constant and equal to one. Hence, we implicitly assume the utility of people to depend on the amounts of expenditures on consumption and environmental effort.

With input elasticities $\alpha = 0.82$ $\beta = 0.3$, which satisfy the inverse u-shape of the EKC, our model suggests life expectancy values for 2012 that fit well into the real data. The simulated curve's equation in Figure 3 is $y = 9.057\ln(x) - 20.79$ and is not significantly different from the actual curve ($R^2 = 0.88$ and correlation between life expectancy and the logarithm of income per head is 0.92) (Letting input elasticities differ according to countries' incomes (more weight on consumption for poorer countries) actually increases the fit. See Appendix.) In addition, due to our assumption of constant relative prices, we underestimate life expectancy for poorer countries and overestimate it for richer countries. This could depend on the fact that environmental effort prices actually increase at a higher rate than consumption prices and a poorer country, the utility of which is mostly from consumption, is positively affected by the trend more than a richer country (Qiusheng and Zongchun, 2014, Kim, 2004).

4. Conclusion

This paper develops a theoretical model which explains and successfully estimates the Preston Curve. Thus, it provides a theoretical basis that will allow many interest groups to make more accurate plans for the future. A policymaker with a targeted income growth rate can use our logistic equation to find out the future life expectancy at period 't', based on initial values. Having country specific estimates of input elasticities for different countries may allow policymakers to have more accurate projections for each country as well.

The simple but powerful model developed in this paper could be further developed by letting the relative prices of consumption and environmental effort vary by country and with time. The model could also be integrated with other macroeconomic models in order to study the dynamics of economic growth.

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Notes

(1) Assumed to be 90 years.
(2) Growth rate of logistic function is defined as $r = \delta\bar{L}$.
(3) $0 \leq \alpha, \beta \leq 1$ and to satisfy the inverse u-shape of the EKC, we need $\alpha + \beta \geq 1$. (For more information, see Andreoni and Levinson, 2001, p. 23).
(4) GDP based on purchasing-power-parity.

References

Alho, J.M. (2006). Forecasting Life Expectancy: A Statistical Look at Model Choice and Use of Auxiliary Series. In: Bengtsson, T. (Ed). *Perspectives on Mortality Forecasting.* Social Insurance Studies No. 3. pp. 41-51. Swedish Social Insurance Agency.

Andreoni, J., Levinson, A. (2001). The Simple Analytics of the Environmental Kuznetz Curve. *Journal of Public Economics.* 80 (2). pp. 269-286.

Bengtsson T. (2006). Linear Increase in Life Expectancy: Past and Present. In: Bengtsson, T. (Ed). *Perspectives on Mortality Forecasting.* Social Insurance Studies. No. 3. pp. 83-99. Swedish Social Insurance Agency.

Coelho, E. (2007). Mortality and Longevity Projections for the Oldest-Old in Portugal. Conference of European Statisticians. Bucharest.

Dinda, S. (2004). Environmental Kuznets Curve Hypothesis: A Survey. *Ecological Economics.* 49(4). pp. 431-455.

Kim, J.D. (2004). A Guideline for the Measurement and Reporting of Environmental Costs. In Bennet, M., Bouma, J.J, Wolters, T. (Eds.). Environmental Management Accounting: Informational and Institutional Developments. pp. 51-65. Kluwer Academic Publishers.

Kirkwood, C.W. (1998). System Dynamics Methods. http://www.public.asu.edu/~kirkwood/ sysdyn/ SDRes.htm.

Oeppen, J. (2006). Life Expectancy Convergence among Nations since 1820: Separating the Effects of Technology and Income. In: Bengtsson, T. (Ed). Perspectives on Mortality Forecasting, *Social Insurance Studies*. No. 3. pp. 55-82. Swedish Social Insurance Agency.

Preston, S.H. (1975). The Changing Relation between Mortality and Level of Economic Development. *Population Studies.* 29(2). pp. 231-248.

Qiusheng, S., Zongchun, L. (2014). A Research on the Management of Enterprise Environmental Costs. In Ren P, Du Z., *Information Technology and Industrial Engineering*. pp. 179-184. WIT Press.

Sterman, J.D. (2000). *Business Dynamics.* McGraw-Hill.

Vallin, J., France, M. (2010). Will Life Expectancy Increase Indefinitely by Three Months Every Year. *Population and Societies.* No. 473. pp. 1-4

World Data Bank, World Development Indicators. (n.d.). Retrieved from http://databank. worldbank.org/data/views/reports/tableview.aspx#

World Economic Outlook Database 2015. Retrieved from http://www.imf.org/external/pubs/ ft/weo/2015/01/weodata/index.aspx

Appendix

A) Health Stages (Bengtsson, 2006).

1st stage 1700-1800: Reduction in volatility/epidemics

2nd stage 1800-1900: Reduction in infectious diseases (influenza, pneumonia, bronchitis, TB, and smallpox)

3rd stage 1900-1960: Reduction in other infectious diseases

4th stage 1960-…: Reduction in chronic diseases

B) Derivation of optimized utility:

$$U(C, P) = C^{\alpha}E^{\beta}$$

$$P_C C + P_E E = M$$

$$\mathcal{L} = C^{\alpha}E^{\beta} + \lambda(M - P_C C - P_E E)$$

$$\frac{\partial \mathcal{L}}{\partial C} = \alpha C^{\alpha-1}E^{\beta} - P_C\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial E} = \beta C^{\alpha}E^{\beta-1} - P_E\lambda = 0$$

$$\lambda = \frac{\alpha}{P_C}C^{\alpha-1}E^{\beta} = \frac{\beta}{P_E}C^{\alpha}E^{\beta-1}$$

$$\frac{\alpha}{P_C}E = \frac{\beta}{P_E}C$$

$$E = \frac{P_C}{P_E}\frac{\beta}{\alpha}C$$

$$P_C C + P_E\frac{P_C}{P_E}\frac{\beta}{\alpha}C = P_C C + P_C\frac{\beta}{\alpha}C = M$$

$$P_C C\left(1 + \frac{\beta}{\alpha}\right) = M$$

$$C^{*} = \frac{M}{P_C\left(1 + \frac{\beta}{\alpha}\right)} = \frac{\alpha}{P_C(\alpha + \beta)}M$$

$$\mathrm{E}^* = \frac{\mathrm{P_C}}{\mathrm{P_E}}\frac{\beta}{\alpha}\mathrm{C}^* = \frac{\mathrm{P_C}}{\mathrm{P_E}}\frac{\beta}{\alpha}\frac{\alpha}{\mathrm{P_C}(\alpha+\beta)}\mathrm{M} = \frac{\beta}{\mathrm{P_E}(\alpha+\beta)}\mathrm{M}$$

$$\mathrm{U}^*(\mathrm{C},\mathrm{P}) = \mathrm{C}^\alpha\mathrm{E}^\beta = \left(\frac{\alpha}{\mathrm{P_C}(\alpha+\beta)}\right)^\alpha\left(\frac{\beta}{\mathrm{P_E}(\alpha+\beta)}\right)^\beta\mathrm{M}^{\alpha+\beta} = \left(\frac{\alpha}{\mathrm{P_C}}\right)^\alpha\left(\frac{\beta}{\mathrm{P_E}}\right)^\beta\left(\frac{\mathrm{M}}{\alpha+\beta}\right)^{\alpha+\beta}$$

$$\mathrm{U}^*(\mathrm{C},\mathrm{P}) = \mathrm{U}^*(\alpha,\beta,\mathrm{P_C},\mathrm{P_E},\mathrm{M})$$

C) Countries used

Algeria	Brazil	Australia
Bangladesh	Bulgaria	Canada
Bolivia	Chile	Denmark
Cameroon	Greece	France
Central African Republic	Hungary	Germany
China	Kazakhstan	Ireland
Colombia	Libya	Italy
Egypt	Malaysia	Japan
India	Mexico	Korea
Indonesia	Panama	Netherlands
Kenya	Poland	New Zealand
Morocco	Portugal	Spain
Pakistan	Romania	United Kingdom
Paraguay	Russia	United States
Peru	Slovak Republic	
Philippines	Slovenia	
Rwanda	Turkey	
Uganda	Venezuela	
Ukraine		
Uzbekistan		
Vietnam		
Income <14000	14000< Income < 30000	30000< Income
α^l=0,98 β=0,3	α^m=0,85 β=0,3	α^h=0,78 β=0,3

D) Input elasticities based on incomes:

When $\alpha = 0.98,\ \beta = 0.3$ for income (real purchasing power) < \$14000, $\alpha = 0.85,\ \beta = 0.3$ for \$14000 < income < \$30000, and $\alpha = 0.78,\ \beta = 0.3$ for income > \$30000, the fit of our simulated curve increases ($y = 8.251\ln(x) + 5.458,\ R^2 = 0.8307$ and correlation between life expectancy and the logarithm of income per head is 0.91). It is again not significantly different from the actual curve. (See Figure 4.)

Figure 4. *Simulation with changing input elasticities*