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A common agency within bureaucracy

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Abstract. *We explore the welfare implications of multiple principals in the top level of bureaucracy. An agent has to carry out two separate tasks, which can either be organized by two separate principals, or combined under one principal. The relationship between the top level (the principals) and the lower level (agent) of bureaucracy is a "principal-agent problem". The presence of multiple principals generates a "common agency". We show that the optimal bureaucratic structure depends on the existence of rents from office: the two systems produce equally welfare-efficient outcomes if there are no rents, a single-principal model dominates common agency otherwise.*

Keywords: principal-agent, bureaucracy, incentives, public goods.

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1. Introduction

It is well known that private provision of public goods is generally inefficient, which in turn necessitates state intervention in these areas. However, suboptimal provision of public goods reduces private effort and incentives, and eventually causes economic downturn. The success or failure of government policies having vital impacts on the real economy and social welfare prepared grounds for a controversial debate on the solutions to improve the performance of public sector. There are certainly numerous potential factors claimed to influence the efficiency in this area, but here we focus on a particular one: how best to design the state bureaucracies?

One common design problem is to find the best organizational structure in which different bureaucratic actors are responsible for different political actions -"separation of powers", as known in the political science literature. Persson and Tabellini (2000) argue that separation of powers increases the voters' ability to impose accountability on politicians and limit the equilibrium rents. In their model, an executive proposes a tax rate to the legislature, who would then announce the level of public good spending, as well as a rent allocation that the executive might veto. This arrangement illustrates, for example, the separation of political powers between the president and Congress in a presidential democracy, or between the different standing committees in a congressional setting. It could also apply to a parliamentary setting where different ministries exist, but let's extend this structure one step further and consider the following example: two ministries are responsible for the production of one type of public good, e.g. the minister of energy attempting to have the local municipality built a power plant on the one hand, and the minister for environment who tries to provide a certain level of environmental quality on the other. The case of a municipality which has to take into account the concerns of different ministers while taking an action is an example of a system with multi-principals, a so-called common agency.

In a common agency, a single agent works for several top-level bureaucrats; i.e. the principals, each with different (sometimes even conflicting) objectives. Political economists have already recognized the relevance of principal-agent modeling by identifying voters as principals and politicians as their agents. Yet agency problem is also applicable to multi-tiered governments, where policy implementation is complicated for the top-level bureaucrats, so that middle-level bureaucrats administer the policies that are decided by the top-level. A close look to the literature surprisingly reveals that relatively few attempts have been made to view bureaucracy as a common agency problem in spite of numerous practices relating to the field. Here I present a model that allows such multiplicity of principals in top-level bureaucracy to explore its welfare impacts.

The common agency game I propose involves two top-level bureaucrats (the principals), a middle-level bureaucrat (the agent) who is in charge of producing public goods, and additionally a third party the representative citizen, who consumes the public good. The key aspect of the model is that the principals value only one type of public good and hence contract with the common agent on the production of this public good. The agent's actions determine the probability distribution of the quality of the public goods and the related costs are compensated by the principal whose budget is determined by the citizen. The citizen faces a moral hazard problem: the agent's actions are private information to the bureaucracy. On top of that, she can have perfect information neither about the transfers within the bureaucracy, nor about the cost structures in public good production (adverse selection). This might lead to "corruption" in equilibrium.

The model provides several interesting results. I find that the optimal bureaucratic system is determined by the existence of the "rents" from public office. Rent in this setting is the potential non-transferrable gain from keeping the office which can be secured if and only if the principal succeeds in inducing her agent to produce a high quality public good. If such rents are present, the citizen is better off under a single-principal model rather than in a common agency. One reason is the cost advantage: the citizen can always reduce the optimal transfer which is paid when both goods are supplied with high quality, by a fraction of the rent unless the rent is too large. The competition between the principals in common agency, on the other hand, limits the ability of the citizen to reduce the cost of providing incentives in a similar manner. However, we will later discuss that cooperation in common agency can be sustained under particular conditions and potentially improve the overall welfare.

Agency problems within governments might account for state failures as Dixit (2006) points out. Recognizing the relationship across different levels of bureaucracy as a principal-agent problem, he argues that the degree of benevolence both at the top-level and middle-level might have a role over bureaucratic efficiency. In this context, interactions arise endogenously: as informational limitations become more severe, it gets more likely that a "predatory" ruler hires selfish bureaucrats whereas a collaborative relationship between a benevolent ruler and a "caring" bureaucrat arises. Although the idea of modeling the agency within bureaucracy is similar, Dixit (2006) does not consider a common agency within bureaucracy and address the issues that I do in my model.

Common agency problems frequently occur in real life. Administrative agencies who are basically responsible to the lawmakers, yet are practically influenced by courts, media, and various interest groups, or European Union where several sovereign governments deal with a common entity in policy making are just a few examples. Dixit et.al (1997) highlight the existence of a common agency in economic policy making in a setting where a subset of all citizens are allowed to lobby the government and promise contributions in return for policy favors. They notice that extending the standard set-up by allowing multiple principals introduces new issues and affect the feasibility of an efficient outcome for all parties. Grossman and Helpman (2001) use the same framework to model the competition between different actors for political influence. As several interest groups confront the policy maker with offers of campaign contributions in exchange for various policies, an equilibrium containing a set of contribution schedules that are mutually best responses can be characterized.

The common agency problem, as a special case of multi-contracting, has been studied in many contexts in other fields of standard economics although it has been novel to the political economy literature. As a typical example, Martimort and Stole (2003) consider the case of two retailers distributing the output of a common manufacturer and compete in the same final market. Such structure emphasizes direct externalities as the price of one retailer directly affects the price faced by the other. Introducing task complementarity where agent chooses his level of performance in two different tasks performed for different principals, Mezzetti (1997) finds that independent contracting under common agency generates more total surplus than cooperation.

In public good production the efficiency of a common agency is highly controversial. Laussel and Le Breton (1996) present results favorable for a common agency where a private firm producing the public good for n consumers is compensated for the associated costs with monetary contributions. They show that free-riding problem is fixed and efficiency can still be sustained under truthful Nash equilibria. However, the very same structure might cause ineffiencies as Martimort and Moreira (2006) illustrate in a slightly different setting: it is not the contributors who try to misreport their valuations for the public good but it is the common agent who has an incentive to claim that the others have a lower level of willingness to pay than what they actually have. The problem of the principals is then to assess the agent's market information which, in turn, creates inefficient outcomes.

The presentation in this paper is as follows. Next section describes the model and derives the equilibria in single principal and common agency models. The third section is a comparative assessment of the two bureaucratic systems in terms of citizen's welfare. We will discuss the cooperative common agency in the fourth section and finally conclude in the last section.

2. The Model

There are two public goods both of which are produced by the "agent"; i.e., the middle-level bureaucrat. The quality of the public good could be either high or low; for good i, $q_i^k \in \{q_i^h, q_i^l\}$. For simplicity, q_i^l is normalized to zero. The agent's effort choices determine the quality of the public goods and induce production costs according to the function

$$C(e_1, e_2) = \delta(e_1 + e_2 + \gamma e_1 e_2)$$

where e_i denotes the unverifiable effort spent in the production of public good i. $\delta > 0$ is a productivity parameter and is perfectly known only by the bureaucracy. $\gamma \in [0,1]$ is a parameter for the agent's preferences over multitasking. The agent's production costs are compensated through transfers by the top-level bureaucracy whose budget is determined by the citizen. However, there is limited liability: the transfers must be non-negative. The agent should also be as well of as he could be if he rejects the contract; i.e. he should be paid enough such that he gets at least his reservation payoff, which is normalized to zero without loss of generality.

The top-level bureaucracy consists of two principals in common agency whereas in single-principal model it does only one. In common agency, each principal is in charge of producing one type of public good. If she succeeds in inducing her agent to produce high quality public good of which she is in charge, she keeps the office; otherwise she has to resign and loses the rent from keeping the public office. In single-principal model, the principal is in charge of both public goods and can keep the office if and only if both public goods are of high quality. The principal's payoff depends on the net monetary transfer; i.e. the difference between the budget she receives from the citizen and the transfer she pays to the agent, plus the rent from public office. The transfers inside the bureaucracy cannot be observed by the citizen. This, together with unobservable productivity parameter δ, puts the citizen into the most disadvantageous position in terms of informational asymmetries.

The citizen values the consumption of the public good of both types, yet at different degrees. For all $k \in \{h, l\}$, the utility she derives from public good 1 of quality k is αq_1^k. Similarly, the utility she derives from public good 2 of quality k is $(1-\alpha)q_2^k$. The parameter $\alpha \in [0,1]$ indicates the relative taste of the citizen over the two public goods.

The timeline is defined as follows. First, the citizen decides on the monetary transfer she would pay for good i of quality k, for all $i \in \{1,2\}$ and $k \in \{h, l\}$. Given the contract offered by the citizen, the principal (principals, in common agency) chooses the transfers to be paid to the agent. The agent makes effort desicions if he accepts the contract offered by a principal. We start with considering the agent's problem.

2.1 . Agent's Problem

If the agent accepts the contract, he privately decides on the effort levels for both tasks, $e_i \in \{0,1\}, i = 1,2$. The effort choices of the agent determine the probability distribution of the public good quality in the following way: if he exerts e_i in task i, the probability that the public good type i will be of high quality (i.e. the "success" in task i) is equal to e_i. We assume that the success in task i and the success in task j are independent events; i.e. the probability of the event with "success in both tasks" is equal to $e_1 e_2$. The probabilities of the other events, meaning "success only in one task" and "success in none of the two tasks" are determined in a similar manner.

The agent is compensated by the top-level bureaucracy through quality-contingent transfers in the following way: if there is success only in public good i, he receives t_i; but if he succeeds in both tasks, the award is t^*, where $t^* \geq t_1 + t_2$. He will be paid zero otherwise (doing so will not be optimal from the top-level's point of view).

The agent will maximize his expected payoff:

$$EU_A(e_1, e_2) = e_1 e_2(t^* - t_1 - t_2) + e_1 t_1 + e_2 t_2 - \delta(e_1 + e_2 + \gamma e_1 e_2) \qquad (1)$$

The term $t^* - t_1 - t_2$ in equation (1) can be interpreted as a bonus payment. If the contract pays a nonzero bonus, then it means the reward paid when both tasks are completed with success; i.e., t^* is higher than the sum of individual payments that are paid when only one success is observed; i.e., $t_1 + t_2$. However, this case can only arise in single-principal model as the bonus is optimally determined by the principal whereas in common agency t^* is exactly the sum of t_i's which are determined separately by the two principals.

The agent chooses his optimal effort choice (e_1^*, e_2^*) to maximize his expected payoff in equation (1). For example, he will supply $(e_1, e_2) = (1,1)$ if and only if the expected payoff of supplying this is greater than of supplying (1,0), (0,1) or (0,0). Then, given the transfers t^*, t_1 and t_2, his strategy is characterized as follows

(i) $(e_1^*, e_2^*) = (1,1)$ if and only if: $t^* - t_1 \geq \delta(1+\gamma)$; $t^* - t_2 \geq \delta(1+\gamma)$ and $t^* \geq \delta(2+\gamma)$. Or equivalently, if and only if

$$t^* \geq max\{t_1 + \delta(1+\gamma), t_2 + \delta(1+\gamma), \delta(2+\gamma)\}$$

(ii) $(e_1^*, e_2^*) = (1,0)$ if and only if: $\delta(1+\gamma) > t^* - t_1$, $t_1 > t_2$, and $t_1 \geq \delta$. Or equivalently, if and

only if

$$t_1 > max\{t_2, t^* - \delta(1+\gamma)\} \ and \ t_1 \geq \delta$$

(iii) $(e_1^*, e_2^*) = (0,1)$ if and only if: $\delta(1+\gamma) > t^* - t_2, t_2 > t_1$ and $t_2 \geq \delta$. Or equivalently, if and only if

$$t_2 > max\{t_1, t^* - \delta(1+\gamma)\} \ and \ t_2 \geq \delta.$$

(iv) $(e_1^*, e_2^*) = (0,0)$ if and only if: $\delta > t_1, \delta > t_2$, and $\delta(2+\gamma) > t^*$. Or equivalently, if and only if

$$\delta > max\{t_1, t_2, \frac{t^*}{2+\gamma}\}$$

The agent is indifferent between (1,0) and (0,1) if and only if: $t_1 = t_2 \geq \delta, t^* - t_1 < \delta(1+\gamma)$, and $t^* - t_2 < \delta(1+\gamma)$.

Finally, the agent will accept the contract if and only if it yields his reservation payoff. Then his strategy also takes into account his participation constraint: $EU_A(e_1^*, e_2^*) \geq 0$.[(1)]

2.2. Equilibrium in Single-Principal Model

2.2.1. Principal's Problem

The top-level bureaucracy consists of one principal who is in charge of both public goods. The transfers determined by the citizen are similar to those within the bureaucracy: a transfer T_i is paid if high quality is achieved only in public good *i* but $T^* \geq T_1 + T_2$ is paid when high quality is observed in both tasks. Nothing is paid when both goods are of low quality (doing so will not be optimal from the citizen's point of view). The rent from the public office, denoted as *R*, could be extracted only when both goods are of high quality. Therefore, the expected payoff of the principal can be expressed as:

$$EU_P(t^1, t^2, t^*) = e^1 e^2 \big(R + T^* - T^1 - T^2 - (t^* - t^1 - t^2)\big) +$$

$$+ e_1(T_1 - t_1) + e_2(T_2 - t_2) \qquad (2)$$

Again the term $T^* - T_1 - T_2$ has a similar interpretation to the bonus in agent's contract. Given the contract, (T^*, T_1, T_2) offered by the citizen, the principal chooses (t^*, t_1, t_2) to maximize her expected payoff in (2) while taking into account the agent's strategy as derived in (i)-(iv). She knows that the agent should be paid at least $t^* = \delta(2+\gamma)$ to exert (1,1). Similarly, $t_i = \delta$ is required to induce either (1,0) or (0,1), and zero for (0,0). Therefore, she determines her strategy, (t^*, t_1, t_2), in the following way:

(i) Offer $t^* = \delta(2+\gamma)$, and any $t_1, t_2 < \delta$ if and only if

$$R + T^* - \delta(2+\gamma) \geq max\{T_1 - \delta, T_2 - \delta, 0\} \tag{3}$$

(ii) Offer $t_1 = \delta$, any $t_2 < t_1$ and $t^* < \delta(2+\gamma)$ if and only if

$$T_1 > max\{R + T^* - \delta(1+\gamma), T_2\} \text{ and } T_1 \geq \delta \tag{4}$$

(iii) Offer any $t^* < \delta(2+\gamma)$, and $t_2 = \delta$, and any $t_1 < t_2$ if and only if

$$T_2 > max\{R + T^* - \delta(1+\gamma), T_1\} \text{ and } T_2 \geq \delta \tag{5}$$

(iv) Offer any $t^* < \delta(2+\gamma)$, and $t_1 = t_2 = \delta$, if and only if

$$T_1 = T_2 = T \geq \delta \text{ and } R + T^* - \delta(1+\gamma) < T \tag{6}$$

(v) Offer any $t^* < \delta(2+\gamma)$ and any $t_1, t_2 < \delta$ if and only if

$$T_1, T_2 < \delta \text{ and } R + T^* - \delta(2+\gamma) < 0 \tag{7}$$

To understand her strategy consider, for example, (i): the contract (t_1, t_2, t^*), such that $t^* = \delta(2+\gamma)$ and any $t_1, t_2 < \delta$, implements the outcome (1,1). It is optimal to implement (1,1) if and only if the payoff from this outcome, $R + T^* - \delta(2+\gamma)$ is greater than the payoffs resulting from the contracts that would implement the outcomes (1,0), (0,1) and (0,0); i.e. $T_1 - \delta$, $T_2 - \delta$ and zero, respectively. Therefore, the principal optimally implements (1,1) if and only if $R + T^* - \delta(2+\gamma) \geq max\{T_1 - \delta, T_2 - \delta, 0\}$. Similarly, the contract in (ii) implements the outcome (1,0) and is optimal if and only if condition (4) is satisfied. Likewise, the contract in (iii) induces the outcome (0,1) and is payoff-maximizing if and only if condition (5) holds, whereas the contract in (iv) implements either (1,0) or (0,1) and the principal is indifferent between the two outcomes if and only if (6) holds. Finally, the contract in (v) implements the outcome (0,0) and it is optimal to do so if and only if condition (7) is satisfied.

Since the citizen can neither observe the transfers between the principal and the agent, nor the true value of δ, there is a potential welfare loss that might arise due to corruption. In other words, such informational imperfections might jointly lead to ex-post *leakages* within the system. It follows that corruption takes place in equilibrium, for example, in case (i) if the true value of δ is small enough. Or specifically, if $\delta < T^*/2 + \gamma$ and condition (3) holds, then the principal implements the outcome (1,1) at a cost of $t^* = \delta(2+\gamma)$ and keeps the excess $T^* - t^*$ for herself. The same issue arises in cases (ii), (iii) and (iv) if the condition $\delta < T_1$, $\delta < T_2$ and $\delta < T_1 = T_2$ hold respectively. Therefore, corruption appears as the cost of an adverse selection problem between the citizen and bureaucracy, which can be minimized once such informational imperfections are removed.

2.2.2. Citizen's Problem

Recall that the citizen values the consumption of k-quality public good type 1 by αq_1^k and type 2 by $(1-\alpha)\, q_2^k$ for all $k \in \{h, l\}$ and also that the level of low quality is normalized to zero; i.e, $q_i^l = 0$ for all $i \in \{1,2\}$. We assume that the citizen values high quality public good type 1 slightly more than the other; however, the value she assigns to the other is relatively not too low.(2)

Assumption 1. $\alpha q_1^h > (1-\alpha) q_2^h \geq \frac{\gamma+2}{2\gamma+3} \alpha q_1^h$.

The citizen offers transfers (T^*, T_1, T_2) to the principal and expects the payoff:

$$EU_C^{SP} = e_1(\alpha q_1^h - T_1) + e_2((1-\alpha) q_2^h - T_2) - e_1 e_2 (T^* - T_1 - T_2) \qquad (8)$$

The productivity parameter δ is private information of both the principal and the agent but the citizen knows that δ is uniformly distributed on $[0, \bar{\delta}], \bar{\delta} \geq \alpha q_1^h$. Then the citizen determines her strategy to maximize the expected payoff in (8) by taking into account how the principal, who observes the true value of δ, would respond to a given contract. In doing so, she considers two things.

First, the transfers (T^*, T_1, T_2) determine which effort-outcomes are feasible to the principal. An outcome is feasible if and only if it leaves a non-negative payoff to the principal. To start with, outcome (0,0) is always feasible regardless of the size of δ: from equation (1), inducing (0,0) is costless although it yields a payoff of zero. On the other hand, to induce outcome (1,0), the principal has to offer a transfer equal to δ to the agent, therefore it is feasible if and only if $\delta \leq T_1$. However, outcome (1,1) is feasible if and only if $\delta(2+\gamma) \leq T^*$; or equivalently $\delta \leq \frac{T^*}{2+\gamma}$. Since the citizen does not know the true δ, she can only evaluate the probability distribution of the feasible outcomes for a given contract option. For example, a contract option such that $T_1 < T_2 < \frac{T^*}{2+\gamma}$, the probability distribution of the feasible outcomes is as follows: with probability $F(T_1)$, any outcome is feasible; but with $F(T_2) - F(T_1)$, all outcomes except (1,0); with probability $F(\frac{T^*}{2+\gamma}) - F(T_2)$ only outcomes (1,1) and (0,0); and finally with $1 - F(\frac{T^*}{2+\gamma})$ only outcome (0,0) is feasible.

Second, given a contract, the citizen considers how the principal who perfectly observes δ would optimally respond by choosing which outcome to implement among the feasible set. To do so, she takes this into account the principal's strategy as derived in equations (3)-(7). This helps to calculate the citizen's expected payoff from a given contract as illustrated in Example 1 below for a randomly selected contract.

Example 1: Consider a contract option

$$(T^*, T_1, T_2) \text{ such that } \frac{T^*}{2+\gamma} < T_2 < T_1 \qquad (9)$$

We will calculate the expected payoff to the citizen from the contract in (9). But first we need to identify the feasible outcome set for a given δ, and then which outcome the principal implement at the optimum (also, see Figure 1):

i. $\delta > T_1$: δ is too large relative to the principal's budget and hence she cannot induce another outcome than (0,0). *Therefore, the outcome (0,0) will be implemented for all* $\delta \in (T_1, \bar{\delta}]$. Note that this is the only range of δ where this outcome is implemented at the optimum since all other outcomes, when feasible, yield a non-zero hence greater payoff to the principal.

ii. $\frac{T^*}{2+\gamma} < \delta \leq T_1$: When $T_2 < \delta \leq T_1$, outcomes (1,0) and (0,0) are both feasible but only the former yields a non-zero payoff to the principal. However, when $T^*/2 + \gamma < \delta \leq T_2$, outcome (0,1) is also feasible but since $T_2 < T_1$ and both outcomes has the same implementation cost (i.e. a transfer of δ to the agent), implementing (1,0) yields a higher net payoff to the principal than implementing (0,1). *Therefore, outcome (1,0) will be implemented for all* $\delta \in (T^*/2 + \gamma), T_1]$.

iii. $\delta \leq \frac{T^*}{2+\gamma}$: Any outcome is feasible. By the same argument, outcome (0,0) yields the lowest payoff to the principal and outcome (1,0) is preferred over outcome (0,1). Then to determine whether outcome (1,1) or outcome (1,0) will be implemented, the citizen takes into account the principal's incentive compatibility. First, recall that the principal's payoff is $R + T^* - \delta(2 + \gamma)$ if she implements outcome (1,1) and it is $T_1 - \delta$ if she implements (1,0). Comparing the two payoffs is equivalent to comparing δ with $\frac{R+T^*-T_1}{1+\gamma}$. Therefore, there are two cases:

Case a: $\frac{T^*}{2+\gamma} \leq \frac{R+T^*-T_1}{1+\gamma}$. In that case, $\delta \leq \frac{T^*}{2+\gamma}$ implies $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$, hence the outcome (1,1) yields a higher payoff to the principal. *Therefore, outcome (1,1) is implemented for all* $\delta \in (0, \frac{T^*}{2+\gamma})$.

Combining i., ii., iii.a., the citizen's expected payoff from the contract that satisfies condition (9) is :

$$F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - F(\frac{T^*}{2+\gamma})](\alpha q_1^h - T_1)$$

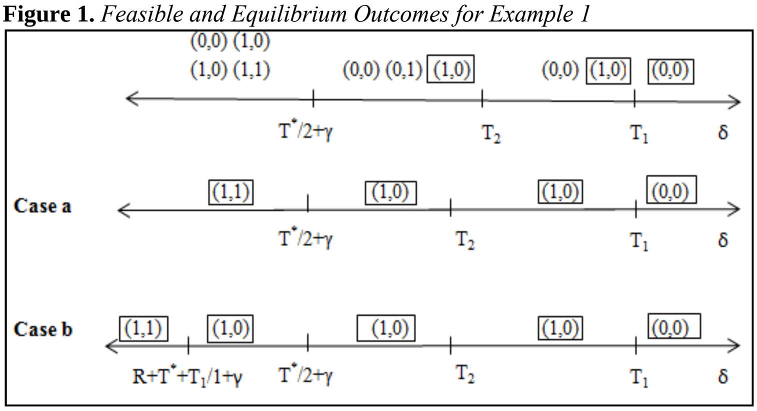
Case b: $\frac{R+T^*-T_1}{1+\gamma} < \frac{T^*}{2+\gamma}$. In that case, incentive compatibility steps in. If $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$, the outcome (1,1) yields a higher payoff to the principal but if $\frac{R+T^*-T_1}{1+\gamma} < \delta \leq \frac{T^*}{2+\gamma}$, it is (1,0). *Therefore, outcome (1,0) is implemented for all* $\delta \in \left(\frac{R+T^*-T_1}{1+\gamma}, \frac{T^*}{2+\gamma}\right)$ *and (1,1) for all* $\delta \in (0, \frac{R+T^*-T_1}{1+\gamma})$.

Combining i., ii., iii.b., the citizen's expected payoff from the contract in condition (9) is:

$$F(\frac{R+T^*-T_1}{1+\gamma})(\alpha q_1^h+(1-\alpha)q_2^h-T^*)+[F(\frac{R+T^*-T_1}{1+\gamma})-F(\frac{T^*}{2+\gamma})](\alpha q_1^h-T_1)$$

As illustrated, for a given contract (T^*, T_1, T_2) it is the relative sizes of T_1, T_2 and $\frac{T^*}{2+\gamma}$ that determine the set of feasible outcomes for different ranges of δ. Given the feasible outcomes for a given δ the principal chooses her payoff-maximizing outcome to implement that, in turn, determines the expected payoff to the citizen who does not observe the true value δ. Therefore, as a first step to characterize the citizen's strategy, we will consider the possible contract options and derive the conditions under which they are optimal. For example, we will first ask when a contract option such that $\frac{T^*}{2+\gamma} < T_2 < T_1$ is optimal. Then we solve for the optimal transfers T_1, T_2 and T^* that maximizes the citizen's expected payoff. Of course, the citizen has several contract options to consider. But we first show that some options are never optimal. Particularly, as Lemma 1, optimal T^* cannot be too high relative to T_1 and T_2.

Figure 1. *Feasible and Equilibrium Outcomes for Example 1*



Note: Equilibrium outcomes are marked in boxes.

Lemma 1. The optimal contract sets $max(T_1, T_2) > \frac{T^*}{2+\gamma}$.

Proof. Assume that the citizen chooses (T_1, T_2, T^*) such that

$$T_1, T_2 \leq \frac{T^*}{2+\gamma} \qquad (10)$$

Obviously, if $\delta > \frac{T^*}{2+\gamma}$, the principal cannot afford to induce any outcome except (0,0). Hence, the citizen expects to receive a zero payoff with probability $1 - F(\frac{T^*}{2+\gamma})$. It is easy to check that with probability $F(\frac{T^*}{2+\gamma})$ outcome (1,1) is implemented and citizen receives $\alpha q_1^h + (1-\alpha)q_2^h$. To see that let $T_1 < T_2$, arbitrarily. For all $\delta \in (T_2, \frac{T^*}{2+\gamma})$, only outcomes (1,1) and (0,0) are feasible and the former yields a higher payoff hence it is implemented. However, for all $\delta \in (T_1, T_2)$, (0,1) is also feasible. Note that $T_2 \leq \frac{T^*}{2+\gamma}$ implies $T_2 \leq \frac{R+T^*-T_1}{1+\gamma}$, therefore the principal strictly prefers outcome (1,1) over (0,1). Similarly for all $\delta < T_1$, outcome (1,0) is also feasible but again $T_1 \leq \frac{T^*}{2+\gamma}$implies $T_1 \leq \frac{R+T^*-T_1}{1+\gamma}$: principal also strictly prefers outcome (1,1) over (1,0). Therefore, for all $\delta < \frac{T^*}{2+\gamma}$ the principal optimally implements outcome (1,1). Note that this result does not depend on our arbitrary assumption of $T_1 < T_2$. Hence, the contract (10) yields the citizen an expected payoff of

$$F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) \tag{11}$$

Now we show that the contract (10) is never optimal; i.e., it is possible to find a contract that yields a higher payoff. Recall that we have assumed $\alpha q_1^h > (1-\alpha)q_2^h$. Now fix T^*. The citizen is willing to pay at most $\alpha q_1^h + (1-\alpha)q_2^h$ when both goods are of high quality. Therefore, optimal T^* satisfies $T^* \leq \alpha q_1^h + (1-\alpha)q_2^h$. Since $\alpha q_1^h > (1-\alpha)q_2^h$ by assumption, at the optimum we have $T^* < 2\alpha q_1^h$. This implies $\frac{T^*}{2+\gamma} < \alpha q_1^h$. Then, we can always find a higher transfer for good type 1 $\widetilde{T_1} > T_1$, such that $\frac{T^*}{2+\gamma} < \widetilde{T_1} \leq \alpha q_1^h$ and $\widetilde{T_1} < \frac{R+T^*-\widetilde{T_1}}{1+\gamma}$, both hold. The former inequality implies that for all $\delta \in (\frac{T^*}{2+\gamma}, T_1)$ the principal implements (1,0) as it yields a higher payoff then the other feasible outcome (0,0). The latter inequality guarantees that the principal prefers to implement outcome (1,1) over (1,0) when both are feasible; i.e. when $\delta < \frac{T^*}{2+\gamma}$. Therefore, the citizen continues to receive the payoff in (11) (since we have fixed T^*) but now she is receiving an additional αq_1^h with probability $F(\widetilde{T_1}) - F(\frac{T^*}{2+\gamma})$ by paying $\widetilde{T_1} \leq \alpha q_1^h$. Therefore, the contract in (10) is not optimal. ■

Lemma 1 rules out the contract options in which T^* is too large relative to T_1 and T_2. Then, in equilibrium, the contract sets at least one of the two transfers above $\frac{T^*}{2+\gamma}$. There remains to show the particular relationship between the two transfers, T_1 and T_2 at the optimum. First, note that a contract that sets $T_1 > T_2$ always assigns zero probability to the outcome (0,1). To see this, observe that outcome (0,1) is feasible only if outcome (1,0) is feasible and from the principal's perspective, it is enough to offer δ to induce either of the two outcomes. Then $T_1 > T_2$ implies the principal receives a higher payoff from outcome (1,0) (i.e., T_1–δ) than from (0,1) (i.e., T_2–δ). Similarly, a contract which satisfies $T_2 > T_1$, implies that the outcome (1,0) is never implemented at the optimum. However, Lemma 2 proves that the latter contract is never optimal under our assumption that the citizen values public good type 1 relatively higher than public good type 2.

Lemma 2. The optimal contract sets $T_1 > T_2$.

Proof. Suppose that the citizen offers T_1 and T_2 such that

$$T_1 = \widehat{T_1} \text{ and } T_2 = \widehat{T_2}, \widehat{T_1} > \widehat{T_2} \quad (12)$$

By Lemma 1, $\widehat{T_1} > \frac{T^*}{2+\gamma}$ should hold at the optimum. Now consider the equilibrium outcomes. With probability $1 - F(\widehat{T_1})$, outcome (0,0) will be implemented. However, outcome (0,1) will never be implemented: $\widehat{T_1} > \widehat{T}_2$ implies that outcome (0,1) is feasible only if (1,0) is feasible and the principal receives a higher payoff from implementing outcome (1,0) (i.e., $T_1 - \delta$) than from (0,1) (i.e., $T_2 - \delta$). Furthermore, when outcomes (1,0) and (1,1) are both feasible; i.e. when $\delta \leq \frac{T^*}{2+\gamma}$, again incentive compatibility steps in and the relative size of $R + T^* - T_1/1 + \gamma$ determines which outcome will be implemented. There are two options for the citizen:

Option 1: $\frac{R+T^*-\widehat{T_1}}{1+\gamma} < \frac{T^*}{2+\gamma}$. In that case, with probability $F(\widehat{T_1}) - F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$, outcome (1,0) will be implemented and the citizen receives αq_1^h. With probability $F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$, outcome (1,1) will be implemented and the citizen will receive $\alpha q_1^h + (1-\alpha)q_2^h$. Now fix T^*.

Step 1. Consider a change in transfers such that $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_1} = \widehat{T_2}$ and $\widetilde{T_2} = \widehat{T_1}$. So now the contract pays more for good type 2 instead. Since T^* is fixed, the citizen still receives $\alpha q_1^h + (1-\alpha)q_2^h$ with probability $F\left(\frac{R+T^*-\widetilde{T_2}}{1+\gamma}\right)$. However, now she receives $(1-\alpha)q_2^h$ with probability $F(\widetilde{T_2}) - F\left(\frac{R+T^*-\widetilde{T_2}}{1+\gamma}\right) = F(\widehat{T_1}) - F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$. Observe that, since $\alpha q_1^h > (1-\alpha)q_2^h$, the contract with $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ yields a lower payoff than the contract in (12). Therefore, it is not optimal to set $T_1 < T_2$.

Step 2. Consider a change in transfers such that $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_1} = \widetilde{T_2} = \widehat{T_1}$. So the contract pays the same transfer for both type of goods. Then, with probability $F(\widehat{T_1}) - F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$., the principal will be indifferent in implementing the two outcomes, (1,0) and (0,1), but strictly prefers to implement (1,1) with probability $F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$. However, since $\alpha q_1^h > (1-\alpha)q_2^h$, the citizen is not indifferent between the two outcomes and setting $T_1 > T_2$ as in contract (12) yields a higher payoff. Therefore, it is not optimal to set the two transfers equal, $T_1 = T_2$.

Option 2: $\frac{R+T^*-\widehat{T_1}}{1+\gamma} \geq \frac{T^*}{2+\gamma}$. In that case, with probability $F(\widehat{T_1}) - F(\frac{T^*}{2+\gamma})$, outcome (1,0) will be implemented and the citizen receives αq_1^h. With probability $F(\frac{T^*}{2+\gamma})$,, outcome (1,1) will be implemented so the citizen will receive $\alpha q_1^h + (1-\alpha)q_2^h$. Again fix T^*.

Step 1. Consider a change in transfers such that $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ where $\widetilde{T_1} = \widehat{T_2}$ and $\widetilde{T_2} = \widehat{T_1}$. Since T^* is fixed, the citizen still receives $\alpha q_1^h + (1-\alpha)q_2^h$ with probability $F(\frac{T^*}{2+\gamma})$. However, she will receive $(1-\alpha)q_2^h$ with probability $F(\widetilde{T_2}) - F(\frac{T^*}{2+\gamma}) = F(\widehat{T_1}) - F(\frac{T^*}{2+\gamma})$. Observe that, since $\alpha q_1^h > (1-\alpha)q_2^h$, the contract with $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ yields a lower payoff than the contract in (12).

Step 2. Consider a change in transfers such that $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_1} = \widetilde{T_2} = \widehat{T_1}$. Again, such contract yields (1,1) with probability $F(\frac{T^*}{2+\gamma})$, but will leave the principal indifferent implementing the two outcomes, (1,0) and (0,1) with probability $F(\widehat{T_1}) - F(\frac{T^*}{2+\gamma})$. However, the citizen is not indifferent between the two outcomes and setting $T_1 > T_2$ as in contract (12) yields a higher payoff. ■

Intuitively, since the citizen derives a higher benefit from high quality public good type 1 alone than public good type 2 alone, she is willing to pay more for good type 1 in equilibrium. From Lemma 1 and Lemma 2, we have an idea about how the optimal contract looks like but Propositions 1 fully characterizes it. First we define two critical values for the rent *R*:

$$R_L \equiv \frac{(\gamma+1)\alpha q_1^h - (1-\alpha)q_2^h}{2\gamma+3}$$

$$R_H \equiv \frac{(2\gamma+3)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7}$$

Observe that Assumption 1 implies $R_H > R_L$.

Proposition 1. ***i.*** If $R > R_H$, the optimal contract is

$$(T_1, T_2, T^*) = (\frac{2(2+\gamma)\alpha q_1^h + (1-\alpha)q_2^h}{4\gamma+7}, 0, \frac{(2+\gamma)\alpha q_1^h + 2(1-\alpha)q_2^h}{4\gamma+7})$$

ii. If $R_L \leq R \leq R_H$, then the optimal contract is

$$(T_1, T_2, T^*) = (\frac{\alpha q_1^h + (1-\alpha)q_2^h + (2\gamma+3)R}{2(2+\gamma)}, 0, \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2})$$

iii. If $R < R_L$, then the optimal contact is

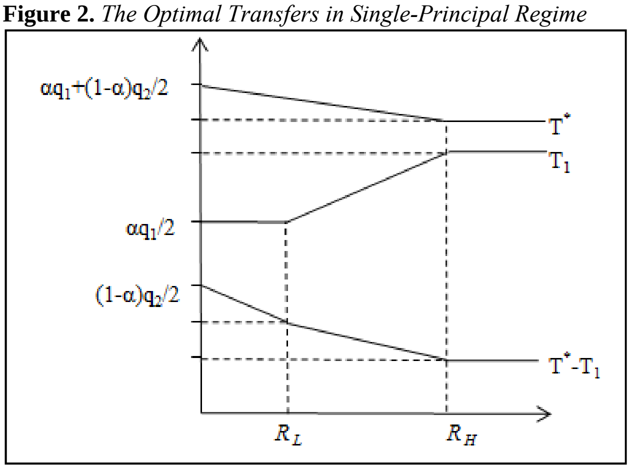
$$(T_1, T_2, T^*) = (\frac{\alpha q_1^h}{2}, 0, \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2})$$

Proof. See Appendix.

Proposition 1 shows that a bonus contract is payoff-maximizing for the citizen; i.e. $T^* > T_1 - T_2$ in all cases. In Figure 2 we illustrate the optimal contract as a function of *R*. Observe that optimal T^* decreases as *R* increases. Intuitively, as the rents from office, that could be earned only when success in both tasks occurs, increases, the optimal incentives to be provided by T^*decreases. In other words, it becomes cheaper to induce success in both tasks when the reward from keeping the office increases. However, optimal T_1 increases as R increases. Since as R gets larger, the principal is more likely to induce outcome (1,1), ceteris paribus. Then the citizen needs to cope with the decrease in incentives to produce outcome (1,0) with a higher T_1. Notice that, in Figure 2, this increase in T_1 offsets the decrease in T^* as R gets larger, and hence the optimal bonus pay $T^* - T_1$ decreases. The reason is that the bonus affects the incentive compatibility of the principal who will be additionally compensated by an extra R when outcome (1,1) is produced. Therefore, for a larger R, the cost-minimizing contract requires a lower bonus to satisfy the incentive compatibility constraint.

It is also possible to show that if the citizen's valuation towards high quality public good 2 is relatively quite low and hence Assumption 1 violated, a bonus contract is not optimal when R is high. Therefore, the citizen simply sets $T^* = T_1$ in that case. Intuitively, if the citizen does not care much about public good type 2, a bonus is relatively expensive to trigger high quality production for this type public good. Moreover, as the rents from office gets larger, which could act as a costless incentive device to increase quality in public good type 2, a bonus pay is no longer pay-off maximizing.

Figure 2. *The Optimal Transfers in Single-Principal Regime*



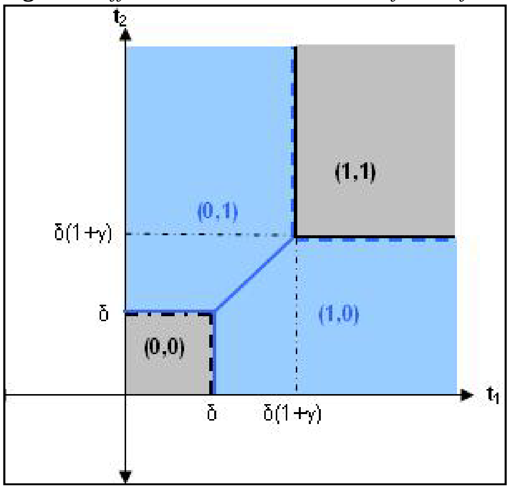
2.3. Equilibrium in Common Agency

In this section we deviate from the previous model by assuming that there are now two principals who are responsible for a single task. They simultaneously offer a contract to the agent: t_i is paid by principal i if success in task i (i.e. high quality of public good type i) is realized, nothing is paid otherwise. Given (t_1,t_2), the agent chooses optimal effort (e_1^*, e_2^*), where $e_i \in \{0,1\}, \forall i \in \{0,1\}$. The agent's strategy, (e_1^*, e_2^*) is derived in Section 2.1, but note that unlike in the previous model where a single principal has an option to offer a bonus contract, in common agency there is no such option: the agent earns $t^* = t_1 + t_2$ if he succeeds in both tasks. Therefore, we modify the agent's strategy, as derived in (i)-(iv), by substituting $t^* = t_1 + t_2$:

$$(e_1^*, e_2^*) = \begin{cases} (1,1) & \text{if} \quad min\{t_1, t_2\} \geq \delta(1+\gamma) \\ (1,0) & \text{if} \quad t_2 < min\{\delta(1+\gamma), t_1\} \text{ and } t_1 \geq \delta \\ (0,1) & \text{if} \quad t_1 < min\{\delta(1+\gamma), t_2\} \text{ and } t_2 \geq \delta \\ \text{either}(1,0) \text{ or } (0,1) & \text{if} \quad t_1 = t_2 = \delta \\ (0,0) & \text{if} \quad \delta > max\{t_1, t_2\} \end{cases} \quad (13)$$

Note that the agent is indifferent between (1,0) and (0,1) if $t_1 = t_2 = \delta$. Figure 3 illustrates the effort choices of the agent, given the contract offers by the principals. If both transfers are sufficiently high, he will work hard for both tasks (in the upper grey-shaded region). If he is not paid enough for both tasks, then he will shirk in both (the lower grey-shaded region). For the remaining cases where only one of the transfers is large enough to compensate the agent; he works only for the task that pays enough (blue shaded regions).

Figure 3. *Effort Choices as a Function of Transfers*



Note: The pair (.,.) in given region corresponds to the optimal effort choice of the agent.

2.3.1. Principal's Problem

The principal i receives T_i from the citizen and offers t_i to the agent, which will be paid only if public good type i is produced with high quality. If the agent is successful in task i, principal i enjoys the rent from office, $R/2$. Therefore, the expected utility of the principal i can be expressed in the reduced form:

$$EU_{P_i}(t_i) = e_i(R/2 + T_i - t_i)$$

The problem of principal i is to choose $t_i \leq T_i$ to maximize EU_{P_i} given the agent's strategy in (13). However, principal i's strategy also takes into account principal -i's offer, t_{-i} for a given T_{-i}. Therefore, (t_1^*, t_2^*) will be a Nash equilibrium in which the principals best-respond to each other. To characterize the principals' strategies, we consider several cases, each with a given (T_1, T_2).

Case 1: $T_i \geq \delta(1+\gamma), \forall i \in \{1,2\}$. Given the agent's strategy in (13) this is the only case where outcome (1,1) follows: if $T_i < \delta(1+\gamma)$, but $T_{-i} \geq \delta(1+\gamma)$, principal i's offer $t_i \leq T_i$ will be too small to induce the agent's effort in both tasks. Therefore payoff maximizing transfers are: $t_i^* = \delta(1+\gamma), \forall i \in \{1,2\}$.

Case 2: $T_i \geq \delta(1+\gamma)$ but $T_{-i} < \delta(1+\gamma)$. Principal i can attract the agent with any $t_i = T_{-i} + \varepsilon$, then she will offer $t_i = T_{-i} + \varepsilon, \varepsilon \in [0, T_i - T_{-i})$. We assume that the equilibrium best responses are $t_i^* = T_{-i}$ and $t_{-i}^* = 0$, and that as a part of the agent's strategy he picks (1,0) when $i = 1$ and (0,1) when $i = 2$.

Case 3: $T_{-i} < \delta$ but $T_i \geq \delta$. The principal *-i*'s budget is too small to attract agent's effort in task *-i*. Then, principal i's best response is to offer $t_i^* = \delta$. The agent will pick (1,0) if $i = 1$ and (0,1) if $i = 2$.

Case 4: $T_i < \delta, \forall i \in \{1,2\}$. None of the principals can compensate the agent's effort; the outcome (0,0) follows and with no loss generality, we assume that the optimal offers are $t_i^* = 0, \forall i \in \{1,2\}$.

Case 5: $\delta \leq T_i < \delta(1+\gamma), \forall i \in \{1,2\}$, and $T_i > T_{-i}$. The principal i offers $t_i = T_{-i} + \varepsilon, \varepsilon \in [0, T_i - T_{-i}]$. Again the same reasoning as in Case 2 applies: the best responses are $t_i^* = T_{-i}$ and $t_{-i}^* = 0$ and as a part of his strategy the agent chooses (1,0) for $i = 1$ and (0,1) for $i = 2$.

Case 6: $\delta \leq T_i < \delta(1+\gamma), \forall i \in \{1,2\}$ and $T_i = T_{-i}$. Competition would wipe away the surplus in the principals' budgets: the equilibrium transfers are $t_i^* = T_i$, $\forall i \in \{1,2\}$, and the agent is indifferent between spending effort in task 1 or 2.

It is easy to check that (t_1^*, t_2^*) in all cases above satisfy the agent's participation constraint, $EU_A \geq 0$.(3)

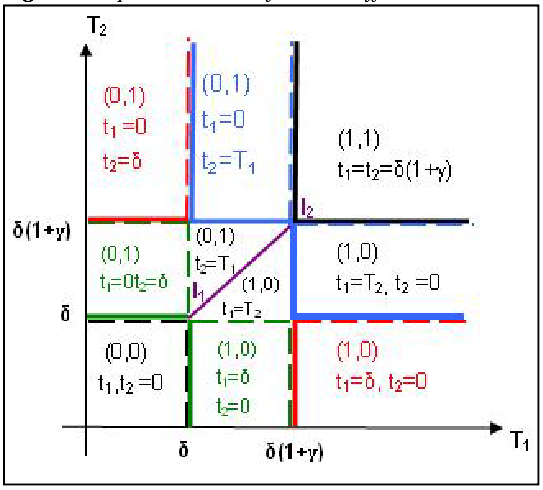
As in the previous section, we have a similar definition for corruption where the principal pockets some portion of his budget. It is clear that corruption is likely in all cases except in Case 4 and 6. In case 1, if $T_i > \delta(1+\gamma)$, both principals are corrupt if $T_i > \delta(1+\gamma), \forall i \in \{1,2\}$; only principal i if $T_i > \delta(1+\gamma), T_{-i} = \delta(1+\gamma)$. In cases 2, 3 and 5, principal i is corrupt if $T_i > \delta$.

Figure 4 represents the equilibrium transfers, (t_1^*, t_2^*) for a given (T_1, T_2), as well as the corresponding outcome that will be implemented. The equilibrium outcome is:

(1,1)	if	$min\{T_1, T_2\} \geq \delta(1+\gamma)$	
(1,0)	if	$T_1 \geq \delta \; and \; T_2 \leq \delta(1+\gamma)$	
(0,1)	if	$T_2 \geq \delta \; and \; T_1 \leq \delta(1+\gamma)$	(14)
either (1,0) or (0,1)	if	$\delta(1+\gamma) > T_1 = T_2 \geq \delta$	
(0,0)	if	$max(T_1, T_2) < \delta$	

Note that if $\delta(1+\gamma) > T_1 = T_2 \geq \delta$, the total budget is not sufficient to induce agent's effort on both tasks and competition between the principals results in $(t_1^*, t_2^*) = (\delta, \delta)$, where the agent will be indifferent spending effort either on task 1 or task 2. This case represented by the line segment I_1I_2 in Figure 4.

Figure 4. *Equlibrium Transfers and Effort Outcomes in Common Agency*



2.3.2. Citizen's Problem

We maintain our assumptions about the citizen's preferences over the public goods as in the single-principal model. If the agent succeeds in task 1, the citizen enjoys a high quality public good that she values $\alpha \in [0,1]$, and receives αq_1^h utility, where αq_1^h is the quality level of high quality public good type 1. Similarly she receives $(1-\alpha)q_2^h$ utility if a high quality public good type 2 is produced. Also we preserve Assumption 1, so that the citizen's taste favors public good type 1 over type 2 but the superiority of good 1 is bounded from below. If the agent fails in a task, a low quality public good with quality level zero is realized.

The expected payoff of the citizen can be expressed as follows:

$$EU_C^{CA} = e_1(\alpha q_1^h - T_1) + e_2((1-\alpha)q_2^h - T_2) \qquad (15)$$

The optimal contract (T_1, T_2) maximizes the expression in (15). Again, because of the assumption that the citizen is asymmetrically informed about the cost parameter δ and the transfers within the bureaucracy, she constructs her strategy by taking into account the possible outcomes that will arise as a result of her contract offers. In doing so, she considers the principals' strategies and the resulting outcome for a given (T_1, T_2) as derived in (14). We now use this insight to derive the optimal contract in Proposition 2.

Proposition 2. The optimal contract in common agency is $(T_1, T_2) = (\alpha q_1^h/2, (1-\alpha)q_2^h)/2))$.

Proof. See Appendix.

Comparing the two bureaucratic regimes, the total transfer which is paid when both type of public goods are of high quality, i.e. $T_1 + T_2$, is higher in common agency than the corresponding transfer in single-principal model, i.e. T^*: observe that the optimal T^* in Figure 2 is always below the sum of equilibrium transfers, $T_1 + T_2$, in Proposition 2. In other words, common agency overpays to achieve high quality in both type of public goods at the same time. The reason is that, in single-principal model the citizen is able to decrease optimal T^* by a fraction of the benefit from keeping the office, R. However, this advantage disappears in common agency since best responses of the principals do not depend on R as they compete to attract the agent's effort. Such competition, in turn, increases the agent's bargaining power, and requires a higher total compensation to induce his effort in both tasks. Therefore, common agency is not cost-efficient in terms of achieving high quality for both public goods.

Focusing on the optimal transfer which is paid only when one type of public good is produced with high quality, T_1 we observe that common agency pays less except when R is too low, $R < R_L$, and the optimal transfers are equal under the two regimes. The reason is, as R increases, the citizen reduces optimal T^* since it becomes less costly to incentivize the principal to exert her agent spending effort in both tasks. This induces the citizen to offer a higher T_1 at the optimum to increase her expected payoff.

3. A Comparative Assessment: Single or Multiple Bureaucrats at the Top-level?

So far we have analyzed the equilibrium separately in the two bureaucratic systems. Now we are ready to compare the efficiency of the two systems. We define the *efficient* bureaucratic system as the one which maximizes the citizen's welfare. Therefore, we compare the expected payoffs of the citizen in the two systems to identify the efficient regime.

Theorem 1. If there are positive rents from public office, a single-principal model is efficient against a common agency. If there are no such rents, the two systems are equally efficient.

Proof. Define $(R) \equiv EU^C_{SP} - EU^C_{CA}$, where EU^C_{SP} and EU^C_{CA} denote the equilibrium payoffs to the citizen in single-principal model and common agency, respectively. EU^C_{CA} is calculated in (28). But EU^C_{SP} depends on R. If $R > R_H, EU^C_{SP}$ is as calculated in (22). Then

$$\Phi(R) = \frac{(\gamma+1)(\alpha q_1^h)^2 + 4(\gamma+1)\alpha q_1^h(1-\alpha)q_2^h - 3((1-\alpha)q_2^h)^2}{4(\gamma+1)(4\gamma+7)\bar{\delta}}$$

Since $\alpha q_1^h > (1-\alpha)q_2^h$, $\Phi(R) > 0$. However, if $R_L < R < R_H$, then EU^C_{SP} is given in (21). Therefore

$$\Phi(R) =$$

$$= \frac{2R(\gamma+1)((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h) - ((1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h)^2 - (4\gamma+7)(\gamma+1)R^2}{4(\gamma+1)(\gamma+2)\bar{\delta}}$$

and

$$\Phi'(R) = \frac{2(\gamma+1)((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h) - 2(4\gamma+7)(\gamma+1)R}{4(\gamma+1)(\gamma+2)\bar{\delta}}$$

Note that $\Phi'(R) > 0$ for $R < R_H$. Also, we have

$$\Phi(R_L) = \frac{(\gamma+1)\alpha q_1^h + (4\gamma+5)(1-\alpha)q_2^h}{4(\gamma+1)(2\gamma+3)^2\bar{\delta}} > 0$$

then we conclude that $\Phi(R) > 0$, if $R_L < R < R_H$. Finally, if $R < R_L$, then EU_{SP}^C is as calculated in (26) and we have

$$\Phi(R) = \frac{R^2 + 2R(1-\alpha)q_2^h}{4(\gamma+1)\bar{\delta}}$$

Therefore $\Phi(R) > 0, \forall R > 0$. However, $\Phi(R) = 0$ when $R = 0$. ■

We conclude that, from the citizen's point of view, a single-principal model is favorable over a common agency unless there are no extra benefits to the principal(s) from keeping the public office. In a single-principal model, the rents from office, accruing only when high quality is achieved in both tasks, helps the citizen to decrease the optimal transfer paid in that instance. Common agency, on the other hand, creates a welfare loss by relatively overpaying the principals in total when they both succeed in producing high quality public good. The point is that since the principals compete to receive the agent's effort, their best responses are independent of the rent from public office and the citizen takes the best responses of the principals into account when determining the optimal transfers. Therefore, the citizen loses the opportunity to reduce the optimal transfer paid when high quality occurs in both tasks by fraction of the rent. If there are no rents, the cost advantage of the single-principal model in providing incentives disappears; i.e. the cost to incentivize the single principal to produce high quality in both tasks increases. In that case, the optimal transfers under the two regimes are equal and the two bureaucratic systems yield the same payoff to the citizen in equilibrium.

4. Discussion: Cooperation for Efficiency?

Our analysis so far has revealed that a multiplicity at the top level bureaucracy is not favorable for the citizen's welfare. A single-principal system; however, might not be feasible due to the additional costs as the number of tasks on which the top level has to deal with the middle-level increases. Such argument might account for the frequently observed common-agency systems; yet we illustrate in this section that cooperation in the top-level can potentially improve the welfare over what could be achieved under a single-principal system.

Suppose that we focus on a case where the citizen's benefits from a high quality public good type 2 is relatively quite high as we impose a stronger condition $\alpha q_1^h > (1-\alpha)q_2^h \geq \frac{(\gamma+2)\alpha q_1^h}{2\gamma+2}$ and the citizen offers the contract in Proposition 2.(4) By using our analysis in common agency, it is easy to derive the equilibrium outcomes and payoffs of the principals as a function of δ. Given that the citizen offers the contract in Proposition 2, we would like to see whether the principals prefer to cooperate in dealing with the middle-level bureaucrat in a way to affect the distribution of final outcomes in the public good production. This could be the case, for example, cooperation would yield outcome (1,1), instead of outcome (1,0) for a given δ. Specifically, under non-cooperative common agency equilibrium, outcome (1,0) follows for all $\delta \in (\frac{(1-\alpha)q_2^h}{2(\gamma+1)}, \frac{(1-\alpha)q_2^h}{2})$, and Principal 1 receives $\frac{\alpha q_1^h-(1-\alpha)q_2^h+R}{2}$, where Principal 2 receives zero. The same outcome also occurs in equilibrium for all $\delta \in (\frac{(1-\alpha)q_2^h}{2}, \frac{\alpha q_1^h}{2})$, but Principal 1 receives this time $\alpha q_1^h + R - \delta$. Under particular conditions, as we will shrotly illustrate, Principal 1 might find it profitable to cooperate with Principal 2 in contracting with the middle-level agent and induce outcome (1,1) instead of outcome (1,0), that could in turn results in higher payoff both for the principals and the citizen, and hence overall higher welfare.

As the principals jointly contract with the bureaucrat and can implement outcome (1,1) at a lower cost, a cooperation produces a pie of $S \equiv (\alpha q_1^h + (1-\alpha)q_2^h)/2 - \delta(\gamma+2)$ to divide. Let β be the Principal 1's share of the "pie" , and D_1 and D_2 the disagreement outcomes, of Principal 1 and 2, that could be achieved under non-cooperative equilibrium. Furthermore, suppose they split the pie over Nash-bargaining. Then β maximizes the Nash product

$$N = (R/2 + \beta S - D_1)(R/2 + (1-\beta)S - D_2)$$

Note that S must be non-negative for cooperation to take place. The solution is

$$\beta^* = \frac{2S + R + \alpha q_1^h - (1-\alpha)q_2^h}{4S}$$

Then evaluating the principals' surpluses at β, we find that both gain

$$\frac{1}{2}\left((1-\alpha)q_2^h + \frac{R}{2} - \delta(\gamma+2)\right) \tag{16}$$

Therefore, two conditions must be satisfied for cooperation to be sustained. First,

$$\delta \leq \frac{\alpha q_1^h + (1-\alpha)q_2^h}{2(\gamma+2)} \tag{17}$$

so that S is non-negative. Second

$$\delta \leq \frac{2(1-\alpha)q_2^h + R}{\gamma+2} \tag{18}$$

so that condition (16) holds and cooperation is attractive for both parties. In other words, via cooperation, outcome (1,1) replaces outcome (1,0), for all $\delta < \frac{\alpha q_1^h+(1-\alpha)q_2^h}{2(\gamma+2)}$ if

$R > \alpha q_1^h - (1-\alpha)q_2^h$. But if $R < \alpha q_1^h - (1-\alpha)q_2^h$, the same is true for all $\delta \leq \frac{2(1-\alpha)q_2^h+R}{\gamma+2}$. Note that

for all $\delta \in (min\left\{\frac{\alpha q_1^h+(1-\alpha)q_2^h}{2(\gamma+2)}, \frac{2(1-\alpha)q_2^h+R}{\gamma+2}\right\}, \frac{\alpha q_1^h}{2})$, cooperation cannot be sustained, therefore non-cooperative outcome, (1,0) prevails for this interval. Therefore, calculating the citizen's payoff yields

$$EU_C^{CA} = \frac{(\gamma+2)(\alpha q_1^h)^2+((1-\alpha)q_2^h)^2+2\alpha q_1^h(1-\alpha)q_2^h}{4(\gamma+2)\bar{\delta}} \qquad if\ R > \alpha q_1^h - (1-\alpha) \qquad (19)$$

and

$$EU_C^{CA} = \frac{\left(2(1-\alpha)q_2^h+R\right)\left(\alpha q_1^h+(1-\alpha)q_2^h\right)+\left((\gamma+1)\alpha q_1^h-(1-\alpha)q_2^h\right)\alpha q_1^h}{4(\gamma+2)\bar{\delta}} \quad if\ R < \alpha q_1^h - (1-\alpha)q_2^h \qquad (20)$$

Now we would like to see whether the citizen benefits from a potential cooperation at the top-level. Recall that, by Proposition 1, if $R < R_L$, the citizen receives the payoff EU_C^{SP4} under single-principal system. Note that $\alpha q_1^h - (1-\alpha)q_2^h < R_L$. Then comparing the payoffs, it is easy to check that EU_C^{SP4} is lower than the payoffs in (19) and (20) under the condition $(1-\alpha)q_2^h \geq \frac{(\gamma+2)\alpha q_1^h}{2\gamma+2}$. However, again by Proposition 1, if $R_L < R < R_H$, the optimal contract yields EU_C^{SP2} and, if $R > R_H$, it yields EU_C^{SP3}. But we observe that the payoff in (19) is lower than both EU_C^{SP2} and EU_C^{SP3}, hence, one more time, a common agency is not favorable against a single principal system.

This partial analysis illustrates that the rents from office, R and the citizen's relative benefit from the two public goods have an impact on the likelihood of cooperation. Given that the benefits from the two public goods is close, although one type always provides the highest benefit, cooperative common agency is likely to deliver higher welfare if R is small enough. Recall that the superiority of a single-principal system comes from the availability of a bonus contract that drastically reduces the cost of incentive provision to the top-level due to the fact that lower incentives are required when there are positive rents from office. This cost advantage becomes less significant as R decreases (see the optimal bonus in Figure 2) and the cooperative outcome can potentially produce favorable results. If we concentrate on the cooperative case alone, a larger R increases the likelihood that it will be sustained; specifically when the term in (16) is positive: cooperation seems attractive for principal 2 to secure the rents from public office which is less likely to accrue under non-cooperative equilibrium. This is indeed the case when δ is relatively high and only principal 1, with the higher transfer offer, can induce high quality public good production. Such coalition is also profitable for principal 1 provided that the size of total transfer increases sufficiently when they cooperate, i.e. when condition (17) holds, so that she can get a larger slice in case of high quality production in both goods.

5. Concluding Remarks

The political economy literature points out the agency problem between the voters as principals and the politicians as their agents: the voters face the problem of designing the best incentive scheme for elected politicians who might not be acting in voters' interest when there are informational asymmetries. Dixit (2006) incorporates this conventional modelling of political agency problem with the one which exists within bureaucracy. My model takes this three-tier structure one step further by allowing a multiplicity in the middle-tier; i.e. a common agency. One practical example for such bureaucratic system is the case of a municipality which has to take into account the concerns of different ministers while taking an action. I attempt to explore the welfare implications of a common agency within bureaucracy in comparison to a single-principal model where there is only one principal acting as a top-level bureaucrat.

The model involves two principals at the top level of bureaucracy and, at the lower level, their common agent who is in charge of producing public goods. The public goods, which could be either high or low quality depending on the agent's unobservable effort, are consumed by the citizen. There are two sets of contracts; one is offered by a principal to the agent, the other is offered by the citizen to a principal. The problem that the contract between the agent and the principal being unobserved by the citizen creates room for corruption.

I first consider the model with a single principal. The single-principal model allows a bonus contract in equilibrium where the citizen pays a higher transfer to the principal, when both goods are of high quality, than the sum of individual transfers when only one type of good is high quality. I find that the optimal contract offered by the citizen depends on the rents from public office which accrues to the principal only when she succeeds in inducing her agent to produce both public goods with high quality. Furthermore, as the rents get larger, the citizen offers a lower bonus transfer but a higher transfer which is paid only if one type of good is of high quality. Common agency, however, produces higher costs to the citizen since the optimal transfer; i.e. the sum of transfers paid when both goods are high quality, cannot be reduced in the same way as in a single-principal model. Since a common agency has a cost disadvantage in this sense, a single-principal model is favorable from the citizen's point of view. However, if there are no rents from public office, the two regimes are equally efficient. This analysis points out an important policy implication: different decision-making bodies within bureaucracy should cooperate in policy implementation to achieve welfare-optimal outcomes. Any coordination failure would drive up the costs in public good production and lead to suboptimal transfers, hence a welfare loss.

Notes

(1) It is obvious that the reservation payoff coincides with the payoff that the agent would receive by exerting (0,0). Hence if (1,1) or (1,0) or (0,1) is optimal, then the payoff from this effort choice yields at least the reservation payoff; i.e participation constraint is satisfied. If (0,0) is the optimal effort choice, the agents exactly receives the reservation payoff; i.e. participation constraint is again satisfied.

(2) Relaxing the second inequality in Assumption 1 yields two more cases to study in the citizen's optimization problem without adding much insight since single-principal model always benefits the citizen more. But it also trivially undermines the performance of the common agency against the single-principal system. We will show that under Assumption 1, cooperation in common agency is sustained and potentially produces higher welfare relative to the single-principal system.

(3) The participation constraint will be satisfied in all cases. See footnote 1.

(4) Note that this is a stronger condition than Assumption 1, as we have already assumed. This assumption guarantees that cooperation can both be sustained and achieve higher payoff for the citizen.

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Appendix

Proof of Proposition 1. By Lemma 1 and Lemma 2, optimal contract sets $T_2 < T_1$ and $\frac{T^*}{2+\gamma} < T_1$. Then, by Remark 1 outcome (0,1) will never be implemented. Regarding the two outcomes, (1,0) and (1,1), both are feasible if $\delta \leq \frac{T^*}{2+\gamma}$. When this is the case, as illustrated in Example 1, the relative size of $\frac{R+T^*-T_1}{1+\gamma}$ with respect δ is important to determine which outcome will be implemented. Therefore, in Step 1 below, we consider different contract options and in each we determine the best contract. In Step 2, we characterize the optimal contract.

Step 1. There are two options.

Option 1: Suppose that the citizen offers T^* and T_1 such that $\frac{T^*}{2+\gamma} \leq \frac{R+T^*-T_1}{1+\gamma}$. The feasible outcomes and the induced outcome for a given range of δ are as illustrated in Option 1 in Figure 5. If $\delta \leq \frac{T^*}{2+\gamma}$, then $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$ also holds, i.e. the principal strictly prefers outcome (1,1) over outcome (1,0) when both are feasible. If $\frac{T^*}{2+\gamma} < \delta \leq T_1$, however, the only feasible outcomes are (1,0) and (0,0), and the former outcome will be implemented since it yields a higher payoff to the principal. If $\delta > T_1$, the principal will implement (0,0).Therefore, Option 1 induces the outcome (1,1) with probability $F(\frac{T^*}{2+\gamma})$, (1,0) with probability $F(T_1) - F(\frac{T^*}{2+\gamma})$, and (0,0) with probability $1 - F(T_1)$. Clearly, we set $T_2 = 0$ at the optimum. Then optimal (T_1, T^*) that maximizes the citizen's expected payoff can be expressed as follows:

$$EU_C^{SP} = F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - F(\frac{T^*}{2+\gamma})](\alpha q_1^h - T_1)$$

such that

$\frac{T^*}{2+\gamma} \leq \frac{R+T^*-T_1}{1+\gamma}$ and $T^* \geq T_1$

Let $\mathcal{L}(T^*, T_1, \lambda)$ denote the Lagrangian function where λ is a Lagrange multiplier.

$$\begin{aligned}\mathcal{L}(T^*, T_1, \lambda) \ = \ & F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - F(\frac{T^*}{2+\gamma})](\alpha q_1^h - T_1) \\ & + \lambda((2+\gamma)R + T^* - (2+\gamma)T_1) + \mu(T^* - T_1)\end{aligned}$$

The Kuhn-Tucker conditions for this constrained maximization problem are:

$$\frac{1}{\delta}((2+\gamma)\alpha q_1^h - 2(2+\gamma)T_1 + T^*) - (2+\gamma)^2\lambda - (2+\gamma)\mu \ = \ 0$$

$$\frac{1}{\delta}((1-\alpha)q_2^h - 2T^* + T_1) + (2+\gamma)(\lambda + \mu) \ = \ 0$$

and

$(2+\gamma)R + T^* - (2+\gamma)T_1 \ \geq \ 0,\ \lambda \geq 0$ and $\lambda((2+\gamma)R + T^* - (2+\gamma)T_1) = 0$

$T^* - T_1 \ \geq \ 0, \mu \geq 0$ and $\mu(T^* - T_1) = 0$

For this option, we will consider possible cases for whether one or two constraints are binding to investigate whether they arise in equilibrium under which conditions.

Case 1. $\lambda, \mu > 0$. In that case, both constrains should be binding, i.e., $(2+\gamma)R + T^* - (2+\gamma)T_1 = 0$,and $T^* = T_1$. Using the Kuhn-Tucker Conditions we get

$$T_1 = T^* = \frac{\gamma+2}{\gamma+1}R \quad \text{(C1)}$$

$$\lambda \ = \ \frac{1}{(\gamma+1)(\gamma+2)\bar{\delta}}((\gamma+2)\alpha q_1^h + (1-\alpha)q_2^h - \frac{2(\gamma+2)^2 R}{\gamma+1}$$

$$\mu \ = \ \frac{1}{(\gamma+1)\bar{\delta}}(\frac{3\gamma+5}{\gamma+1}R - \alpha q_1^h - (1-\alpha)q_2^h)$$

The contract in (C1) satisfies the constraint $\mu > 0$ if and only $if\ R \geq \tilde{R}$ where $\tilde{R} \equiv \frac{(\gamma+1)(\alpha q_1^h + (1-\alpha)q_2^h)}{3\gamma+5}$. However, if $R \geq \tilde{R}, then\ \lambda < 0$; i.e., these inequalities cannot hold at the same time since $(1-\alpha)q_2^h > \frac{\gamma+2}{2\gamma+3}\alpha q_1^h$. Therefore, there exists no R such that the contract in (C1) is a solution; i.e. both constraints cannot be binding at the same time.

Case 2. $\lambda > 0, \mu = 0$. Then the condition $(2+\gamma)R + T^* - (2+\gamma)T_1 = 0$ must hold in equlibrium. The first-order conditions yield

$$T_1 \ = \ \frac{\alpha q_1^h + (1-\alpha)q_2^h + (2\gamma+3)R)}{(2(\gamma+2)} \ , T^* \ = \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2} \quad \text{(C2)}$$

$$\lambda \ = \ \frac{(2\gamma+3)\alpha q_1^h - (1-\alpha)q_2^h - (4\gamma+7)R}{\bar{\delta}(4\gamma+7)};\ \mu = 0$$

λ is positive if $R < R_{H.}$ Also, the contract C2 satisfies the constraint $T^* > T_1$ if $R < R_H$. Therefore, C2 is a solution if $R < R_H$.

Case 3. $\lambda = 0, \mu > 0$. This implies $T_1 = T^*$should hold in equlibrium. Imposing this and the specified λ and μ into first-order conditions yield $T_1 = T^* = \frac{(\gamma+2)\alpha q_1^h + (1-\alpha)q_2^h}{2(\gamma+2)}$

$$\mu = \frac{(\gamma+2)\alpha q_1^h - (2\gamma+3)(1-\alpha)q_2^h}{2(\gamma+2)\bar{\delta}}$$

Therefore, this case can never arise in equilibrium.

Case 4. $\lambda = 0, \mu = 0$. Imposing these conditions into first-order conditions, we get:

$$T_1 \ = \ \frac{2(\gamma+2)\alpha q_1^h + (1-\alpha)q_2^h}{4\gamma+7} , T^* \ = \ \frac{(\gamma+2)\alpha q_1^h + 2(1-\alpha)q_2^h}{4\gamma+7} \quad \text{(C3)}$$

Notice that contract C3 satisfies the constraint $(2+\gamma)R + T^* - (2+\gamma)T_1 > 0$ if $R > R_H$. Also, Assumption 1ensures that C3 satisfies the constraint $T^* > T_1$.

Then, restricting the contract set to *Option 1*, the best contract is C2 if $R \leq R_H$, and it is C3 if $R > R_H$. The citizen's expected payoffs from contracts C2 and C3 are respectively:

$$EU_C^{SP2} = \frac{(\alpha q_1^h + (1-\alpha)q_2^h)^2 + 2R((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h) - (4\gamma+7)R^2}{4(2+\gamma)\bar{\delta}} \tag{21}$$

$$EU_C^{SP3} = \frac{(\gamma+2)(\alpha q_1^h)^2 + \alpha(1-\alpha)q_1^h q_2^h + ((1-\alpha)q_2^h)^2}{(4\gamma+7)\bar{\delta}} \tag{22}$$

Option 2. Suppose that the citizen offers T_1 and T^* such that $\frac{R+T^*-T_1}{1+\gamma} \leq \frac{T^*}{2+\gamma}$. The feasible outcomes and the induced outcome for a given range of δ are the same as illustrated in *Option 2* in Figure 5. Therefore, the outcome (1,1) will be implemented with probability $F(\frac{R+T^*-T_1}{1+\gamma})$; (1,0) with probability $[F(T_1) - F(\frac{R+T^*-T_1}{1+\gamma})]$ and (0,0) with probability $[1 - F(T_1)]$. T^* and T_1 will be chosen to maximize the expected payoff $EU_C^{SP} = F(\frac{R+T^*-T_1}{1+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - F(\frac{R+T^*-T_1}{1+\gamma})](\alpha q_1^h - T_1)]$

such that

$\frac{T^*}{2+\gamma} \geq \frac{R+T^*-T_1}{1+\gamma}$ and $T^* \geq T_1$

The Lagrangian function for this optimization problem is

$$\mathcal{L}(T^*, T_1, \lambda) = F(\frac{R+T^*-T_1}{1+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - \frac{R+T^*-T_1}{1+\gamma}](\alpha q_1^h - T_1) - \lambda((2+\gamma)R + T^* - (2+\gamma)T_1) + \mu(T^* - T_1)$$

The Kuhn-Tucker conditions for this constrained maximization problem are:

$$\frac{1}{\bar{\delta}(\gamma+1)}(2T_1 - 2T^* + (1-\alpha)q_2^h - R) - \lambda + \mu = 0$$

$$\frac{1}{\bar{\delta}(\gamma+1)}(R + (1+\gamma)\alpha q_1^h + 2T^* - 2(2+\gamma)T_1 - (1-\alpha)q_2^h) + (2+\gamma)\lambda - \mu = 0$$

and

$(2+\gamma)R + T^* - (2+\gamma)T_1 \leq 0, \lambda \geq 0, and\ \lambda((2+\gamma)R + T^* - (2+\gamma)T_1) = 0$

$T^* - T_1 \geq 0, \mu \geq 0\ and\ \mu(T^* - T_1) = 0$

For this option, again, we will consider possible cases for whether one or two constraints are binding to investigate whether they arise in equilibrium under which conditions.

Case 1. $\lambda, \mu > 0$. Since this case coincides with the case in *Option 1* when we have binding constraints, payoff maximizing (T_1, T^*) is the same as derived in C1. Using the Kuhn-Tucker conditions, we get

$$\lambda = \frac{2(\gamma+2)R-(\gamma+1)\alpha q_1^h}{\bar{\delta}(\gamma+1)^2} \tag{23}$$

$$\mu = \frac{(3\gamma+5)R-(\gamma+1)(\alpha q_1^h+(1-\alpha)q_2^h)}{\bar{\delta}(\gamma+1)^2} \tag{24}$$

Observe that if $R > \tilde{R}$, where $\tilde{R} > R_H$, then (23) implies $\lambda, \mu \geq 0$. Therefore C1 and (23) could be a solution to the maximization problem if and only if $R > \tilde{R}$.

Case 2. $\lambda > 0, \mu = 0$. This implies $(2+\gamma)R + T^* - (2+\gamma)T_1 = 0$ must hold in equilibrium. This, together with first-order conditions yield again contract C2:

$$(T_1, T^*) = \left(\frac{\alpha q_1^h+(1-\alpha)q_2^h+(2\gamma+3)R}{2(\gamma+2)}, \frac{\alpha q_1^h+(1-\alpha)q_2^h-R}{2}\right)$$

$$\lambda = \frac{(2\gamma+3)R-(\gamma+1)\alpha q_1^h+(1-\alpha)q_2^h}{\bar{\delta}(\gamma+1)(\gamma+2)}$$

Again, λ is positive if and only if $R > R_L$. Also, contract C2 satisfies the optimization constraint $T^* > T_1$ if $R < \tilde{R}$. Thefore, C2 could be a solution is if and only if $R_L < R < \tilde{R}$.

Case 3. $\lambda = 0, \mu > 0$. Imposing this case into the first order condtions, we get $T_1 = T^* = \frac{\alpha q_1^h}{2}$, and $\mu = \frac{R-(1-\alpha)q_2^h}{\bar{\delta}(\gamma+1)}$, which could be a solution if and only if $R > (1-\alpha)q_2^h$. However, notice that plugging (T_1, T^*) back into the optimization contraint, $(2+\gamma)R + T^* - (2+\gamma)T_1 < 0$, implies that $R < \frac{(\gamma+1)\alpha q_1^h}{2(\gamma+2)} < (1-\alpha)q_2^h$. Therefore, we conclude that this case cannot arise in equilibrium.

Case 4. $\lambda = 0, \mu = 0$. Once again, from the first order conditions, we get

$$T_1 = \frac{\alpha q_1^h}{2}, \quad T^* = \frac{\alpha q_1^h+(1-\alpha)q_2^h-R}{2} \tag{C4}$$

Observe that only if $R < (1-\alpha)q_2^h$, the contract C4 satisfies the optimization constraint, $T^* > T_1$. Also, to satisfy the constraint $(2+\gamma)R + T^* - (2+\gamma)T_1 < 0$, $R < R_L$ must hold. Since $R_L < (1-\alpha)q_2^h$, contract C4 is a solution if and only if $R < R_L$.

Summarizing Cases 1-4: Restricting the contract set to Option 2, the best contract is C1 if $R > \tilde{R}$, and it is C2 if $R_L < R < \tilde{R}$, and finally C4 if $R < R_L$. The citizen's expected payoffs from contracts C2 as calculated in (21) and contracts C1 and C4 are respectively:

$$EU_C^{SP1} = \frac{R((\gamma+2)(\gamma+1)\alpha q_1^h+(\gamma+1)(1-\alpha)q_2^h-(\gamma+2)^2R)}{(\gamma+1)^2\bar{\delta}} \tag{25}$$

$$EU_C^{SP4} = \frac{(R+(1-\alpha)q_2^h)^2+(\gamma+1)(\alpha q_1^h)^2}{4(\gamma+1)^2\bar{\delta}} \tag{26}$$

Step 2. We compare the expected payoffs from contracts C1 through C4, i.e. EU_C^{SP1}, EU_C^{SP2}, EU_C^{SP3}, and EU_C^{SP4}, respectively. The optimal contract maximizes the expected payoff of the citizen. Now if $R < R_L$, Option 1 yields EU_C^{SP2} and Option 2 yields EU_C^{SP4}. Define $F(R) \equiv EU_C^{SP4} - EU_C^{SP2}$. Then $F(R)$ can be simplified as:

$$F(R) = \frac{((2\gamma+3)R + (1-\alpha)q_2^h - (\gamma+1)\alpha q_1^h)^2}{4(\gamma+1)(\gamma+2)\bar{\delta}}$$

Observe that $F(R)$ is convex and $F'(R) = 0$ at $R = R_L$. Therefore, $F(R) > 0$ for all $R < R_L$; i.e., Option 2 yields a higher expected payoff when R is sufficiently small. Hence, optimal contract is C4 if $R < R_L$.

For all $R \in (R_L, R_H)$, Option 1 and Option 2 both yields EU_C^{SP2}.

For all $R \in (R_H, \tilde{R})$, Option 1 yields EU_C^{SP3} and Option 2 yields EU_C^{SP2}. First observe that EU_C^{SP2} is concave in R and $\partial EU_C^{SP2}/\partial R < 0$ for all $R > R_H$ and $\partial EU_C^{SP2}/\partial R = 0$ at $R = R_H$. Therefore, EU_C^{SP2} is maximized at $R = R_H$. Simple calculation yields that EU_C^{SP2}, evaluated at $R = R_{M2}$ is equal to EU_C^{SP3}. This, together with that EU_C^{SP3} is independent of R, implies $EU_C^{SP2} \leq EU_C^{SP3}$ for all R. Then we conclude that *Option 1* yields a higher expected payoff and thus C3 is the optimal contract for all $R \in (R_H, \tilde{R})$.

Finally if $R > \tilde{R}$, *Option 1* yields EU_C^{SP3} whereas *Option 2* yields EU_C^{SP1}. Again, observe that EU_C^{SP1} is concave in R and maximized at $R = \hat{R} < \tilde{R}$, where $\hat{R} \equiv \frac{(\gamma+1)(\gamma+2)\alpha q_1^h + (\gamma+1)(1-\alpha)q_2^h}{2(\gamma+2)^2}$. Then EU_C^{SP1}, evaluated at value-maxizing R is

$$EU_C^{SP1} = \frac{(\gamma+2)\alpha q_1^h + (1-\alpha)q_2^h)^2}{4\bar{\delta}(\gamma+2)^2}$$

Clearly, $EU_C^{SP1} < EU_C^{SP3}$. Again, since EU_C^{SP3} is independent of R, we conclude that $EU_C^{SP1} < EU_C^{SP3}$ for all R. Therefore, *Option 1* yields a higher expected payoff if $R > \tilde{R}$ and hence contract C3 is optimal. ∎

Proof of Proposition 2. Step 1. There are three contract options.

Option 1: Suppose the citizen offers (T_1, T_2) such that $T_1 > T_2$. Then $T_1 > \frac{T_2}{\gamma+1}$. From (14), the outcome will be (0,0) if $\delta > T_1$; (1,0) if $\frac{T_2}{\gamma+1} < \delta \leq T_1$, and (1,1) if $\delta < \frac{T_2}{\gamma+1}$. Figure 5 illustrates the equilibrium outcome as δ changes. Then the citizen's expected payoff is:

$$EU_C^{CA}(T_1, T_2) = F\left(\frac{T_2}{\gamma+1}\right)(\alpha q_1^h + (1-\alpha)q_2^h - T_1 - T_2) + \left(F(T_1) - F\left(\frac{T_2}{\gamma+1}\right)\right)(\alpha q_1^h - T_1)$$

First order conditions yield

$$(T_1, T_2) = \left(\frac{\alpha q_1^h}{2}, \frac{(1-\alpha)q_2^h}{2}\right) \qquad (27)$$

The citizen's expected payoff from contract in equation (27) is:

$$EU_C^{CA}(T_1, T_2) = \frac{(\gamma+1)(\alpha q_1^h)^2 + ((1-\alpha)q_2^h)^2)}{4(\gamma+1)\overline{\delta}} \quad (28)$$

Option 2: Suppose the citizen offers (T_1, T_2) such that $T_2 > T_1$. Then $T_2 > \frac{T_1}{\gamma+1}$. From (14), the outcome will be (0,0) if $\delta > T_1$; (0,1) if $\frac{T_1}{\gamma+1} < \delta \leq T_2$, and (1,1) if $\delta \leq \frac{T_1}{\gamma+1}$. Figure 5 illustrates the equilibrium outcome as δ changes. Then the citizen's expected payoff is:

$$EU_C^{CA}(T^1, T^2) = F(\frac{T^1}{\gamma+1})(\alpha q_1^h + (1-\alpha)q_2^h - T_1 - T_2) + (F(T_2) -$$

$$-F(\frac{T_1}{\gamma+1})((1-\alpha)q_2^h - T_2)$$

Using first order conditions, the solution is again (27). However, under our Assumption 1, the contract in (27) does not satisfy the constraint $T_2 > T_1$. Therefore, this case cannot arise.

$$EU_C^{CA}(T_1, T_2) = \frac{(\gamma+1)((1-\alpha)q_2^h)^2 + (\alpha q_1^h)^2}{4(\gamma+1)\overline{\delta}} \quad (29)$$

Option 3: Suppose the citizen offers (T_1, T_2) such that $T_1 = T_2 \equiv T$. From (14), the outcome will be (0,0) if $\delta > T$; (1,0) or (0,1) if $\frac{T}{\gamma+1} < \delta \leq T$, and (1,1) if $\delta \leq \frac{T}{\gamma+1}$. Figure 5 illustrates the equilibrium outcome as δ changes. Then the citizen's expected payoff is:

$$EU_C^{CA2}(T^1, T^2) = F(\frac{T}{\gamma+1})(\alpha q_1^h + (1-\alpha)q_2^h - 2T) + (F(T)$$

$$-F(\frac{T}{\gamma+1})(\frac{\alpha q_1^h + (1-\alpha)q_2^h}{2} - T)$$

First order conditions yield

$$T = \frac{\alpha q_1^h + (1-\alpha)q_2^h}{2} \quad (30)$$

The citizen's expected payoff from this contract is:

$$EU_C^{CA3}(T_1, T_2) = \frac{(\gamma+2)(\alpha q_1^h + (1-\alpha)q_2^h)^2}{16(\gamma+1)\overline{\delta}} \quad (31)$$

Step 2. The contract in (30) is not optimal under Assumption 1: the expected payoff in equation (31) is less than in equation (28). Therefore, the equilibrium payoff of the citizen is as derived using *Option 1* in (28). ■

Figure 5. *Equilibrium Outcomes for Options 1-3 in Proposition 2*