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Predicting probability of default of Indian companies: A market based approach

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Abstract. *The paper models default probabilities for Indian companies in Black-Scholes-Metron (BSM) framework. The objective Probability of Default (PD) estimates are found to be higher for firms registered with Board of Industrial and Financial Reconstruction (BIFR). The proposed method can be applied to obtain direct PD estimates of companies to track their default status, calculate credit capital and corporate pricing by investors and financial institutions.*

Keywords: Credit Risk, BSM Model, Indian Companies, Probability of Default.

JEL Classification: G33, G32.

1. Introduction

Assessing financial position of borrower or group of borrower is a crucial task for lender and investors. There is always chance a borrower willingly or unwillingly will not meet its contractual obligation in a credit transaction. Since Beaver (1966), there has been an enormous development in the literature of bankruptcy prediction. Bankruptcy prediction literature is developed in the two directions. First, the models based upon accounting based information, namely, Altman (1966), Ohlson (1980), Zmijewski (1984) etc. Second, models based on structural and reduced form approach. The structural models are based upon Black and Scholes (1973) option pricing theory which was extended by Metron (1974) to model default. In this setting a firm can default on its debt obligation only at the time of maturity. Some of the notable study on BSM framework was done by Agarwal and Taffler (2008), Wu, Gaunt and Gray (2010), Hillegeist et al. (2004), Bharath and Shumway (2008). Later, some extension is done by allowing default to occur prior to the date of maturity. These models were introduced by Black and Cox (1976), Lonfstaff and Schwartz (1995), Leland and Toft (1996). On the other hand reduced form models or hazard models focus over modelling default explicitly as an intensity or compensator process. This approach was first introduced by Jarrow and Turnbull (1995) and further extended by Duffie and Singleton (1999) and Lando (1994).

In India there is limited research done on bankruptcy predictions because of absence of bankruptcy law and lack of market based financial information on the firms. BIFR was formed in the year 1987 where the firm can registered sick if their accumulated losses exceeds its net worth. Default prediction study in India uses BIFR reference to identify distress firms. Some of the notable market based studies in India based upon BIFR reference is Varma and Raghunathan (2000), Kulkarni et al (2005). The current study is an attempt to apply BSM framework to model default probabilities on larger samples of Indian listed companies using BIFR reference.

The remainder of the paper is organized as follows. Section 2 provides information on data and methodology. Empirical results and model validation is covered in section 3 and the study concludes with section 4.

2. Data and methodology

The study uses BIFR reference to identify distressed firms from the list of firm's registered sick firms during 2006 to 2014. The study uses a total of 80 companies comprising 30 distressed and 50 non-distressed firms. Financial information of the companies is collected from their balance sheet and income statements at the end of each year from their respective website. The data on stock prices of the listed companies are taken from Bombay Stock Exchange (BSE). Information related to risk free return is collected from Reserve Bank of India (RBI) publication.

The BSM model is based on the option pricing theory of Black and Scholes (1973). In the model the value of the equity of firms' assets is modelled as a call option, with the face value of debt as exercise price and debt maturity as the option's time to maturity, and the value of the debt is modelled as a put option. This model is based on certain assumptions: (i) The firm only default at the time of maturity T. (ii) Firm's asset values follow lognormal distribution. (iii) The firm's assets V is financed by equity, E and debt F, where the total value of the firm's debt (D) consists of one non-callable zero-coupon bond with face value F. Hence, if at maturity T, the firm's asset value V is enough to pay back the face value of the debt F, i.e. if V > F, the firm does not default and the shareholders receive V-F. Otherwise, if V < F, the firm defaults, bondholders takes control of the firm, and the shareholders receive nothing.

From above underlying assumptions, the value of equity in the Black-Scholes-Merton (BSM) framework is given by:

E = max (0, V-F).

Apart from other parameters in order to calculate distance to default (DD) and Probability of default (PD) we need to find out the total market value of the firm and its volatility. The total market value of the firm is defined as the sum of the market value of the firm's debt and the value of its equity. In market equity values are readily available, reliable data on market value of debt is generally unavailable. The BSM model solves this problem by assuming the total value of a firm follows a geometric Brownian motion:

$$dV = \mu V dt + \sigma_V V dZ \,, \qquad (1)$$

Where:
V is the total value of the firm;
μ is expected continuously compounded return on V;
σ_V is volatility of firm value;
dZ is a standard Wiener process.

Again, by Black-Scholes formula equity value of a firm as a call option is given as:

$$E = VN(d_1) - Fe^{-rT} N(d_2) \qquad (2)$$

Where, $$d_1 = \frac{\ln(V/F) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \qquad (3)$$

$$d_2 = \frac{\ln(V/F) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \qquad (4)$$

Or, $d_2 = d_1 - \sigma_V \sqrt{T}$

r and *T* are risk-free rate and time to maturity respectively. *N ( )* is the cumulative standard normal distribution function.

Under the risk neutral probability measure, the default probability (PD) is given by:

$$N(-d_2) = N\left[-\frac{\ln(V/F) + (r - \sigma_V^2/2)T}{\sigma_V\sqrt{T}}\right]. \tag{5}$$

If the risk free interest rate r is replaced in (5) with the expected return on the asset value or the 'drift' of the asset value, μ_V, the distance to default measures can be obtained, which is:

$$DD = \frac{\ln(V/F) + (\mu_V - \sigma_V^2/2)T}{\sigma_V\sqrt{T}}. \tag{6}$$

And the corresponding probability of default (PD) of the firm as per Black-Scholes-Merton (BSM) model is:

$$N(-\hat{d}_2) = N\left[-\frac{\ln(V/F) + (\mu_V - \sigma_V^2/2)T}{\sigma_V\sqrt{T}}\right]. \tag{7}$$

The equation (7) shows that the probability of bankruptcy is a function of the distance between the current values of the firm's assets and face value of its liabilities (V/F) adjusted for the expected growth in asset values $(\mu_V - \sigma_V^2/2)$ relative to asset volatility (σ_v). The state of default occurs when, at maturity, the value of the firm is below the face value of debt, that is when $V \leq F$. Equation (2) is derived under the assumption of risk-neutrality where all assets are expected to grow at the risk-free rate, hence PD based on the risk free rate will provide the risk neutral probability of default as shown in equation (5). However, the probability of bankruptcy depends upon the actual distribution of future asset values, which is a function of the expected return on asset values (μ_V). When the objective is to assess credit risk of various positions, and not to price contingent claims, the objective or 'real' default probability has to be used. Hence, the prime objective of credit risk model is to find the real or objective probabilities of default.

Equation (7) includes three unknowns, namely, V, σ_V and μ_V. We need to find all three parameters in order to obtain an estimate of (7). In order to identify the above unknowns, the model invokes the Weiner process to model equity value, E.

$$E = \mu_E E dt + \sigma_E E dZ. \tag{8}$$

Where, μ_E is expected continuously compounded return on E, σ_E is the volatility of equity value and dz is a standard Weiner process. By the Ito's lemma, we can also represent the process for equity as:

$$dE = \left(\frac{\partial E}{\partial t} + \mu_V V\frac{\partial E}{\partial V} + \frac{1}{2}\sigma_V^2 V^2\frac{\partial^2 E}{\partial V^2}\right)dt + \sigma_V V\frac{\partial E}{\partial V}dZ \tag{9}$$

Since the diffusion terms in the equity process in (8) and (9) are equal, we can write the following relationship:

$$\sigma_E E = \sigma_V V \frac{\partial E}{\partial V} = \sigma_V VN(d_1) . \quad (10)$$

Equation (2) and (10) complete the system of two simultaneous nonlinear equations with two unknowns, *V* and σ_V ; and the k, parameters are *E*, σ_E , *r*, *F* and *T.* Hence the above two unknowns can be obtained by solving two equations (2) and (10) simultaneously using the solver routine in Microsoft excel.

Having found asset value *V, and* its volatility σ_V , the next step is finding the drift of asset value, which is expected market. Return on asset (μ_V). μ_V is estimated from market value of asset of current year estimated by solving equation 2 and 10 and market value of asset of previous year. In many cases, the actual return on assets is negative. As mentioned in Hillegeist et al. (2004), since expected returns cannot be negative, we can set the expected growth rate equal to the risk-free rate in such cases. Hence he suggested that μ_V can be calculated as:

$$\mu_v(t) = \max\left[\frac{V(t) - V(t-1)}{V(t-1)}, r\right] . \quad (11)$$

Now using the known parameters (*E*, σ_E , *r*, *F* and *T*) and estimated unknown parameters (*V,* σ_V and μ_V), one can estimate real or objective probability of default using equation (7).

3. Empirical results

The summary statistics and default probabilities for all defaulted and non-defaulted firms are reported in Table 1. Risk free rate of return (r) is measured as the 10-year government securities yield. Equity return volatility (σ_E) is calculated as the annualized standard deviation of daily returns during the given year.

Table 1. *BSM Model Summary - All Companies*

Variable	Mean	Standard deviation	Minimum	Maximum
Market value of equity (E)	196.752	478.733	1.742	3141.926
Face value of debt (F)	260.686	350.815	7.080	1760.195
Risk free rate (r)	0.077	0.004	0.071	0.084
Equity return volatility (σE)	0.667	0.349	0.279	3.360
Market value of firm asset (V)	457.437	695.076	16.439	4675.481
Asset volatility (σ_V)	0.272	0.294	0.029	2.480
Expected return on asset (μ_V)	0.208	0.291	0.071	1.529
Probability of default (PD)	5.111	11.762	0.000	98.960

Source: Author's estimation.

It is clear from Table 1 wide range of firms are considered in the study in terms of their market value of equity and face value of debt. The firms with maximum and minimum market value of equity and face value of debt are 3141.96 and 1760.195, and 1.742 and 7.080 respectively. Market value of equity, face value of debt and market value of firm asset are found to be highly volatile. The mean objective PD for all firms is 5.111 with 0.000 minimum and 98.960 maximum values respectively.

Table 2. *BSM Model Summary - Distressed Companies*

Variable	Mean	Standard deviation	Minimum	Maximum
Market value of equity (E)	20.329	22.068	1.742	100.3254
Face value of debt (F)	279.402	378.137	14.640	1760.195
Risk free rate (r)	0.078	0.004	0.071	0.084
Equity return volatility (σE)	0.751	0.458	0.545	3.360
Market value of firm asset (V)	299.731	391.730	16.439	1801.563
Asset volatility (σ_V)	0.072	0.049	0.013	0.194
Expected return on asset (μ_V)	0.078	0.004	0.071	0.084
Probability of default (PD)	10.439	16.053	2.990	98.960

Source: Author's estimation.

Tables 2 and 3 shows default probabilities for firms registered with BIFR and non-distressed firms respectively. From Table 2 the firms filed with BIFR or defaulted firms have higher PD than non-defaulted firms. The average PD reported for defaulted group is 10.439 with 2.990 minimum and 98.960 maximum values respectively. Even in the case of distressed group market value of equity, face value of debt and market value of firm asset is found to be highly volatile.

From Table 3 the non-defaulted firms have mean objective PD of 0.752 with 0.000 minimum and 3.910 maximum values respectively. Again in the case of non-distressed firms market value of equity, face value of debt and market value of firm asset is found to be highly volatile.

Table 3. *BSM Model Summary- Non-distressed Companies*

Variable	Mean	Standard deviation	Minimum	Maximum
Market value of equity (E)	341.097	610.973	11.827	3141.926
Face value of debt (F)	245.372	330.438	7.080	1533.555
Risk free rate (r)	0.079	0.005	0.0712	0.084
Equity return volatility (σE)	0.543	0.114	0.256	0.783
Market value of firm asset (V)	586.469	851.382	19.906	4675.481
Asset volatility (σ_V)	0.116	0.061	0.457	0.557
Expected return on asset (μ_V)	0.314	0.359	0.071	1.529
Probability of default (PD)	0.752	1.091	0.000	3.910

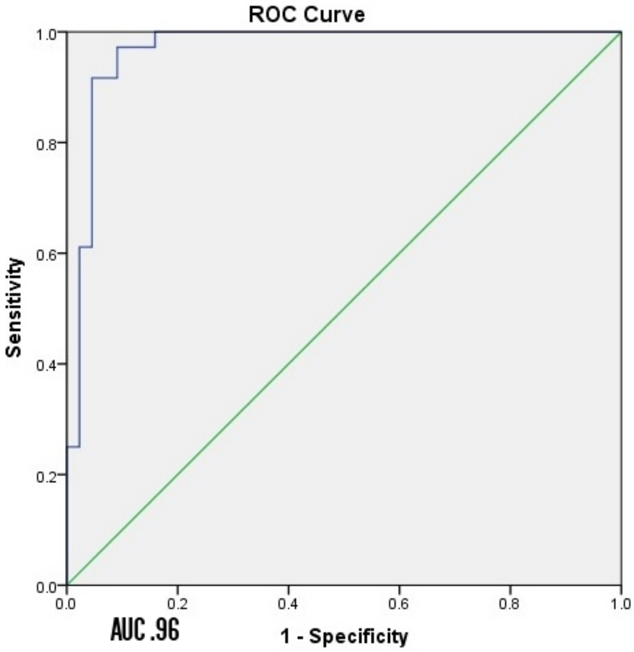
Source: Author's estimation.

From Table 2 and 3 it can be seen mean value of PD for distressed companies (10.44%) is significantly higher than the non-distressed companies (0.75%).The result shows the healthy firms have very less likelihood of default. Hence, as expected, the BSM model predicts a higher PD for distressed companies and lower PD for non-distressed companies.

After estimating objective PD of the companies the Receiver Operating Characteristic (ROC) analysis is performed as a diagnostic check for the estimated PD by BSM model for defaulted and non-defaulted firms. ROC is one of the widely used diagnostic test for model evaluation to visualize the performance of a binary classifier. The accuracy of the test depends upon how well it classifies between the groups. An ROC with AUC 1 represents the perfect test, whereas AUC with 0.5 represents worthless test.

Fig 1 shows the area under the ROC curve for estimated PD from BSM model. The value of AUC is .96, which is in between.9 to 1. Hence, the test is excellent for BSM model which have a good balance of specificity and sensitivity.

Figure 1. *Shows the area under the Receiver Operating Characteristic curve (AUROC) .96 of BSM model*



Source: Author’s estimation.

4. Conclusions

This paper derives a risk-neutral (and objective) indicator of credit risk that can be used to assess financial distress of firms based upon BSM framework. The empirical finding shows the mean PD estimated from BSM for distressed group (10.43%) is higher than mean probability of default estimated from non-distressed (0.75%) companies. The model can be applied to calculate direct PD estimates which can be used by investors and customers to take an informed decision on whether to do business with such companies which are likely to default in the near future. Banks and the financial institutions can use the model to predict whether a company is going to default before sanctioning the credit.

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