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Job complexity and wage bargaining

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Abstract. *The objective of this paper is to qualify and discuss the effects of public policies according to bargaining solution used between workers and firms. We compare the effects of three solutions, Nash, Kalai-Smorod-insky and Equal-Sacrifice in a matching model where the job complexity is endogenous and workers are differentiated by their qualification level. We show that the choice of solution is extremely important since the effects of unemployment benefits on unemployment, labor market participation and on the job complexity can be completely opposite.*

Keywords: Bargaining, Nash, Kalai-Smorodinsky, Equal-Sacrifice, Complexity, Unemployment benefits.

JEL Classification: J64, J68.

1. Introduction

Despite the relevance of matching models to study the dynamics of the labor market (Pissarides, 2000), it is legitimate to ask the question on the fact that these models use systemically Nash as a rule for surplus sharing. The only use of this solution could skew the results by overestimating or underestimating the effects of the policies studied (Gerber and Upmann, 2006; Anbarci et al., 2002; Laroque and Salanié, 2004). There are other solutions that can provide more appropriate responses to the realities of the labor market. Among them we can include the Equal-Sacrifice (ES) and the Kalai-Smorodinsky solution (KS) (Kalai-Smorodinsky, 1975).

Our paper fits into this line and aims to study the effects of increased unemployment benefits in a matching model where the job complexity is endogenous. In this framework, workers are vertically differentiated by their qualification level and the productivity depends both on the degree of complexity and on the ability (i.e. qualification level) of the worker who is occupying it. This matching model was developed by Amine and Lages (2010). The effects of unemployment benefits are analyzed according to the bargaining solution used (Nash, KS and ES). We show that the increase in unemployment benefits has the effect of making more complex jobs when using Nash and the opposite when wages are negotiated according to KS and ES solutions.

The rest of paper is organized as follows. The model is presented in Section 2. Then, the bargaining solutions are discussed in Section 3. The solution to the model and the results of our analysis are presented in Section 4.

2. The model

As mentioned in the introduction, we use, in this article, the matching model developed by Amine and Lages (2010) and in which we consider an economy populated by a large exogenous number of workers and a large endogenous number of firms. Workers are vertically differentiated by their qualification level, denoted by z, and have an infinite horizon. z is distributed according to a continuous distribution, $G(z)$, with support $z \in [z_{min}, Z]$. The density of $G(z)$ is denoted by $g(z)$. Each firm offers a single job. All firms and workers are risk neutral and have the same rate of time preference denoted by r. In line with Amine and Lages (2010), each firm i in this economy requires a minimal ability, called $\hat{z}_i$, for her future worker. In this perspective, we assume that the productivity of a job-*i*, noted $y_i(a_i, z)$, depends both on the degree of complexity and on the ability (*i.e.* qualification level) of the worker who is occupying it. Formally, $y_i(a_i, z)$ is written as follows: $y_i(a_i, z) = A(a_i) + a_i z$. The endogenous variable a_i ($a_i \geq 0$) measures the degree of complexity of the job offered by firm *i*. Considering that an increase in complexity must raise skilled workers productivity (decrease unskilled workers productivity), $A(a_i)$ is a decreasing and concave function.

Meetings between firms with a vacant job and *active* unemployed workers are summarized by a constant-returns matching function (Pissarides 2000). Let, θ, represent the labour market tightness (*i.e.* the ratio of vacant jobs, V , to *active* unemployed workers U). Formally, the matching function, noted $\pi(V,U)$, is an homogenous function of degree 1 and is increasing function in V and U. Thus, the probability for a firm to meet an *active* worker satisfies: $q=\pi(V,U)/V=\pi(1,1/\theta)=q(\theta)$. Concerning workers, the hiring probability, noted p, is written as follows: $p(\theta)=\theta q(\theta)$. Contrary to $q(\theta)$, the probability that an *active* worker finds a job is an increasing function of θ.

Let's consider the expected lifetime utility of an *active* worker. At the stationary state, if this worker holds job-i, his/her lifetime utility $W_i(z)$ depends on his wage $w_i(z)$ and on the destruction rate s. We assume that unemployed workers income is composed of unemployment benefits b. Their expected lifetime utility $W_u(z)$, depends on the probability $p(\theta)$ of finding a job. As a result, in the stationary state, utilities $W_i(z)$ and $W_u(z)$ satisfy:

$$rW_i(z)=w_i(z)-s\big(W_i(z)-W_u(z)\big) \tag{1}$$

$$rW_u(z)=b+p(\theta)\big(W(z)-W_u(z)\big) \tag{2}$$

where $W(z)$ is the lifetime utility of workers with ability z when holding a job different from i.

We consider that the firms jobs are either vacant or filled. The value of job-i filled with a worker with ability z, $J_i(z)$, depends on the net instantaneous income ($y_i(a_i,z)-w_i(z)$) and on the future profits taking into account that the firm can die at any time with probability s. It satisfies:

$$rJ_i(z)=y_i(a_i,z)-w_i(z)-s\big(J_i(z)-J_{vi}(z)\big) \tag{3}$$

As long its job is not filled, firm i must invest c to create this job and to look for a worker. Furthermore, opening a new job is more profitable if the probability $q(\theta)$ is high. The value J_{vi} satisfies:

$$rJ_{vi}=-c+\frac{q(\theta)}{1-G(\hat{z})}\int_{\hat{z}_i}^{Z}(J_i(z)-J_{vi})\,g(z)\,dz \tag{4}$$

Taking into account of the free-entry assumption, we admit that new jobs will be created until the optimal value of a job vacant be equal to zero. In addition, the expected value for any variable in the model satisfy the following expression:

$$\bar{x}_i=\frac{1}{1-G(\hat{z}_i)}\int_{\hat{z}_i}^{Z}x_i(z)\,g(z)\,dz \tag{5}$$

In the stationary equilibrium, the unemployment rate u is a decreasing function of the recruiting probability $p(\theta)$ (for a given level of s):

$$u=\frac{s}{s+p(\theta)}\,. \tag{6}$$

3. Wage bargaining and surplus sharing

In this section, we'll see how the employee-employer couple will share the surplus created using three solutions: Nash generalized, Kalai-Smorodinsky and Equal-Sacrifice.

3.1. Nash solution

Using Nash solution, the surplus created by a firm/worker is divided between the two agents according to their respective bargaining strength. In fact, if β $(0<\beta<1)$ represents the workers bargaining strength, the maximization program for firm *i* is:

$$Max \quad (W_i(z)-W_u(z))^{\beta}(J_i(z)-J_{vi})^{(1-\beta)} \tag{7}$$

Solution of this program implies that global surplus is divided between the two agents as follows:

$$W_i(z)-W_u(z)=\beta(W_i(z)-W_u(z)+J_i(z)-J_{vi}) \tag{8}$$

$$J_i(z)-J_{vi}=(1-\beta)(W_i(z)-W_u(z)+J_i(z)-J_{vi}) \tag{9}$$

3.2. Kalai-Smorodinsky solution

The Kalai-Smorodinsky solution is to set the maximum values for the employer and for the employee. The maximization program is written as follows:

$$\phi^{KS}=(J_i(z)-J_{vi})(W_i(z)^{max}-W_u(z))-(J_i(z)^{max}-J_{vi})(W_i(z)-W_u(z))=0 \tag{10}$$

The maximum utility of a worker $W_i(z)^{max}$ and the maximum value of filled-job $J_i(z)^{max}$ satisfy:

$$rW_i(z)^{max}=y_i(z)-s\big(W_i(z)-W_u(z)\big) \tag{11}$$

$$rJ_i(z)^{max}=y_i(z)-b-s\big(J_i(z)-J_{vi}(z)\big) \tag{12}$$

Given the last two expressions, we get:

$$W_i(z)-W_u(z)=\frac{W_u(z)-y_i(z)}{J_{vi}(z)+b-y_i(z)}(J_i(z)-J_{vi}) \tag{13}$$

3.3. Equal-Sacrifice solution

This solution (Aumann and Maschler 1985) equalizes the sacrifice from the maximum feasible payoff net of the threat point. Bargaining under this rule is then written as follows:

$$W_i(z)^{max}-W_i(z)=J_i(z)^{max}-J_i(z) \tag{14}$$

Using the equations (11) and (12), we establish the following expression:

$$y_i(z)-w_i(z)=w_i(z)-b \tag{15}$$

4. Model equilibrium

Solving the model consists of establishing interactions at the stationary equilibrium, between labor market tightness θ, degree of complexity a and ability threshold $\hat{z}$. We first study and specify the optimal degree of job complexity, and then describe wage setting and job creation processes.

4.1. Optimal job complexity

When entering the labor market, firm *i* decides on the degree of complexity of the created job (a_i) and on the ability threshold ($\hat{z}_i$). These optimal choices result from maximization of the asset value J_{vi} with respect to both variables with the constraint that $\hat{z}_i$ is not lower than $\hat{z}$.

Using equations (3) and (4), we show that J_{vi} can be written as a function of the pair $(a_i, \hat{z}_i)$. We obtain the following expression:

$$rJ_{vi} = -c + \frac{q(\theta)(1-\beta)}{(r+s)(1-G(\hat{z}))}\int_{\hat{z}_i}^{Z}[y_i(a_i, z) - r(W_u(z) + J_{vi})]g(z)dz \tag{16}$$

In the symmetric equilibrium, the first order conditions can be written as follows:

$$\frac{1}{1-G(\hat{z})}\int_{\hat{z}}^{Z} zg(z)dz = -A'(a) \tag{17}$$

$$y(a,\hat{z}) = A(a) + a\hat{z} = b \tag{18}$$

Taking into account of the function $A(a)$ concavity, we obtain that the derivative of job complexity a with respect to the ability threshold $\hat{z}$ is positive. We show then that an increase in ability threshold ($\hat{z}$) leads to a rise in job complexity (a) thus reducing the labor market participation. Moreover, equation (18) shows that a worker is considered *active*, in this economy, only if his productivity (*i.e.* his qualification level) allows to obtain a positive or null global surplus.

Note that these two optimization results (17) and (18) have already been established by Amine and Lages (2010). So we remember them only for use in our analysis but the demonstration and interpretations related to this optimization are available in the article cited above.

4.2. Job creation process

4.2.1. Nash solution

We use equations (8) and (9) in order to establish interactions, at the stationary equilibrium, between labor market tightness, degree of complexity and ability threshold. We obtain an expression of the workers' share in the global surplus:

$$W(z)-W_u(z)=\frac{\beta}{1-\beta}\frac{y(a,z)-w(z)}{r+s} \tag{19}$$

Using equations (1) and (2), we deduce a second expression for the workers' share in the global surplus:

$$W(z)-W_u(z)=\frac{w(z)-b}{r+s+p(\theta)} \tag{20}$$

From these two expressions for the workers' share $(W(z)-W_u(z))$, we establish the wage setting equation :

$$\overline{w}=\overline{y}-\frac{(1-\beta)(r+s)(\overline{y}-b)}{r+s+\beta p(\theta)} \tag{21}$$

Since we assume firm free entry, we can establish a second expression of the average wage:

$$\overline{w}=\overline{y}-\frac{(r+s)c}{q(\theta)} \tag{22}$$

Substituting equation (21) in (22), we obtain an other expression between endogenous variables of the model:

$$-c+\frac{q(\theta)(1-\beta)(\overline{y}-b)}{r+s+\beta p(\theta)}=0 \tag{23}$$

4.2.2. KS solution

Substituting equations (2) and (3) into equation (13) yields:

$$W(z)-W_u(z)=\frac{(y(z)-w(z))(b-y(z))}{(b-y(z))(r+s)-p(\theta)(y(z)-w(z))} \tag{24}$$

Using the expressions of utilities (1) and (2) a second expression of $(W(z)-W_u(z))$ is obtained:

$$W(z)-W_u(z)=\frac{w(z)-b}{r+s+p(\theta)} \tag{25}$$

Using the two equations of $(W(z)-W_u(z))$, one establishes a second expression between the model equilibrium variables:

$$\frac{\overline{w}-b}{r+s+p(\theta)}=\frac{(\overline{y}-\overline{w})(b-\overline{y})}{(b-\overline{y})(r+s)-p(\theta)(\overline{y}-\overline{w})} \tag{26}$$

4.2.3. Equal sacrifice solution

Using the expression (15), we establish an expression of the average wage:

$$\overline{y}-\overline{w}=\overline{w}-b \tag{27}$$

Since we assume firm free entry, we can establish a second expression of the average wage:

$$\overline{w}=\overline{y}-\frac{(r+s)c}{q(\theta)} \tag{28}$$

By replacing the last two expressions we get :

$$\overline{w}-b=\frac{(r+s)c}{q(\theta)} \tag{29}$$

This equation determines ability threshold $\hat{z}$ as implicit function of labor market tightness θ and job complexity a.

4.3. Numerical analysis

In this section we will analyze the effects of increased unemployment benefits on the job complexity and on the labor market participation. These effects will be qualified according to bargaining solution (Nash, KS, ES). Given the ambiguity of some theoretical results, we will use a numerical analysis. Indeed, this model is calibrated in order to represent a situation similar to the French economy.

In accordance with current real rate, the annual rate of time preference is estimated at 5%. The job destruction rate is fixed at 0.15 in order to represent an employment flow compatible with the French situation.

We use a Cobb-Douglas function of the form $\pi(\theta)=h\theta^{\eta}$ to represent the matching function. In accordance with standard matching models (Cahuc and Lehmann 2000), we choose an elasticity, noted η, of this function with respect to vacant job equal to 0.5. The bargaining power is fixed at 0.5. The Hosios condition is hence satisfied. Unemployment benefits, which are funded by a neutral tax, represent 50% of wage. In this economy, for an unemployment benefits level equal to 0.5, unemployment rate reaches 11.46%.

For these simulations, we assume that the complexity function $A(a)$ is quadratic ($A(a)=-a^2$). Qualification level is distributed among workers according to a continuous distribution, $G(z)$, with $G(z)=\alpha(z-z_{min})$. The labor market participation rate, noted τ, satisfies: $\tau=1-G(\hat{z})$. The following table summarizes the effects of rising unemployment benefits.

Table 1. *The unemployment benefits effects depending on the bargaining solution*

	θ	$\hat{z}$	a	$\overline{y}$	$\overline{w}$	$p(\theta)$	$q(\theta)$	u	$\tau=1-G(\hat{z})$
b_{nash}	–	+	+	+	+	–	+	+	–
b_{ks}	+	–	–	–	–	+	–	–	+
b_{es}	+	–	–	–	–	+	–	–	+

According to the results in the table above, the effects of increased unemployment benefits on variables model are standard and have already been demonstrated in several articles (Amine and Lages, 2010). Indeed, in a matching model, when wages are negotiated by the Nash rule, an increase in unemployment benefits has the effect of making jobs more complex and therefore firms more selective. This accentuation of the complexity and selectivity results in an increase in productivity and wage. However, job creation is becoming less profitable for firms as their share in the overall surplus decreases as the workers' bargaining power is strengthened with higher unemployment benefits.

However, when the surplus sharing is done according to the KS or ES solution, we establish the inverse of the effects obtained with the Nash solution. Indeed, an increase in unemployment benefits, in the case of KS and ES, has the effect of making jobs less complex and therefore firms less selective. This decrease complexity and selectivity results in a deterioration of productivity and hence the average wage. Despite the increase of b, firms are encouraged to create more jobs, hence the increase in θ, as their share in the surplus increased. Unemployment decreases and participation in the labor market also increases.

5. Final remarks

As we explained in the introduction, the aim of this paper is to show that the use of the generalized Nash rule as surplus sharing solution between workers and firms is not always relevant. In other words, we have shown that the choice of the solution may distort results about effects of public policies.

Using a matching model in which the choice of complexity of jobs is endogenous, we have shown that higher unemployment benefits can have two completely opposite effects on labor market performance based on the bargaining solution (Nash, KS, ES). Indeed, papers that focus on matching models should qualify their results and conclusions and explain why Nash is used systematically.

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