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Limitation of ARIMA models in financial and monetary economics

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Abstract. *Abandoning the classical econometric modeling approach which consists in using explanatory variables (suggested by economic theory for prediction), we choose instead to use a sophisticated method developed by Box and Jenkins (1970) based solely on the past behavior of the variable being modeled/forecast.*
As we are in a data-rich environment and the economies and financial markets are more integrated than ever before, the quantitative methods in business and finance has increased substantially in recent years.
This paper investigates the limitation of autoregressive integrated moving average (ARIMA) models in financial and monetary economics using the behavior of BET Index and EUR/RON exchange rates, respectively. Two important features discovered in the analysis of financial time series in this paper are fat-tails (large losses or gains are coming at a higher probability than the normal distribution would suggest) and volatility clustering, these empirical properties can't be captured by integrated ARMA models, hence the limitation of these models.

Keywords: Time Series Analysis, Autoregressive Integrated Moving Average, Forecast, BET Index, Exchange Rates, Box-Jenkins approach.

JEL Classification: C13, C22, C51, C52, C55, C58.

1. Introduction. Literature review

Unsurprisingly, Financial Economics represents the most empirical discipline of the economics/social science branches, showing a rapid increase of its usage in practice as well as stabilizing and facilitating growth of global economy. On the other hand, a strong economy is based on a well-established stock market, which is volatile by nature. Volatility affects, directly or indirectly, many factors such as: interest rates, currency exchange rates, imports, exports, GDP, and so forth. For example, a change in monetary policy leads to a change in the interest rates which affects the stock market return.

Also, risk managers are interested in forecasting the evolution of prices and risk factors. This can be done using only current and past observations which represents a key topic in time series analysis. Volatility as a phenomenon as well as a concept remains central to modern financial markets and academic research.

Researches regarding the predictability of exchange rates and financial asset returns have been under examination over the years and are still being studied stoutly:

It was shown by Akaike, H. (1976) that monetary approach can outperform the Random Walk model in an out-of-sample forecast exercise by incorporating a money demand function with a partial adjustment mechanism.

Starting with 1983 (Meese and Rogoff) forecasts of exchange rates using models based on Random Walk exceeded models based on the macroeconomic indicators, but this efficiency is lost when we are dealing with a time horizon that is extended by more than 12 months. It seems that empirical models used in seventies for exchange rates prediction fit well in sample, but we can't say the same thing if we consider an out-of-sample forecast.

Mark, N.C. (1995) presents evidence that long-horizon changes in the logarithm of spot exchange rates are predictable, motivated by a monetary model of exchange rate determination and is defined to be a linear combination of log relative money stocks and log relative real incomes. So, he found that the empirical exchange rate models were helpful in predicting long-horizons by investigating the movements of the U.S. Dollar price against four major currencies: the Canadian Dollar, Deutsche Mark, Swiss Franc, and the Japanese Yen, in a time period of 18 years (quarterly observations, from 1973:2 to 1991:4). In addition, subsequent works have questioned on whether exchange rates can be forecast at long-horizons, which leads to a weak consensus that models are not very helpful in predicting.

The economists Bellgard C. and Goldschmidt P. (1999) tried in nineties to forecast exchange rates using integrated *ARMA* models, but they have found these models are not suitable.

Weisang, G. and Awazu, Y. (2008) used integrated $ARMA$ models for modeling USD/EUR exchange rate. They found that the series of monthly USD/EUR exchange rate for the period 1994:01 to 2007:10 was best modeled by a linear relationship between the current value and its preceding three values. They also lead to the conclusion that $ARIMA(1,1,1)$ is the most adequate model for the prediction of the analyzed time series.

Finn, D. B. (2010) compares two monetary models: the flexible-price against rational-expectations, and conclude that the second one performs just as good as the Random Walk model. Another aspect presented in "Structural Time Series Model for the Analysis of exchange Rate of Naira" was about nominal exchange rate whose behavior was thought for long time to be described good by the Naïve Random-Walk (NRW) model. This leads to the conclusion of the inexistence of certain systematic economic forces in determining the foreign exchange rates.

Another example of a study using Box-Jenkins methodology is for instance the paper "Exchange-rates Forecasting: Exponential Smoothing Techniques and ARIMA Models", in which the authors investigated the behavior of daily exchange rates of the RON versus the most important currencies in terms of international trade, namely the Euro, United States Dollar, British Pound, Japanese Yen, Chinese Renminbi and the Russian Ruble. (Făt and Dezsi, 2011).

Begu, L.S., Spătaru, S. and Marin, E. (2012) investigated the volatility of foreign exchange rates taking into account the daily RON/EUR exchange rates from 05.01.2009 to 12.10.2012. Trying to capture the key features of the analyzed data, they used several models: $ARCH$, $GARCH$, $EGARCH$ and $TGARCH$. "The empirical results suggested that, for modeling the volatility of returns, the estimated $GARCH$ models fit the sample data good enough. In practice, the $GARCH(1,1)$ process generally seems to work reasonably well." (Begu et al., 2012, pp. 38-39).

Another example of a study using the Box-Jenkins approach in forecasting exchange rates is "The prediction of exchange rates with the use of Auto-Regressive Integrated Moving Average models" in which Spiesová, D. (2014) confirms that to predict the conditional variance and then to estimate the future values of exchange rates (the Czech Koruna, Swedish Krona, British Pound, Polish Zloty, Hungarian Forint and the Romanian Leu vs. Euro), it is adequate to use the $ARIMA(1,1,1)$ model without constant, or $ARIMA[(1,7), 1, (1,7)]$ model, where in the long-term, the square root of the conditional variance inclines towards stable value. The author also concluded that using integrated $ARMA$ models presented certain problems in estimating and validating the model and those models are more effective in interpretation of the medium-term value.

The same can be seen in researches regarding financial asset returns. According to Fama, E.F. (1965) all theories are based on the same assumption, the history of a time series data is rich in information – "History repeats itself in that "patterns" of past price behavior will tend to recur in the future." (Fama, 1965, p. 34). He used daily prices, from 1957 to 1962:09:26 (the starting date varies from share to share from 1965:01 to 1958:04), for each of the 30 shares of the Dow-Jones Industrial Average Index, and pointed out an interesting characteristic given by the signs of the autocorrelation coefficients. For the daily differences, he finds out that 23 out of 30 first-order autocorrelations coefficients are positive, while for the four-day differences and nine, respectively, he finds that 21 of the autocorrelation coefficients and 24, respectively, are negative.

In his paper, Kon, S.J. (1984) explains fat-tails and positive skewness regarding daily returns of stocks and indexes making use of a discrete mixture of normal distribution. In financial theory the most important assumptions with respect to asset prices are the multivariate normal distribution and stationarity of the parameters. Moreover, the normality and stationarity represent required assumptions to most econometric models.

Lo, A.W. and MacKinlay, A.C. (1988) applying a test relied on variance estimators provide additional evidence regarding the Non-Random Walk evolution of the stock prices. Another interesting thing shown in the paper is given by the positive autocorrelation of the weekly returns on portfolios and the negative one for the individual securities.

Conrad, J., Kaul, G. and Nimalendran, M. (1991) show interest in some scheming properties of time series behavior of short-horizon asset returns and portfolios revealed in many papers regarding the positively autocorrelated portfolio returns and the negatively autocorrelated short-horizon individual asset returns. The first one implies that returns are foreseeable and the rank of the positive autocorrelation is inversely proportional to the size of the company.

Later on, Chang, K-P. and Ting, K.-S. (2000) applied the methodology of Lo and MacKinlay on the weekly Taiex Index (Taiwan composite value-weighted stock market index) for the period 1971-1996 and concluded the movements do not fit a Random Walk. It is likely that using the same methodology on more current data it can lead to different results since the Taiwan investment climate has changed a lot from 1971.

Cont, R. (2001) underlines some basic properties regarding the assets, price variations and other Financial Markets analysis. These properties are better shaped using volatility clustering, conditional heavy tails and linear autocorrelation. In this case, the abnormality of the residues shows that the traditional tools used in time series modeling ($ARMA$ models) cannot be used to predict asset returns. In this case, nonlinear measures of dependence should be taken into account.

Christoffersen, P. (2003) also finds that the distribution of asset prices (returns) is characterized by fat-tails whose capture is decisive for a relevant analysis.

Unlike Chang and Ting's (2000) study upon Taiex Index, extending this study for the period 1996-2006, Lock, D.B. (2007) obtained that the weekly movements of Taiex Index from 1971 to 2006 follow a Random Walk. The gap in results may be thanks to the nature of the market, it being in its early stages till the 1980s and later reaching out maturity.

Tinca, A. (2013) highlights the underlying properties of financial markets using results like conditional heavy tails, negative asymmetry, the aggregational gaussianity is more pronounced for monthly returns compared to weekly returns, volatility clustering, negative correlation between volatility and returns, positive correlation between volatility and trading volume, low significance of the mean of the daily returns. Asset pricing models tend to fail when normality assumptions are taken into account.

2. Specification of the ARMA model

Time series represents a source of information for analysis and economic forecast, and reveals knowledge items which are useful for research or economic activity. We may consider time series as a raw material which processed by statistical or econometric methods can highlight recurring issues, analogies, conditionings or benchmarks. More specifically, time series passed through the filter of specific analysis/forecast methods can provide information on: the existence of a dominant evolutionary direction which applies particularly to conditions of normality of the process, appearance of systematic periodic oscillations with chances of repeating as effect and scale in the future, evolution's inertial character of some processes or relatively predictable appearance of the evolution of some processes as a response to some deviations from the past.

The dependence of the adjoining observations represents an inherent characteristic of time series and is of practical significance. Techniques for analyzing this dependence require "the development of stochastic and dynamic models for time series data and the use of such models in important areas of application" (Box et al., 2015, p. 7).

The analysis of time series can be divided in two types of methods, namely:

Spectral analysis, based on frequency components of a data series:

$$y_t = \mu + \sum_{i=1}^{n} [y_{1i} \, cos(2\pi f_i t) + y_{2i} \, sin(2\pi f_i t)] \tag{2.1}$$

where:
y_1 and y_2 are uncorrelated random variables, having zero variance $\sigma^2(f_i) = 0$;
the set $\{f_i\}_{\, i=\overline{1,T}}$ denotes the frequency at which y_t is evaluated.

The purpose of the analysis is to show at which frequencies the variable is active.

Time-domain analysis, based on the evolution of a variable with respect to time. It consists of modeling directly the relationships with delay of a series and its history.

The main aim of time series analysis is to evolve mathematical and statistical tools that reflect reality with respect to the analyzed sample data, thus providing plausible description for the population in cause.

Box and Jenkins, who developed in 1970 the technique for modeling time series known as the Box-Jenkins approach, stated that financial time series are not stationary since the mean changes over time, so they proceeded to differentiate the data in order to obtain stationarity. Thus, if the initial data was considered as being described by the random variable Y, the input for the Box-Jenkins technique would be described by the new variable Y^{BJ}. This model takes into consideration only the past values and past errors of the analyzed variable whose mathematical representation is as follow:

$$Y_t^{BJ} = f\left(Y_{t-1}^{BJ}, \dots, Y_{t-p}^{BJ}\right) + g\left(\varepsilon_{t-1}, \dots, \varepsilon_{t-q}\right) + \varepsilon_t \tag{2.2}$$

hence the difference from the traditional econometric model. The model above is the autoregressive integrated moving average process for Y, noted $ARIMA(p, d, q)$.

We start to describe mathematically the $ARMA(p,q)$ model by introducing those two components, $AR(p)$ process and $MA(q)$ process, presented in "Financial Econometrics: From Basics to Advanced Modeling Techniques".

An autoregressive process, noted as $AR(p)$, represents a process where the present value of a time series is described by a function of its p - past values. The mathematical representation of the $AR(p)$ process is given by the following equation (the p -th order difference equation):

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \varepsilon_t \qquad (2.3)$$

where:
y_t - the realisation of the dependent variable Y at time t;
y_{t-1}, y_{t-2}, ..., y_{t-p} - the realisation of the lagged dependent variables;
α_1, α_2, ..., α_p- the unknown parameters of the model, $\alpha_1 \neq 0$;
ε_t - the value of the disturbance term at time t, i.i.d. $\varepsilon_t \sim N(0,\sigma^2)$;
p - the number of lagged values of Y and represents the order of the process.

Consider L the lag operator, the equation of an $AR(p)$ process is then:

$$\left(1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p\right) y_t = \varepsilon_t \qquad (2.4)$$

or, equivalently

$$\alpha(L) y_t = \varepsilon_t \qquad (2.5)$$

where: $\alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p$ and represents the autoregressive polynomial.

A moving average process, noted as $MA(q)$, represents a process where the present value of a time series is described by a function of its current and q - past disturbances (lagged errors). The mathematical representation of the $MA(q)$ process is given by the following equation:

$$y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots + \beta_q \varepsilon_{t-q} \qquad (2.6)$$

where:
y_t - the realisation of the dependent variable Y at time t;
ε_{t-1}, ε_{t-2}, ..., ε_{t-q} - the realisation of the lagged disturbances;
β_1, β_2, ..., β_q - the unknown parameters of the model, $\beta_q \neq 0$;
ε_t - the current value of the disturbance term, i.i.d. $\varepsilon_t \sim N(0,\sigma^2)$;
p - the number of lagged values of Y and represents the order of the process.

Without losing any generality, in the above representation of a $MA(q)$ process, we assumed that the coefficient of ε_t equals 1. This restriction is important, otherwise being unable to identify the coefficient of ε_t or σ^2. Thus, if we assume β_0 being the coefficient of ε_t, then it is required either restriction $\beta_0 = 1$ or $\sigma^2 = 1$.

In the case of $\beta_0 \neq 1$, we can rewrite the following equation

$$y_t = \beta_0 \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots + \beta_q \varepsilon_{t-q} \qquad (2.7)$$

in terms of standardized disturbances as:

$$y_t = \sigma u_t + \beta_1 \sigma u_{t-1} + \dots + \beta_q \sigma u_{t-q}, \text{ with } Var(u_t) = 1, \tag{2.8}$$

where $u_t = \frac{\varepsilon_t}{\sigma}$ and $Var(\varepsilon_t) = \sigma^2$.

Consider L the lag operator, the equation of a $MA(q)$ process is then:

$$y_t = \left(\beta_0 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q\right)\varepsilon_t \tag{2.9}$$

or, equivalently

$$y_t = \beta(L)\varepsilon_t \tag{2.10}$$

where $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$ and represents the moving average polynomial of degree q with $b_0 \neq 0$.

The two processes discussed above can be seen as particular cases of a mixed autoregressive moving average process, noted $ARMA(p, q)$, that represents the current value of a time series depending upon its past values and on the preceding residual values, given by the following equation:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \tag{2.11}$$

or it can be rewritten in terms of the autoregressive and moving average polynomials as:

$$\alpha(L) y_t = \beta(L)\varepsilon_t. \tag{2.12}$$

The benefit of choosing a mixed autoregressive moving average process instead of simple autoregressive process or moving average process arises from a thrifty model with few unknown parameters.

An autoregressive integrated moving average process, noted as $ARIMA(p, d, q)$, is an $ARMA$ model where instead of Y_t we are dealing with the differenced series of Y_t, and it has the following general form:

$$\alpha(L)\Delta^d y_t = \beta(L)\varepsilon_t, \text{ with } \Delta^0 = 1 \tag{2.13}$$

where:
$\alpha(L)$, $\beta(L)$, ε_t - have the meaning set out in this paper;
$\Delta^d y_t$ - the new time series obtained by differentiating the initial series $\{y_t\}$ d - times.

According to Kennedy (2008), the Box-Jenkins methodology for constructing $ARIMA$ process has three main steps:

Step 1. Identification/model selection – one must determine the order of each component of $ARIMA$ process i.e., p, d, q.

Step 2. Estimation of the model parameters;

Step 3. Diagnostic checking or adequacy of the model. The most important and difficult step is given by the trial and error identification of the model based on correlogram. “In this respect the Box-Jenkins method is an art form, requiring considerable experience for a researcher to be able to select the correct model.” (Kennedy, 2008, p. 298).

3. Data and methodology

Given the growing importance of the current elements of uncertainty and stock exchanges, uncertainty reflected in both the volatility of financial instruments prices (shares, bonds, derivatives), and in interest rates and foreign exchange rates, this paper aims the following aspects:

- presenting the functionality of Box-Jenkins methodology in two case studies: Romanian GDP and weekly United States regular gasoline price, respectively;
- showing the limitation of $ARIMA$ models in financial and monetary economics using on the one hand the BET Index analysis, and on the other hand the EUR/RON exchange rate analysis.

3.1. The functionality of Box-Jenkins methodology

3.1.1. Romanian GDP

The advantages of integrated $ARMA$ models are presented in Stancu, S. (2011) where the analyzed time series is Romanian GDP from 2000 to 2010, quarterly data obtained from the official website of the National Institute of Statistics(1).

To achieve the adequate $ARIMA(p,d,q)$ model, the GDP series was tested for stationarity by applying the unit root tests: Augmented Dickey-Fuller (Dickey and Fuller, 1979) and Phillips-Perron (Phillips and Perron, 1988). Besides tests of stationarity, the chart series and correlogram also indicated that the GDP series is non-stationary and presents serial autocorrelation for the first fourteen lags.

Table 1. *Augmented Dickey-Fuller (ADF) test with a constant and linear trend and Phillips-Perron (PP) test with a constant, respectively, applied on Romanian GDP series*

Indicators		Unit root tests	
		PP Constant	ADF Constant, Linear Trend
t-Statistic (Prob.)		-1.215748 (0.6591)	-3.113834 (0.1174)
t-critical	1%	-3.592462	-4.211868
	5%	-2.931404	-3.529758
	10%	-2.603944	-3.196411

Source: Stancu, 2011.

Since the null hypothesis is not rejected and the GDP series is non-stationary, it is necessary to proceed to its transformation. After the first difference, the autocorrelation coefficients are close to zero which leads to the conclusion that this new series is stationary as can be seen from table below.

Table 2. *Phillips-Perron (PP) test with a constant for the new GDP series (i.e., DGDP)*

Indicators		Phillips-Perron unit root test
		Constant
t-Statistic (Prob.)		-15.90439 (0.0000)
t-critical	1%	-3.596616
	5%	-2.933158
	10%	-2.604867

Source: Stancu, 2011.

The output of Eviews for descriptive statistics presented in Table 3 shows that the stationary series is characterized by: $mean = 3402.426$ million RON in current prices (GDP has evolved from one quarter to another, on average, by 3402.426 million RON in current prices), and we can say that the new series follows an approximately normal distribution, asymmetric and flatter.

Table 3. *Descriptive Statistics of Romanian GDP*

DGDP		DGDP	
Mean	3402.426	Jarque-Bera	23.63671
Median	9998.200	Probability	0.000007
Maximum	25922.10		
Minimum	-64904.80	Sum	146304.3
Std. Dev.	22367.52	Sum Sq. Dev.	2.10E+10
Skewness	-1.597471		
Kurtosis	4.727696	Observations	43

Source: Stancu, 2011.

Next step consists in seasonality detection and its deseasonalisation using the moving average method. The result is a new time series, $DGDPSA$, that is the first difference of GDP seasonal adjustment.

Correlogram analysis reveals that the autocorrelation function (ACF) has the first thirteen values greater than zero and decreasing, and the partial autocorrelation function ($PACF$) has the first five values different from zero. So, the series has a moving average component with the number of lagged values at most equal to 13 and an autoregressive component with the number of lagged equal to 5.

The parameters of the autoregressive moving average model will be estimated using the stationary and seasonally adjusted series. Since "it is therefore recommended not to insist on unambiguous determination of the model order, but to try more models" (Spiesová, 2014, p. 31), using Eviews sotfware were estimated nine models. After estimating the parameters of those nine equations, the output reveals that all the autoregressive models are statistically valid ($F_{statistical} > F_{critical}$ and $Prob(F_{critical}) < 0.05$).

In order to verify whether the model is adequate we proceed to test the residues. Since Durbin-Watson statistics are close to 2, it follows that residues are not serial correlated. Heteroskedasticity of residues is tested using White test, according with there are several valid models: $AR(5)$, $ARMA(4,2)$, $ARMA(3,1)$, $ARMA(3,2)$, $ARMA(6,1)$, $ARMA(5,3)$ and $ARMA(1,1)$ as can be seen from the results presented in Table 4.

Table 4. *Heteroskedasticity of residues using White test*

Model	LM=Obs*R-squared	Heteroskedasticity of residues
AR(5)	$LM = 1.137 < {}^{2}_{0.05,6} = 12.59$	Residues are homoscedastic.
ARMA(4,2)	$LM = 6.593 < {}^{2}_{0.05,7} = 14.07$	Residues are homoscedastic.
ARMA(3,1)	$LM = 2.595 < {}^{2}_{0.05,5} = 11.07$	Residues are homoscedastic.
ARMA(3,2)	$LM = 10.069 < {}^{2}_{0.05,6} = 12.59$	Residues are homoscedastic.
ARMA(6,1)	$LM = 8.869 < {}^{2}_{0.05,7} = 14.07$	Residues are homoscedastic.
ARMA(5,3)	$LM = 11.58 < {}^{2}_{0.05,9} = 16.92$	Residues are homoscedastic.
ARMA(1,1)	$LM = 6.747 < {}^{2}_{0.05,3} = 7.81$	Residues are homoscedastic.

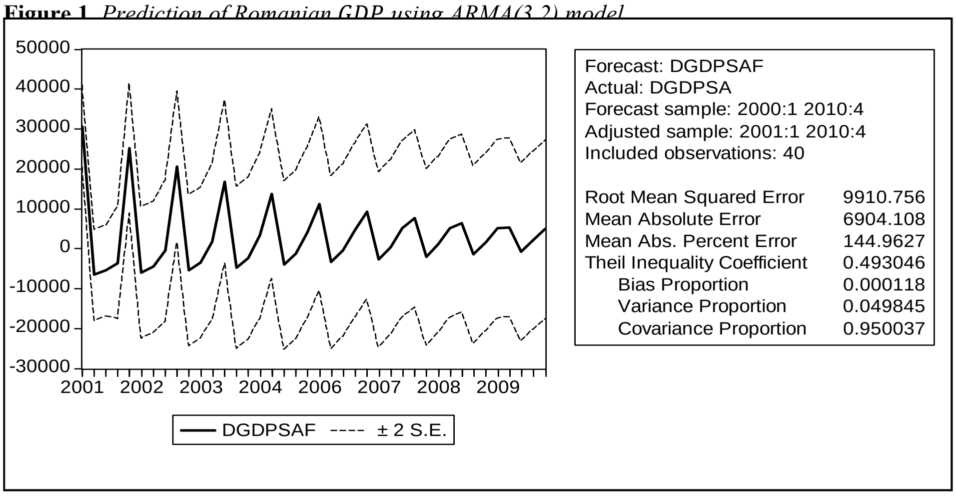
Source: Stancu, 2011.

In *Econometrie. Teorie și aplicații utilizând Eviews,* the Romanian quarterly GDP was predicted with the use of $ARMA(3,2)$, the selection of the best model being achieved based on the classical criteria (the greatest value of adjusted $R-squared$, $\bar{R}^2 = 84.03\%$) and taking into account that all coefficients are significantly different from zero it is given by the following equation:

$$PIB_t = 2860.192 - 0,9635PIB_{t-1} - 0.94734PIB_{t-2} - 0.84796PIB_{t-3} + 1.156\varepsilon_t + 0.9999\varepsilon_{t-1} \quad (3.1.1)$$

The forecast range for Romanian GDP using $ARMA(3,2)$ model, at a 95% interval, is exhibited in Figure 1.

Figure 1. *Prediction of Romanian GDP using ARMA(3,2) model*



Source: Stancu, 2011.

3.1.2. Weekly US regular gasoline price

The weekly U.S. gasoline price (expressed as Dollars per Gallon) is investigated using R software in the paper "An Introduction to Analysis of Financial Data with R" and it is collected from data base of U.S. Energy Information Administration[(2)]. The study period is from January 06, 1997 to September 27, 2010.

Considering the price fluctuation, the analysis begins with the log price series. The time plot of the log price series indicates non-stationarity. After the first difference (logarithmic returns), the autocorrelation coefficients are close to zero which leads to the conclusion that this new series is weakly stationary.

Correlogram analysis reveals the partial autocorrelation function ($PACF$) has the first five values different from zero that indicates an autoregressive component with the number of lags equal to 5. The fitted $AR(5)$ model without the constant is given by the following equation in terms of the autoregressive polynomial:

$$(1 - 0.504L - 0.079L^2 - 0.122L^3 + 0.101L^5)y_t = \varepsilon_t. \quad (3.1.2.1)$$

Since model identification is specified empirically, an alternative is thinking about an $ARMA(1,3)$ model for the weekly growth rates of U.S. gasoline price. The equation of the fitted $ARMA(1,3)$ model, in terms of the autoregressive polynomial, is then:

$$(1-0.633L)y_t = (1-0.127L+0.141L^3)\varepsilon_t. \quad (3.1.2.2)$$

Thus, there are two suitable models for the weekly growth rates of U.S. gasoline price, $AR(5)$ and $ARMA(1,3)$, respectively. Determining which model is kept for forecasting represents the final step of model fitting. The selection of the best model is achieved based on Akaike Info Criterion (AIC). As $AIC_{ARMA(1,3)} = -3704.6$ is bigger than $AIC_{AR(5)} = -3704.96$, the selected model for forecasting is the $AR(5)$ model since having a model with a smaller error variance is better.

Therefore, the $AR(5)$ model stands for predicting the U.S. gasoline price more than 10 days in advance. The results of out-of-sample forecast for the weekly growth rates of U.S. gasoline price, with the forecast origin on January 24, 2003, are presented below:

Table 5. *Out-of-Sample Results using R software*

Particulars	***AR(5)* model**
Root Mean Square Errors (*RMSE*)	0.02171
Mean Absolute Errors (*MAE*)	0.01538

Tsay, R.S. (2013). *An Introduction to Analysis of Financial Data with R.,* New Jersey: John Wiley & Sons, Inc., Hoboken.

3.2. Limitation of ARIMA models in financial and monetary economics

3.2.1. Application of Box-Jenkins methodology using BET Index

Our analysis takes into account Bucharest Exchange Trading Index (BET Index), daily quotations for the time period of September 22, 1997 to May 18, 2015, which involves investigating a time series of 4350 observations, and monthly quotations of the same index for the time period of September/1997 to April/2015 (211 observations). The data were collected from Bloomberg data base.

BET Index represents the benchmark index for the local capital market reflecting the evolution of ten most liquid companies listed on Bucharest Stock Exchange regulated market, except for financial investment companies (SIFs) [(3)].

To achieve the adequate $ARIMA(p,d,q)$ model, daily BET Index series was tested for stationarity through graphical representation, analyzing the behavior of autocorrelation and partial autocorrelation functions and also using the unit root tests (Augmented Dickey-Fuller and Phillips-Perron, respectively).

The evolution of daily BET Index prices from September 22, 1997 to May 18, 2015 is exhibited in Figure 2:

Figure 2. *Daily BET Index prices from September 22, 1997 to May 18, 2015*

Source: Bloomberg.

The evolution of daily BET Index prices indicates that the series has constant and trend (ascending, descending and then ascending again).

Daily BET Index prices correlogram reveals very high autocorrelations for the first two lags, 0.999 respectively, with values decreasing very slowly for the next lags, reaching a value of 0.971 at the 36th lag. This leads to the conclusion that the aforementioned series is non-stationary.

High Q-Stat test values together with $Prob. < 5\%$ to all lags confirms the presence of autocorrelation hence it is therefore a stochastic process without white noise in residues.

Non-stationarity is also highlighted by applying the Augmented Dickey-Fuller and Phillips-Perron tests, as can be seen from the table below:

Table 6. *The results of Augmented Dickey-Fuller and Phillips-Perron tests for daily BET Index prices*

Indicators			Daily BET Index prices		
			Constant	Constant, Linear Trend	None
ADF Test	t-Statistic (Prob.)		**-0.871602 (0.7976)**	**-1.383817 (0.8658)**	**0.541844 (0.8331)**
	t-critical	1%	-3.431670	-3.960149	-2.565494
		5%	-2.862008	-3.410838	-1.940897
		10%	-2.567062	-3.127218	-1.616651
C		t-Statistic (Prob.)	**1.421662 (0.1552)**	**0.735385 (0.4621)**	
@TREND(1)				**1.080134 (0.2801)**	
PP Test	t-Statistic (Prob.)		**-0.930643 (0.7790)**	**-1.470200 (0.8397)**	**0.446824 (0.8106)**
	t-critical	1%	-3.431670	-3.960149	-2.565494

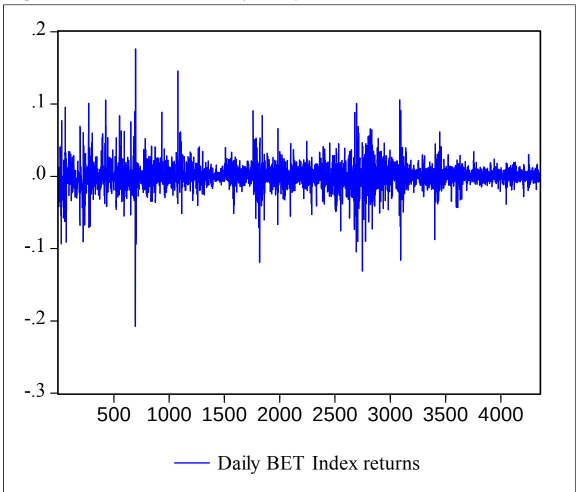
Indicators		Daily BET Index prices		
		Constant	Constant, Linear Trend	None
	5%	-2.862008	-3.410838	-1.940897
	10%	-2.567062	-3.127217	-1.616651
C	t-Statistic (Prob.)	**1.410547 (0.1584)**	**0.753295 (0.4513)**	
@TREND(1)	t-Statistic (Prob.)		**1.023191 (0.3063)**	
Conclusions		The series is non-stationary and the intercept is not significant.	The series is non-stationary and both intercept and trend are not significant.	The series is non-stationary.

Source: Own calculations.

Since the null hypothesis H_0: "Series has a unit root" is not rejected for 1%, 5% and 10% significance level, the non-stationarity of the analyzed time series is removed by transforming it into continuously compounded returns (logarithmic yields). Decision to use daily returns was based on Strong N. (1992) approach according to which logarithmic returns have a good chance of being normally distributed. Likewise, empirical properties such as stationarity and ergodicity are owned by logarithmic returns and not by prices.

The evolution of daily BET Index returns from September 22, 1997 to May 18, 2015, exhibited in Figure 3, indicates that the logarithmic return series appears to be stationarity.

Figure 3. *The evolution of daily BET Index returns*



Source: Own calculations.

The ACF and $PACF$ coefficients of daily returns series are close to zero at all lags, starting from 0.098 at lag 1, and decreasing quickly at lag 2 (0.038 and 0.029, respectively), which leads to the conclusion that the series of daily returns is generated by a Random Walk process. This is also confirmed by statistical ADF and PP tests. Table 7 exposes the findings of the ADF and PP tests which confirms that the return series of the BET Index is stationary for common levels of significance (1%, 5% and 10%, respectively) and $Prob. < 0.05$.

Table 7. *The results of Augmented Dickey-Fuller and Phillips-Perron tests for daily BET Index returns*

Indicators			Daily BET Index returns		
			Constant	Constant, Linear Trend	None
ADF Test	t-Statistic (Prob.)		**-59.73285 (0.0001)**	**-59.72600 (0.0000)**	**-59.70382 (0.0001)**
	t-critical	1%	-3.431670	-3.960149	-2.565494
		5%	-2.862008	-3.410838	-1.940897
		10%	-2.567062	-3.127218	-1.616651
C		t-Statistic (Prob.)	**1.534739 (0.1249)**	**0.807145 (0.4196)**	
@TREND(1)				**-0.046275 (0.9631)**	
PP Test	t-Statistic (Prob.)		**-60.31756 (0.0001)**	**-60.31113 (0.0000)**	**-60.32571 (0.0001)**
	t-critical	1%	-3.431670	-3.960149	-2.565494
		5%	-2.862008	-3.410838	-1.940897
		10%	-2.567062	-3.127218	-1.616651
C		t-Statistic (Prob.)	**1.534739 (0.1249)**	**0.807145 (0.4196)**	
@TREND(1)				**-0.046275 (0.9631)**	
Conclusions			The time series of returns is stationary and the intercept is not significant.	The time series of returns is stationary, and both intercept and trend are not significant.	The time series of returns is stationary.

Source: own calculations.

Table 8 points out descriptive statistics for the two time series, daily BET Index prices and returns. The conclusions drawn from the analysis of descriptive statistics for daily time series are as follows:

- distribution of daily price series reveals positive asymmetry ($\text{Skewness} = 0.138582 > 0$), while the distribution of daily returns shows negative asymmetry ($\text{Skewness} = -0.227395 < 0$);
- distribution of daily prices is platykurtic ($\text{Kurtosis} = 1.877349 < 3$), while the distribution of daily returns is leptokurtic ($\text{Kurtosis} = 15.70472 > 3$);
- Jarque-Bera statistics indicates that both series have abnormal distributions, the time series of returns presenting an extraordinarily high value ($\text{Jarque} - \text{Bera} = 29286.28$), and the probability of accepting the hypothesis of normality is zero in both cases.

Table 8. *Descriptive Statistics of daily time series*

Indicators	BET Index prices	BET Index returns
Skewness	0.138582	-0.227395
Kurtosis	1.877349	15.70472
Jarque-Bera	242.3053	29286.28
Probability	0.000000	0.000000

Source: own calculations.

It is worth noting that most of securities have such distributions as above and also the probability of occurrencing an extreme event into a leptokurtotic distribution is higher than the probability of occurrence in a normal distribution, and vice versa.

The analysis of BET Index returns correlogram reveals that we can try to estimate an $ARMA$ model. Thus, the optimal values of p and q corresponding to $AR(p)$ and $MA(q)$ models is maximum 3 and we proceed to identify the $ARMA$ model by trial-and-error. Using Eviews sotfware five models are estimated: $AR(1)$, $MA(1)$, $MA(2)$, $ARMA(1,1)$

and $ARMA(1,2)$, followed by checking whether each model found is appropriate. This is done applying statistical tests for residues: absence of autocorrelation (residues correlogram, Q-statistics or LM test), homoskedasticity (White Heteroskedasticity test) and normality (Histogram – Normality test).

We predict daily BET Index using $ARMA(1,2)$, the selection of the best model being achieved based on Akaike Info Criterion (we aim finding a model with a smaller error variance, $AIC = -5.179848$), as can be seen from Table 9:

$$BET_t^{daily} = 0.000469 - 0.631639 BET_{t-1}^{daily} + \varepsilon_t + 0.728061\varepsilon_{t-1} + 0.093907\varepsilon_{t-2} \quad (3.2.1.1)$$

Table 9. *Selecting the best model for predicting daily BET Index*

The estimated model	Adjusted R-squared	Akaike info criterion	Schwarz criterion	F-statistic (Prob)
AR(1)	0.009453	-5.178942	-5.176008	42.48333 (0.000000)
MA(1)	0.008819	-5.178365	-5.175432	39.68557 (0.000000)
MA(2)	0.010299	-5.179630	-5.175231	23.62318 (0.000000)
ARMA(1,1)	0.010050	-5.179315	-5.174915	23.06602 (0.0000)
ARMA(1,2)	0.010805	-5.179848	-5.173981	16.82781 (0.000000)

Source: own calculations.

We have found that for all analyzed models the normality assumption for residues is not fulfilled. According to specialized studies that happens often to residues of financial time series, hence the limitation of $ARMA$ models. The next step is to extend the study of indices by applying models of $GARCH$ family.

Table 10. *Statistical tests for residues*

The estimated model	LM Test		Normal distribution
	F-stat (Prob.)	Obs*R-squared (Prob.)	Jarque-Bera (Prob.)
AR(1)	2.584462 (0.075553)	5.167535 (0.075489)	36217.11 (0.000000)
MA(1)	4.223178 (0.014712)	8.437729 (0.014715)	35986.78 (0.000000)
MA(2)	0.791784 (0.453101)	1.584806 (0.452755)	34773.34 (0.000000)
ARMA(1,1)	3.335593 (0.035685)	6.668552 (0.035640)	35483.54 (0.000000)
ARMA(1,2)	1.423534 (0.240974)	2.849102 (0.240616)	34770.81 (0.000000)

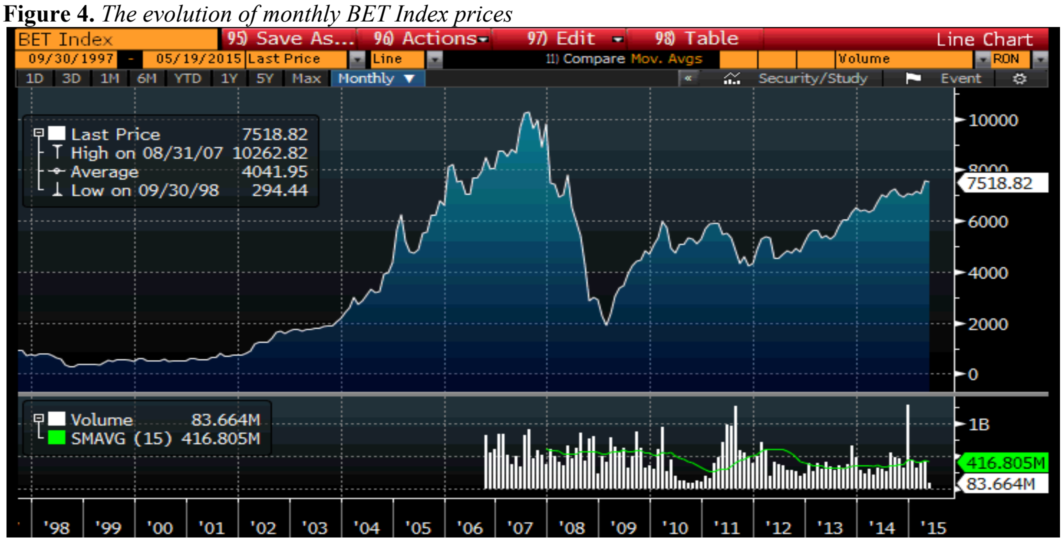
Source: own calculations.

In an analogous way we proceeded to analyze monthly quotations and observed that conclusions outlined for daily series are also preserved for monthly series. Accordingly, we will make a brief presentation of the obtained results.

Monthly BET Index series is tested for stationarity through graphical representation, analyzing the behavior of autocorrelation and partial autocorrelation functions and also using the unit root tests (Augmented Dickey-Fuller and Phillips-Perron, respectively).

The evolution of monthly BET Index prices from September/1997 to April/2015 indicates that the series has constant and trend (ascending, descending and then ascending again), as can be seen in figure below:

Figure 4. *The evolution of monthly BET Index prices*



Source: Bloomberg.

Correlogram of monthly BET Index prices reveals very high autocorrelations for the first two lags, 0.983 respectively, with values decreasing very slowly for the next lags, reaching a value of 0.528 at the 20th lag. This leads to the conclusion that the aforementioned series is non-stationary.

Q-Stat test confirms the non-stationarity of the analyzed time series which is also highlighted by applying the Augmented Dickey-Fuller and Phillips-Perron tests:

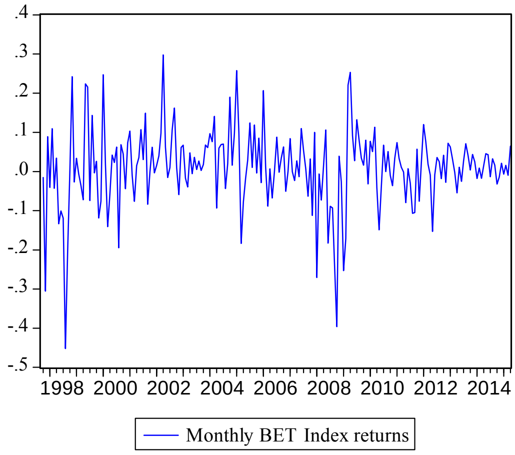
Table 11. *The results of Augmented Dickey-Fuller and Phillips-Perron tests for monthly BET Index prices*

Indicators			Monthly BET Index prices		
			Constant	**Constant, Linear Trend**	**None**
ADF Test	**t-Statistic (Prob.)**		**-0.941107 (0.7736)**	**-1.528782 (0.8168)**	**0.401635 (0.7986)**
	t-critical	**1%**	-3.461327	-4.002142	-2.575968
		5%	-2.875062	-3.431265	-1.942338
		10%	-2.574054	-3.139292	-1.615698
PP Test	**t-Statistic (Prob.)**		-1.119303 (0.7083)	-1.827130 (0.6882)	0.173247 (0.7355)
	t-critical	**1%**	-3.461327	-4.002142	-2.575968
		5%	-2.875062	-3.431265	-1.942338
		10%	-2.574054	-3.139292	-1.615698
Conclusion			The series is non-stationary.		

Source: own calculations.

The non-stationarity of monthly BET Index prices is removed by converting into return series computed as continuously compounded returns. The evolution of monthly BET Index returns from September/1997 to April/2015 is exhibited in the figure below:

Figure 5. *The evolution of monthly BET Index returns*



Source: own calculations.

The ACF and $PACF$ coefficients of monthly returns series are close to zero at all lags, starting from 0.205 at lag 1, and decreasing quickly at lag 2 (0.007 and -0.036, respectively), which leads to the conclusion that the series of monthly returns is generated, most likely, by a Random Walk process. This is confirmed by statistical ADF and PP tests. Table 12 exposes the findings of the ADF and PP tests and confirms that the return series of the BET Index is stationary, since the values of the tests are greater, in absolute value, than the critical values for common levels of significance (1%, 5% and 10%, respectively) and $Prob. < 0.05$.

Table 12. *The results of Augmented Dickey-Fuller and Phillips-Perron tests for monthly BET Index returns*

Indicators			**Monthly BET Index returns**		
			Constant	**Constant, Linear Trend**	**None**
ADF Test	**t-Statistic (Prob.)**		**-11.70085 (0.0000)**	**-11.67268 (0.0000)**	**-11.63434 (0.0000)**
	t-critical	**1%**	-3.461478	-4.002354	-2.576020
		5%	-2.875128	-3.431368	-1.942346
		10%	-2.574090	-3.139353	-1.615693
PP Test	**t-Statistic (Prob.)**		-11.74997 (0.0000)	-11.72206 (0.0000)	-11.72356 (0.0000)
	t-critical	**1%**	-3.461478	-4.002354	-2.576020
		5%	-2.875128	-3.431368	-1.942346
		10%	-2.574090	-3.139353	-1.615693
Conclusion			The series is stationary.		

Source: own calculations.

Table 13 points out descriptive statistics for the two time series, monthly BET Index prices and returns. The conclusions that we can draw from the analysis of descriptive statistics for the monthly time series are:

- distribution of monthly prices reveals positive asymmetry ($Skewness = 0.173164 > 0$), while the distribution of monthly returns shows negative asymmetry ($Skewness = -0.806950 < 0$);
- distribution of monthly prices is platykurtic ($Kurtosis = 1.869338 < 3$), while the distribution of daily returns is leptokurtic ($Kurtosis = 6.527186 > 3$);

- Jarque-Bera statistics indicates that both series have abnormal distributions ($Jarque - Bera = 12.35200$ and 132.2769, respectively), and the probability of accepting the hypothesis of normality is close to zero in both cases.

Table 13. *Descriptive Statistics for monthly BET Index prices and returns*

Indicators	BET Index prices	BET Index returns
Skewness	0.173164	-0.806950
Kurtosis	1.869338	6.527186
Jarque-Bera	12.35200	132.2769
Probability	0.002079	0.000000

Source: own calculations.

We predict monthly BET Index using $ARMA(2,2)$. Selecting the best model using Akaike Info Criterion ($AIC = -1.832137$), we obtain the following equation:

$$BET_t^{monthly} = 0.010599 - 1.259707BET_{t-1}^{monthly} - 0.541348BET_{t-2}^{monthly} + \varepsilon_t + 1.503900\varepsilon_{t-1} + 0.783401\varepsilon_{t-2} \quad (3.2.1.2)$$

Based on BET Index returns correlogram, we estimated the following models: $AR(1)$, $MA(1)$, $ARMA(1,2)$ and $ARMA(2,2)$, followed by checking whether each model found is appropriate. This is done applying statistical tests for residues: absence of autocorrelation (residues correlogram, Q-statistics or LM test), homoskedasticity (White Heteroskedasticity test) and normality (Histogram – Normality test).

Table 14. *Selecting the best model for predicting monthly BET Index*

The estimated model	Adjusted R-squared	Akaike Info Criterion	Schwarz Criterion	F-statistic (Prob.)
AR(1)	0.037605	-1.772704	-1.740826	9.166532 (0.002776)
MA(1)	0.039002	-1.778670	-1.746899	9.522923 (0.002304)
ARMA(1,2)	0.049836	-1.776109	-1.712354	4.654024 (0.003600)
ARMA(2,2)	0.066649	-1.832137	-1.752177	4.713224 (0.001169)

Source: own calculations.

We have found that for all analyzed models the normality assumption for residues is not fulfilled, hence the maintenance of the conclusions drawn for the daily series.

Table 15. *Statistical tests for residues*

The estimated model	LM Test		Normal distribution
	F-stat (Prob.)	Obs*R-squared (Prob.)	Jarque-Bera (Prob.)
AR(1)	1.262292 (0.285184)	2.542447 (0.280488)	88.54354 (0.000000)
MA(1)	0.018358 (0.981811)	0.037392 (0.981478)	92.43055 (0.000000)
ARMA(1,2)	0.967779 (0.381663)	1.941772 (0.378747)	75.54996 (0.000000)
ARMA(2,2)	0.488427 (0.614314)	0.992468 (0.608819)	126.6361 (0.000000)

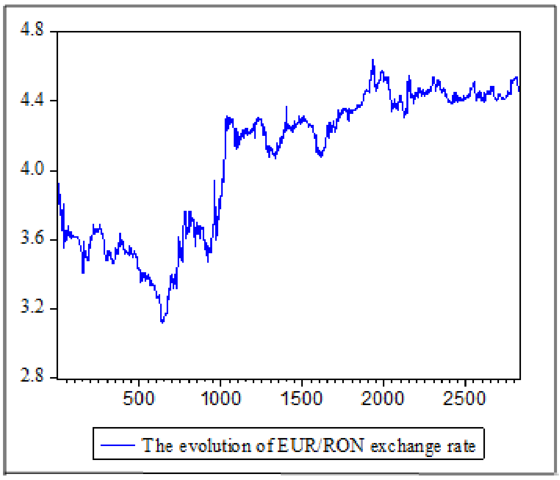
Source: own calculations.

3.2.2. Application of Box-Jenkins methodology using EUR/RON exchange rate

The paper aims to investigate the Box-Jenkins methodology using the EUR/RON exchange rate, daily quotations for the time period from January 03, 2005 to February 19, 2016, which involves investigating a time series of 2829 observations. The data were collected from the official website of the National Bank of Romania(4).

The evolution of the daily EUR/RON exchange rate for the period January 03, 2005 - February 19, 2016 denotes that the series has constant and trend.

Figure 6. *The evolution of daily EUR/RON exchange rate*



Source: own calculations.

Likewise, the correlogram of daily EUR/RON exchange rate reveals very high autocorrelations for all lags, with values decreasing very slowly and reaching a value of 0.963 at the 36th lag. This leads together with High Q-Stat test values ($Prob. < 0.05$) to the conclusion that the aforementioned series is non-stationary.

Withal, the Augmented Dickey-Fuller and Phillips-Perron tests mark that the time series is non-stationary.

Table 16. *The results of Augmented Dickey-Fuller and Phillips-Perron tests for daily EUR/RON exchange rate*

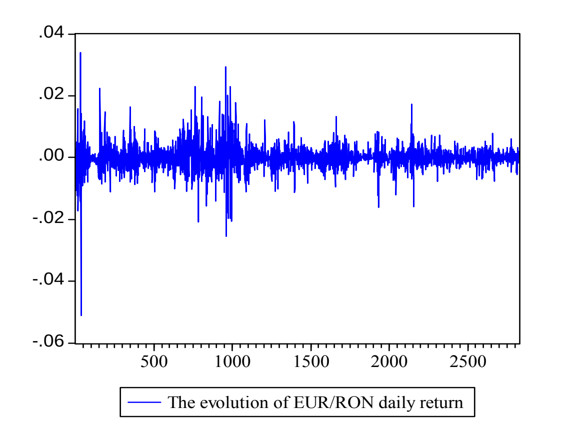
Indicators			Daily EUR/RON exchange rate		
			Constant	Constant, Linear Trend	None
ADF Test	t-Statistic (Prob.)		**-0.740810 (0.8345)**	**-2.473692 (0.3413)**	**0.670140 (0.8606)**
	t-critical	1%	-3.432474	-3.961291	-2.565781
		5%	-2.862364	-3.411398	-1.940936
		10%	-2.567253	-3.127549	-1.616624
PP Test	t-Statistic (Prob.)		**-0.792774 (0.8205)**	**-2.713431 (0.2311)**	**0.554067 (0.8358)**
	t-critical	1%	-3.432471	-3.961287	-2.565780
		5%	-2.862363	-3.411396	-1.940936
		10%	-2.567253	-3.127548	-1.616624
Conclusion			The series is non-stationary.		

Source: own calculations.

The null hypothesis is not rejected, therefore we are trying to remove the non-stationarity using return series computed as the difference of the natural logarithm.

Daily EUR/RON return series for the period from January 03, 2005 to February 19, 2016 is exhibited below:

Figure 7. *The evolution of daily EUR/RON return series*



Source: own calculation.

The ACF and $PACF$ coefficients of daily returns series are close to zero at all lags, starting from 0.177 at lag 1, and decreasing quickly at lag 2 (-0.109 and -0.145, respectively), which leads to the conclusion that the series of daily returns is generated, most likely, by a Random Walk process. Table 17 presents the findings of the ADF and PP tests and confirms that the series of EUR/RON returns is stationary.

Table 17. *The results of Augmented Dickey-Fuller and Phillips-Perron tests for daily EUR/RON exchange rate returns*

Indicators			**Daily EUR/RON Returns**		
			Constant	**Constant, Linear Trend**	**None**
ADF Test	**t-Statistic (Prob.)**		**-35.04889 (0.0000)**	**-35.04831 (0.0000)**	**-35.04506 (0.0000)**
	t-critical	**1%**	-3.432474	-3.961291	-2.565781
		5%	-2.862364	-3.411398	-1.940936
		10%	-2.567253	-3.127549	-1.616624
PP Test	**t-Statistic (Prob.)**		**-43.85462 (0.0000)**	**-43.84877 (0.0000)**	**-43.87804 (0.0001)**
	t-critical	**1%**	-3.432472	-3.961289	-2.565780
		5%	-2.862363	-3.411397	-1.940936
		10%	-2.567253	-3.127549	-1.616624
Conclusion			The series is stationary.		

Source: own calculations.

Table 18 points out descriptive statistics for the two time series, daily EUR/RON exchange rates and returns. The conclusions that we can draw from the analysis of descriptive statistics for the daily time series are:

- negative asymmetry ($Skewness = -0.586219 < 0$) for daily EUR/RON exchange rates and positive asymmetry ($Skewness = 0.033855 > 0$) for returns;

- platykurtic distribution ($Kurtosis = 1.797290 < 3$) for daily EUR/RON exchange rates and leptokurtic distribution ($Kurtosis = 20.00848 > 3$) for returns and it presents fat-tails;
- non-normal distribution for both series ($Jarque - Bera = 332.5394$ and 34088.37, respectively, and the probability of accepting the hypothesis of normality is zero in both cases).

Table 18. *Descriptive Statistics for daily EUR/RON series over the sample period*

Indicators	Daily EUR/RON exchange rates	Daily EUR/RON returns
Skewness	-0.586219	0.033855
Kurtosis	1.797290	20.00848
Jarque-Bera	332.5394	34088.37
Probability	0.000000	0.000000

Source: own calculations.

Using Eviews software nine models are estimated based on the correlogram of daily EUR/RON returns and are statistically valid ($F_{statistical} > F_{critical}$ and $Prob.(F_{critical}) < 5\%$).

Table 19. *Statistically valid models*

The estimated model	Adjusted R-squared	Akaike info criterion	Schwarz criterion	F-statistic (Prob.)
AR(1)	0.030931	-8.215766	-8.211559	91.19959 (0.000000)
AR(2)	0.050647	-8.236318	-8.230005	76.35598 (0.000000)
AR(3)	0.059900	-8.245432	-8.237012	60.97838 (0.000000)
MA(1)	0.038720	-8.221706	-8.217500	114.8710 (0.000000)
MA(2)	0.041741	-8.224501	-8.218192	62.57165 (0.000000)
MA(3)	0.059250	-8.242588	-8.234176	60.34983 (0.000000)
ARMA(1,2)	0.050392	-8.235346	-8.226932	50.98838 (0.000000)
ARMA(2,1)	0.056591	-8.242245	-8.233828	57.48662 (0.000000)
ARMA(2,2)	0.059075	-8.244528	-8.234007	45.34107 (0.000000)

Source: own calculations.

In accordance with Akaike Info Criterion ($minimum\ AIC = -8.245432$), we select the autoregressive model $AR(3)$ to estimate the EUR/RON exchange rate. The equation of $AR(3)$ model is given by:

$$Y_t = 0.0000498 + 0.186885Y_{t-1} - 0.124431Y_{t-2} - 0.100426Y_{t-3} + \varepsilon_t \quad (3.2.2)$$

As in the case of BET Index series, the EUR/RON exchange rate series also reveals the non-normality of residues, therefore the limitation of $ARMA$ models in monetary economics.

Table 20. *Statistical tests for residues*

The estimated model	LM Test		Normal distribution
	F-stat (Prob.)	Obs*R-squared (Prob.)	Jarque-Bera (Prob.)
AR(1)	26.30613 (0.000000)	51.72285 (0.000000)	38145.28 (0.000000)
AR(2)	14.97831 (0.000000)	29.69439 (0.000000)	26570.70 (0.000000)
AR(3)	0.659732 (0.517070)	1.321653 (0.516424)	20539.52 (0.000000)
MA(1)	23.90158 (0.000000)	47.07397 (0.000000)	35406.99 (0.000000)
MA(2)	22.21396 (0.000000)	43.81702 (0.000000)	32554.97 (0.000000)
MA(3)	2.518329 (0.080775)	5.038309 (0.080528)	20948.62 (0.000000)
ARMA(1,2)	13.17244 (0.000002)	26.15661 (0.000002)	27507.68 (0.000000)
ARMA(2,1)	5.672717 (0.003478)	11.32401 (0.003476)	22873.36 (0.000000)
ARMA(2,2)	1.464084 (0.231466)	2.932391 (0.230802)	21177.67 (0.000000)

Source: own calculations.

4. Results and conclusions

Usually in financial time series data we observe asymmetries, sudden outbreak at irregular time intervals and periods of high and low volatility. The exchange rate data strengthens the idea of this kind of behavior. Integrated $ARMA$ models have definite limitations in counterfeiting these properties.

One of the most important features of the integrated $ARMA$ models is the assumption of constant variance, most financial data exhibit changes in volatility and this feature of the data can't be fulfilled under this assumption. The symmetric joint distribution of the stationary $Gaussian\ ARMA$ models does not fit data with strong asymmetry (as we can observe a strong negative asymmetry for daily EUR/RON exchange rate and positive asymmetry for returns).

Due to the assumption of normality, it is more suitable to use these models with data that have only a negligible probability of sudden burst of very large amplitude at irregular time epochs. These limitations of the integrated $ARMA$ models lead us to models where we can retain the general $ARMA$ framework, and allow the White Noise (innovations) to be non-Gaussian or abandon the linearity assumption.

Notwithstanding financial time series usually reveals more complex structures than those offered by $ARMA$ processes, they may serve as a reference against more complex approaches by representing "a first starting point" (Rachev et al., 2007, p. 201). Since we have shown in this paper the limitation of $ARIMA$ models for the analyzed time series, BET Index and EUR/RON exchange rate, respectively, the next step consists in extending the study by applying models of $GARCH$ family.

On the other hand, predicting the future values of a variable based on past and current observations as we have pointed out in this paper in cases of Romanian GDP and United States regular gasoline price represents a key theme in time series analysis. One thing to keep in mind when we think about $ARMA$ models is given by the great power to capture "very complex patters of temporal correlation" (Cochrane, 1997: 25).

Notes

(1) See http://www.inse.ro/cms/rw/pages/PIB-trim.ro.do;jsessionid=0a02458c30d550d7fda54cdc 43fab67a95089cf660 cf.e38QbxeSahyTbi0Se0
(2) See http://www.eia.gov/
(3) See http://www.bvb.ro/FinancialInstruments/Indices/Overview#
(4) See http://www.bnr.ro/Exchange-rates-1224.aspx

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