

## Regulation versus regulated monopolization of a Cournot oligopoly with unknown cost

**Ismail SAGLAM**  
Ipek University, Ankara, Turkey  
saglam@bilkent.edu.tr

**Abstract.** *This paper studies whether a Cournot oligopoly with unknown costs should be left unregulated, or regulated according to the optimal mechanism of Gradstein (1995), or first monopolized and then regulated according to the optimal mechanism of Baron and Myerson (1982). We show that the answer to this question depends on the number of the oligopolistic firms and the size of their fixed costs, as well as on the weight of the producer welfare in the social objective function.*

**Keywords:** Monopoly; Oligopoly; Cournot competition; Regulation; Asymmetric information.

**JEL Classification:** D82; L51.

## 1. Introduction

This paper attempts to answer whether a Cournot oligopoly with unknown costs should be left unregulated, or optimally regulated, or first monopolized and then optimally regulated. While this question is novel, the regulatory design problem when the regulator has incomplete information about the regulated industry has been extensively studied in the economics of regulation since the seminal work of Baron and Myerson (BM) (1982). The wisdom of BM, who dealt with the problem of regulating a natural monopoly with unknown costs, was simply to use the Revelation Principle (Dasgupta, Hammond and Maskin, 1979; Myerson, 1979; Harris and Townsend, 1981), by which they could restrict themselves to incentive-compatible revelation mechanisms, requiring the monopolistic firm to report its unknown cost information and ensuring that it has no incentive for misreporting. This approach allows one to directly calculate the informational rents that should be provided to the regulated firm as a reward for truthful revelation. By deriving the minimal subsidy that will generate these rents to the firm, one can also calculate the net gains of the producer and consumers. Aggregating these net gains by a (generalized) social welfare function, the objective of the regulator was reduced by BM to finding a quantity schedule (a function of cost reports), among several other schedules, that can maximize the expected value of the social welfare with respect to the regulator's beliefs about the private cost information of the regulated firm.

Many extensions of the BM's problem -to deal with regulating a monopolistic industry under various types of asymmetric information- were later studied by Sappington (1983), Baron and Besanko (1984), Riordian (1984), Rochet (1984), Laffont and Tirole (1986), and Lewis and Sappington (1988), among many others. A common feature of all these studies is that the optimal regulatory mechanism must offer to the regulated monopoly some information rents, which create a deadweight loss in welfare unless the social welfare function treats the producer and the consumer welfare equally. A remarkable work of Gradstein (1995) extended the regulatory mechanism design to the case of an oligopoly with asymmetric cost information and showed that the information rents that should be offered to the oligopolistic firms can be balanced to be zero. This exciting result implies that the marginal cost pricing can always be freely implemented irrespective of the definition of the social welfare. However, since these rents cannot be individually rational for some of the regulated firms, it is questionable whether a regulatory authority could force the oligopolistic firms to participate in the regulatory program proposed by Gradstein (1995).

As a partial remedy to the said problem, the transfers proposed by Gradstein (1995) can be balanced to be equal to the aggregate fixed costs of the oligopolistic industry, as we will propose in Section 2.2. This modification may ensure that the Gradstein's (1995) regulatory mechanism would satisfy an industry-wide rationality (or participation) constraint. Although such a modification is likely to increase the applicability of the Gradstein's (1995) optimal regulation program, it is not clear whether the outcome of that program would be welfare superior to the equilibrium outcome of an unregulated Cournot oligopoly. Definitely, consumers would benefit under the optimal regulation program from the increased output implied by the marginal cost pricing; but, they would also be

hurt by the subsidies they need to pay for financing the fixed costs of the regulated firms. This tradeoff, and consequently the ambiguity as to the desirability of oligopoly regulation, can in fact be felt by the whole society under a social welfare function where the welfare of producers has, in general, a lower weight than the welfare of consumers. Moreover, it is also ambiguous, in general, whether regulating an oligopoly according to the Gradstein's (1995) optimal regulation program would be more desirable - in terms of the induced social welfare - than simply monopolizing the oligopoly and regulating it afterwards according to the BM's (1982) optimal mechanism. This second ambiguity is simply caused by the following tradeoff faced by consumers: On the one hand, the regulated monopolization can be unattractive for consumers since the information rents that should be offered to the regulated monopoly as part of the incentive program can be very huge relative to the gross consumer surplus, unlike the informational rents in the case of oligopoly regulation which can be as low as the aggregate fixed costs of the industry. But, on the other hand, the regulated monopolization of an oligopolistic industry with  $N$  firms would also free consumers from the obligation to finance the fixed costs of  $N-1$  firms who are not allowed to operate; hence the tradeoff. Calculating these two tradeoffs with the help of a formal model and investigating how they would be affected by the relevant parameters of the model, we aim to answer whether/when a Cournot oligopoly should be optimally regulated or first monopolized and afterwards optimally regulated.

The rest of the paper is organized as follows: Section 2 presents the model, Section 3 presents our results, and Section 4 concludes.

## 2. Model

We consider an oligopolistic market involving  $N \geq 2$  symmetric firms with unknown costs. Firms are indexed by  $i = 1, 2, \dots, N$ . Firm  $i$  faces the cost function

$$C_i(q_i, \theta) = \begin{cases} K + \theta q_i & \text{if } q_i > 0, \\ 0 & \text{if } q_i = 0, \end{cases} \quad (1)$$

where  $K$  is the known fixed cost and  $\theta$  is the marginal cost, which are the same for all firms. It is assumed that  $\theta$  lies in a known interval  $\Theta = [\theta_0, \theta_1]$ , where  $\theta_1 > \theta_0 \geq 0$ . The total production in the market is denoted by  $Q$ , so  $Q = \sum_i q_i$ . The market faces an inverse demand function which is given by

$$P(Q) = \begin{cases} a - Q & \text{if } 0 \leq Q \leq a, \\ 0 & \text{if } Q > a, \end{cases} \quad (2)$$

where  $a > \theta_1$  is a known constant. The gross value to consumers of a total quantity  $Q$  is denoted by  $V(Q)$  where

$$V(Q) = \int_0^Q P(\tilde{Q}) d\tilde{Q}. \quad (3)$$

Then, the consumer surplus becomes  $V(Q) - P(Q)Q$ .

Everything about the described market other than  $\theta$  is known to the regulator as well as to the firms, while  $\theta$  is only known to the firms. However, the regulator has (commonly known) prior beliefs about  $\theta$ , represented by the probability density function  $f(\cdot)$ , which is positive and continuous over  $\Theta$ . Finally,  $F(\cdot)$  denotes the corresponding cumulative distribution function.

### 2.1. Unregulated Cournot oligopoly

Here, we consider that the firms in the described oligopolistic industry compete à la Cournot (in quantities). Let  $\pi_i(q_i, q_{-i}; \theta)$  denote the operational profits of firm  $i$  with the marginal cost  $\theta$  when it produces  $q_i$  units of output while the quantity decisions of other firms are given by the list  $q_{-i} = (q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_N)$ . Let  $q = (q_i, q_{-i})$  be the quantity profile for the given industry. Then,

$$\pi_i(q; \theta) = P(Q)q_i - \theta q_i - K, \quad (4)$$

using  $Q = \sum_i q_i$ . We say that the quantity profile  $q^*$  is a Cournot (Nash) equilibrium if for all  $i$  it is true that

$$\pi_i(q_i^*, q_{-i}^*; \theta) \geq \pi_i(q_i, q_{-i}^*; \theta) \text{ for all } q_i \geq 0. \quad (5)$$

**Proposition 1.** When the oligopolistic firms' marginal cost is  $\theta$ , the Cournot (Nash) equilibrium is given by the quantity profile  $q^*(\theta)$  that satisfies

$$q_i^*(\theta) = \frac{a - \theta}{N + 1} \text{ for all } i. \quad (6)$$

**Proof.** See the Appendix.

Equation (6) implies the total quantity of output is  $Q^*(\theta) = \sum_i q_i^*(\theta) = (a - \theta)N/(N + 1)$ , while the price is  $P(Q^*(\theta)) = a - (a - \theta)N/(N + 1)$  at the Cournot (Nash) equilibrium. The equilibrium profits of firm  $i$  would then become

$$\pi_i^C(q^*(\theta); \theta) = \frac{(a - \theta)^2}{(N + 1)^2} - K. \quad (7)$$

(The superscript  $C$  points to the unregulated Cournot oligopoly.) Correspondingly, the equilibrium profits for the industry, i.e., the producer welfare, would be

$$\pi^C(\theta, K, N) \equiv \sum_i \pi_i^C(q^*(\theta); \theta) = \frac{(a - \theta)^2 N}{(N + 1)^2} - NK. \quad (8)$$

On the other side, the equilibrium welfare of consumers is given by

$$CW^C(\theta, N) \equiv V(Q^*(\theta)) - P(Q^*(\theta))Q^*(\theta) = \frac{[Q^*(\theta)]^2}{2} = \frac{(a - \theta)^2 N^2}{2(N + 1)^2}. \quad (9)$$

Finally, we denote by  $SW^C(\theta, \alpha, N)$  the equilibrium social welfare at the marginal cost level  $\theta$ , which is the sum of  $CW^C(\theta, N)$  and  $\alpha$  fraction of  $\pi^C(\theta, K, N)$ , with  $\alpha \in [0, 1]$ . Formally,

$$\begin{aligned}
SW^C(\theta, \alpha, K, N) &\equiv CW^C(\theta, N) + \alpha \pi^C(\theta, K, N) = \frac{(a - \theta)^2 N^2}{2(N + 1)^2} + \alpha \frac{(a - \theta)^2 N}{(N + 1)^2} - \alpha NK \\
&= \frac{(a - \theta)^2}{2} \frac{N^2 + 2\alpha N}{(N + 1)^2} - \alpha NK.
\end{aligned} \tag{10}$$

## 2.2. Regulated oligopoly

Here, we will consider the optimal regulation of the oligopolistic industry described at the beginning of this section. To this end, we will focus on the optimal mechanism proposed by Gradstein (1995). This mechanism consists of transfer functions  $T_i: [0, a] \rightarrow \mathfrak{R}$ ,  $i = 1, \dots, N$ , where  $T_i(q)$  is the transfer received by firm  $i$  under the quantity profile  $q$ . Then, the net gain of firm  $i$  when regulated according to this mechanism will be the sum of its operational profits and the transfer it receives. Let  $\pi_i^G(q; \theta)$  denote the net gain of firm  $i$  with the marginal cost  $\theta$  when the output profile is  $q$ . (The superscript  $G$  points to the regulation of the oligopoly according to Gradstein's (1995) mechanism.) Then,

$$\pi_i^G(q; \theta) = P(Q)q_i - \theta q_i - K + T_i(q), \tag{11}$$

using  $Q = \sum_i q_i$ . Resultingly, the industry welfare at the quantity profile  $q$  would be

$$\pi^G(q; \theta) = \sum_i \pi_i^G(q; \theta) = P(Q)Q - \theta Q - NK + \sum_i T_i(q). \tag{12}$$

On the other side, the welfare of consumers under the given mechanism would be

$$CW^G(q; \theta) = V(Q) - P(Q)Q - \sum_i T_i(q). \tag{13}$$

Finally, we denote by  $SW^G(q; \theta)$  the social welfare at the marginal cost level  $\theta$ , which is the sum of  $CW^G(q; \theta)$  and  $\alpha$  fraction of  $\pi^G(q; \theta)$ , with  $\alpha \in [0, 1]$ . Formally,

$$\begin{aligned}
SW^G(q; \theta) &= CW^G(q; \theta) + \alpha \pi^G(q; \theta), \\
&= V(Q) - P(Q)Q - \sum_i T_i(q) + \alpha(P(Q)Q - \theta Q - NK + \sum_i T_i(q)), \\
&= V(Q) - \theta Q - (1 - \alpha)(P(Q)Q - \theta Q + \sum_i T_i(q)) - \alpha NK.
\end{aligned} \tag{14}$$

The mechanism  $(T_i)_{i=1}^N$  satisfies the participation constraint for the oligopoly if

$$\pi^G(q; \theta) \geq 0, \tag{15}$$

for all  $\theta \in \Theta$  and for all feasible profiles  $q$ . On the other hand, a mechanism  $(T_i)_{i=1}^N$  and a quantity profile  $q^S(\theta)$  maximize the social welfare  $SW^G(q; \theta)$  only if the price is always equal to the marginal cost, i.e.,  $P(Q^S(\theta)) = \theta$ , where  $Q^S(\theta) = \sum_{i=1}^N q_i^S(\theta)$  for all  $\theta \in \Theta$ . Clearly,  $q_i^S(\theta) = (a - \theta)/N$  and  $Q^S(\theta) = a - \theta$  for all  $\theta \in \Theta$ . It then follows from (12) and  $P(Q^S(\theta)) = \theta$  that the socially optimal quantity profile  $q^S(\theta)$  satisfies the

participation constraint (15) if  $\sum_i T_i(q^S(\theta)) \geq NK$ . Since maximizing  $SW^G(q; \theta)$  in (14) requires minimizing  $\sum_i T_i(q(\theta))$ , we must have  $\sum_i T_i(q^S(\theta)) = NK$ .

Because the marginal cost is only known to the firms, the regulator cannot directly enforce the socially optimal outcome at the given marginal cost. However, the regulator can implement this outcome by allowing the oligopolistic firms to play a Cournot game under a suitable chosen transfer mechanism  $(T_i)_i^N$ . In that game, firm  $i$  would maximize  $\pi_i^G(q; \theta)$  given by (11). Let  $q^*(\theta)$  denote the Cournot (Nash) equilibrium profile for that game. Then, for all  $i$  it must be true that

$$\pi_i^G(q_i^*(\theta), q_{-i}^*(\theta); \theta) \geq \pi_i^G(q_i, q_{-i}^*(\theta); \theta) \text{ for all } q_i \geq 0. \quad (16)$$

A mechanism  $(T_i)_i^N$  is said to Nash implement the socially optimal outcome for the described oligopolistic industry if  $q^*(\theta) = q^S(\theta)$  for all  $\theta \in \Theta$ . This mechanism also satisfies the participation constraint (15) if  $\sum_i T_i(q^*(\theta)) = NK$  for all  $\theta \in \Theta$ . Using these definitions, we obtain the following result as a corollary to Proposition 1 of Gradstein (1995).

**Proposition 2.** *There exists a (transfer) mechanism that Nash implements the socially optimal outcome for the described oligopolistic industry and also satisfies the participation constraint (15).*

**Proof.** See the Appendix.

Note that at the socially optimal outcome, the social welfare reduces to

$$\begin{aligned} SW^G(\theta, \alpha, K, N) &\equiv SW^G(q^S(\theta); \theta) = V\left(\sum_i q_i^S(\theta)\right) - \theta \sum_i q_i^S(\theta) - NK, \\ &= \frac{[\sum_i q_i^S(\theta)]^2}{2} - NK, \\ &= \frac{(a - \theta)^2}{2} - NK. \end{aligned} \quad (17)$$

In the original mechanism of Gradstein (1995), the transfers are required to be balanced, i.e.,  $\sum_i T_i(q) = 0$  for all feasible profiles  $q$ , implying that the participation constraint for the oligopolistic firms are not taken into consideration. On the other hand, in our modified mechanism, the participation constraint can be satisfied at the industry level, i.e.,  $\sum_i \pi_i^G(q, \theta) = 0$ , implying that some individual firms may obtain negative gains at the implemented social outcome while some others would obtain positive gains. In other words, it is not possible to ensure  $\pi_i^G(q; \theta) \geq 0$  for each  $i$  (at all  $\theta$  and  $q$ ). Definitely, this is a serious weakness, casting doubts on the applicability of the presented implementation mechanism, i.e., the desirability of the oligopoly regulation, irrespective of the induced social welfare.

### 2.3. Regulated monopoly

Here, we consider that the oligopolistic industry is monopolized, with the regulator selecting one of the  $N$  symmetric firms (henceforth the monopolistic firm) as the sole producer. The monopolized market will then be subject to quantity and price regulation according to the direct-revelation mechanism proposed by BM (1982). This mechanism involves four functions  $(r, p, q, s)$  of the cost parameter. After the mechanism is announced, the monopolistic firm is asked to submit a cost report from the set  $\Theta$ . If its report is  $\hat{\theta}$ ,  $r(\hat{\theta})$  is the probability that the monopolistic firm is allowed to produce,  $p(\hat{\theta})$  and  $q(\hat{\theta})$  are the price and quantity of the good respectively, and  $s(\hat{\theta})$  is the subsidy received by the monopolistic firm. In that case, the welfare of the regulated firm would become

$$\pi(\hat{\theta}, \theta) = [p(\hat{\theta})q(\hat{\theta}) - C(q(\hat{\theta}), \theta)]r(\hat{\theta}) + s(\hat{\theta}). \quad (18)$$

A regulatory policy  $(r, p, q, s)$  is called by BM as *feasible* if it satisfies (19)-(22) for all  $\theta \in \Theta$ :

$$(i) \quad 0 \leq r(\theta) \leq 1 \quad (19)$$

$$(ii) \quad p(\theta) = P(q(\theta)) \quad (20)$$

$$(iii) \quad \pi(\theta, \theta) \geq \pi(\hat{\theta}, \theta), \text{ for all } \hat{\theta} \in \Theta \text{ (incentive - compat.)} \quad (21)$$

$$(iv) \quad \pi(\theta, \theta) \geq 0 \text{ (individual - rationality)} \quad (22)$$

Given a feasible regulatory policy  $(r, p, q, s)$  and any cost  $\theta \in \Theta$ , the consumer and the producer welfare would respectively become

$$CW(\theta) = [V(q(\theta)) - p(\theta)q(\theta)]r(\theta) - s(\theta), \quad (23)$$

$$\pi(\theta) \equiv \pi(\theta, \theta) = [p(\theta)q(\theta) - C(q(\theta), \theta)]r(\theta) + s(\theta). \quad (24)$$

The social welfare  $SW(\theta, \alpha)$  is the sum of the consumer welfare and  $\alpha$  fraction of the producer welfare, with  $\alpha \in [0, 1]$ . So,

$$\begin{aligned} SW(\theta, \alpha) &= CW(\theta) + \alpha \pi(\theta) \\ &= [V(q(\theta)) - C(q(\theta), \theta)]r(\theta) - (1 - \alpha) \pi(\theta). \end{aligned} \quad (25)$$

The regulator aims to choose a feasible mechanism  $(r, p, q, s)$  to

$$\text{maximize } \int_{\theta \in \Theta} SW(\theta, \alpha) f(\theta) d\theta \text{ subject to (19) - (22)}. \quad (26)$$

Before presenting the optimal regulatory policy of BM, we introduce the following assumption. (This assumption, which simplifies the optimal regulatory policy and consequently our results in Section 3, was not needed by BM in their characterization result.)

**Assumption 1.**  $F(\theta)/f(\theta)$  is nondecreasing on  $\Theta$ .

The following result is a direct corollary to Proposition 1 of BM (1982).

**Proposition 3.** *Let Assumption 1 hold. Then, the solution to the problem in (26) is  $(\bar{r}, \bar{p}, \bar{q}, \bar{s})$  satisfying equations (27)-(30) for all  $\theta \in \Theta$ :*

$$\bar{p}(\theta) = \theta + (1 - \alpha) \frac{F(\theta)}{f(\theta)} \quad (27)$$

$$P(\bar{q}(\theta)) = \bar{p}(\theta) \quad (28)$$

$$\bar{r}(\theta) = \begin{cases} 1 & \text{if } V(\bar{q}(\theta)) - \bar{p}(\theta)\bar{q}(\theta) \geq K \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

$$\bar{s}(\theta) = [K + \theta\bar{q}(\theta) - \bar{p}(\theta)\bar{q}(\theta)]\bar{r}(\theta) + \int_{\theta}^{\theta_1} \bar{r}(\tilde{\theta})\bar{q}(\tilde{\theta})d\tilde{\theta} \quad (30)$$

**Proof.** See the Appendix.

Given (27)-(30), we can calculate the producer welfare, consumer welfare, and the social welfare as

$$\begin{aligned} \pi^{BM}(\theta, \alpha, K) &= \int_{\theta}^{\theta_1} \bar{r}(\tilde{\theta})\bar{q}(\tilde{\theta})d\tilde{\theta} \\ &= \int_{\theta}^{\theta_1} \bar{r}(\tilde{\theta}) \left( \alpha - \tilde{\theta} - (1 - \alpha) \frac{F(\tilde{\theta})}{f(\tilde{\theta})} \right) d\tilde{\theta}, \end{aligned} \quad (31)$$

$$\begin{aligned} CW^{BM}(\theta, \alpha, K) &= [V(\bar{q}(\theta)) - K - \theta\bar{q}(\theta)]\bar{r}(\theta) - \pi^{BM}(\theta) \\ &= \frac{(\alpha - \theta)\bar{q}(\theta)\bar{r}(\theta)}{2} - K\bar{r}(\theta) - \pi^{BM}(\theta), \end{aligned} \quad (32)$$

$$\begin{aligned} SW^{BM}(\theta, \alpha, K) &= [V(\bar{q}(\theta)) - K - \theta\bar{q}(\theta)]\bar{r}(\theta) - (1 - \alpha)\pi^{BM}(\theta) \\ &= \frac{(\alpha - \theta)\bar{q}(\theta)}{2}\bar{r}(\theta) - K\bar{r}(\theta) - (1 - \alpha)\pi^{BM}(\theta), \end{aligned} \quad (33)$$

respectively. (Here, the superscript *BM* points to the regulation of the monopoly according to the BM mechanism.)

### 3. Results

For some of our results, we will need the following assumption.

**Assumption 2.** The demand parameter  $a$  satisfies

$$a > \sqrt{2K} + \theta_1 + \frac{1 - \alpha}{f(\theta_1)}. \quad (34)$$

When  $\alpha = 1$  and  $K = 0$ , Assumption 2 reduces to  $a > \theta_1$ , which was already assumed in the description of the inverse demand curve in Section 2. Assumption 2, along with Assumption 1, will yield the following corollary (to Proposition 3), ensuring that the monopolistic firm will always be allowed to operate under the BM mechanism.

**Corollary 1.** *Let Assumptions 1 and 2 hold. Then the optimal mechanism in (27)-(30) implies  $\bar{r}(\theta) = 1$  for all  $\theta \in \Theta$ .*

**Proof.** See the Appendix.

The below result shows that when the production requires fixed costs, one can always find a social welfare function with a sufficiently high weight for the producer welfare under which the regulation of the monopoly becomes, for the society, always superior to the regulated oligopoly, while the latter would become superior to the unregulated oligopoly.

**Proposition 4.** *Let Assumptions 1 and 2 hold. Then, for all  $K > 0$  and integers  $N \geq 2$ , there exists  $\bar{\alpha}(K, N) \in [0, 1)$  such that for all  $\alpha \in [\bar{\alpha}(K, N), 1]$ , we have*

$$SW^{BM}(\theta, \alpha, K) > SW^G(\theta, \alpha, K, N) > SW^C(\theta, \alpha, K, N) \text{ for all } \theta \in \Theta.$$

**Proof.** See the Appendix.

The above proposition partially changes when the fixed cost of production is zero. In that case, the highest social welfare can always be attained not only by the regulated monopolization of the oligopoly but also by the regulation of the oligopoly itself.

**Proposition 5.** *Let Assumptions 1 and 2 hold. Then, for all integers  $N \geq 2$ , there exists  $\bar{\alpha}(N) \in [0, 1)$  such that for all  $\alpha \in [\bar{\alpha}(N), 1]$ , we have*

$$SW^{BM}(\theta, \alpha, 0) = SW^G(\theta, \alpha, 0, N) > SW^C(\theta, \alpha, 0, N) \text{ for all } \theta \in \Theta.$$

**Proof.** See the Appendix.

Propositions 4 and 5, both of which are valid when  $\alpha$  is sufficiently close to 1, are silent for an arbitrary value of  $\alpha \in [0, 1)$ . However, we are able to show that for oligopolies with sufficiently high number of firms, a regulated monopoly is always socially more desirable than the regulated or unregulated oligopoly for any  $\alpha \in [0, 1)$ , provided that there are fixed costs of production.

**Proposition 6.** *Let Assumption 1 hold. Then, for all  $\alpha \in [0, 1)$  and  $K > 0$ , there exists an integer  $\bar{N}(\alpha, K) \geq 2$  such that for all integers  $N \geq \bar{N}(\alpha, K)$ , we have*

$$SW^{BM}(\theta, \alpha, K) > SW^C(\theta, \alpha, K, N) > SW^G(\theta, \alpha, K, N) \text{ for all } \theta \in \Theta.$$

**Proof.** See the Appendix.

One should note in the above proposition that the regulated oligopoly is not only inferior to the regulated monopoly but also to the unregulated oligopoly when the number of oligopolistic firms is sufficiently high. This is simply because of the participation constraint that requires consumers to finance the total fixed costs,  $NK$ , in the case of regulation. Since the weight of the consumer welfare is "one" in the social welfare function, the contribution of the total fixed costs to the social welfare becomes equal to  $-NK$  in the case of a regulated oligopoly. On the other hand, the said contribution becomes  $-\alpha NK$  in the case of an unregulated oligopoly, since the fixed costs are incurred by the oligopolistic firms and their welfare weight is  $\alpha$ . The difference of these two

terms,  $-(1 - \alpha)NK$ , is always negative (unless  $\alpha = 1$  or  $K = 0$ ), making the regulated oligopoly inferior to the unregulated oligopoly for sufficiently high values of  $N$ . It follows that when  $\alpha$  is different from 1 and when fixed costs are present in the production, the regulator should leave the oligopoly unregulated, unless, of course, her policy choices involve the regulated monopolization of the oligopoly, which happens to be the first-best policy option. As shown by the next result, Proposition 6 is no longer valid when there are restrictions on the size of the fixed costs.

**Proposition 7.** *Let Assumption 1 hold and consider any  $\alpha \in [0,1)$ . Then, (i) for all  $N \geq 2$  there exists  $K(\alpha, N) > 0$  such that  $SW^G(\theta, \alpha, K, N) > SW^C(\theta, \alpha, K, N)$  for all  $K \in [0, K(\alpha, N)]$  and  $\theta \in \Theta$ , and (ii) there exist  $K(\alpha) > 0$  and integer  $N(\alpha) \geq 2$  such that  $SW^C(\theta, \alpha, K, N) > SW^{BM}(\theta, \alpha, K)$  for all  $K \in [0, K(\alpha)]$ , integers  $N \geq N(\alpha)$ , and  $\theta \in \Theta$ .*

**Proof.** See the Appendix.

Proposition 7 shows that in situations where the fixed costs of production are sufficiently small, regulating an oligopoly is always superior to not regulating it from the viewpoint of the society. Additionally, if in these situations the number of firms is sufficiently large, even the unregulated oligopoly becomes socially superior to the regulated monopoly.

#### 4. Conclusion

In this paper, we have studied whether a Cournot oligopoly with unknown costs should be left unregulated, or optimally regulated, or first monopolized and then optimally regulated. For the given oligopolistic industry, we have first characterized the Cournot (Nash) equilibrium allocation of the quantity competition game (Proposition 1). Next we have presented the equilibrium allocation when the oligopoly is optimally regulated (Proposition 2) according to the mechanism of Gradstein (1995). As the final component of our model, we have also considered the situation when one of the oligopolistic firms is granted the monopoly right and optimally regulated (Proposition 3) according to the mechanism of BM (1982). Calculating the actual social welfare in each of these three cases, we have been able to show that when there are fixed costs in production and also the weight of the producer welfare in the social welfare function, i.e., the  $\alpha$  parameter, takes a sufficiently high value in the unit interval, the realized social welfare is always higher for the optimally regulated monopoly than for the regulated oligopoly, while regulating the oligopoly becomes socially more desirable than not regulating it (Proposition 4). This result slightly changes when fixed costs are absent in production. In that case, the highest social welfare can always be attained not only with an optimally regulated monopoly but also with an optimally regulated oligopoly (Proposition 5).

Next, we have considered oligopolies with large number of firms (or simply large oligopolies), allowing us to make welfare comparisons for values of  $\alpha$  not very close to one. We have showed that for any  $\alpha$  less than one, there always exists a sufficiently large oligopoly such that the outcome obtained from the optimally regulated monopolization of this oligopoly is always (socially) welfare superior to the unregulated (Cournot) outcome, which, on the other hand, is always welfare superior to the regulated outcome of the same

oligopoly (Proposition 6). However, this result dramatically changes if there are size restrictions on the fixed costs. For situations where the fixed costs of production are sufficiently small, we find that optimally regulating a Cournot oligopoly is always superior to not regulating it from the viewpoint of the society (Proposition 7(i)). Additionally, if in these situations the number of firms is sufficiently large, even the unregulated oligopoly may become socially superior to the optimally regulated monopoly (Proposition 7(ii)).

To conclude, this study has revealed that whether and how to regulate an oligopolistic industry cannot be answered independently of the number of the oligopolistic firms and the size of their fixed costs, and the definition of the social welfare function, i.e., the relative weight of the producer welfare. The sum of information rents an optimal regulatory mechanism should offer to the regulated firms can be as low as the total fixed costs of production in the case of an oligopoly, rendering a regulated oligopoly socially more desirable than a regulated monopoly provided that these fixed costs are sufficiently small. Oppositely, in cases where the production processes require large fixed costs, the regulated monopolization of an oligopoly becomes socially superior to the regulation of the oligopoly if the social welfare function does not treat producers and consumers very asymmetrically or if the number of oligopolistic firms is sufficiently large.

---

## References

---

- Baron, D. and Besanko, D., 1984. Regulation, Asymmetric Information, and Auditing, *RAND Journal of Economics*, 15, pp. 447-470.
- Baron, D. and Myerson, R.B., 1982. Regulating a Monopolist with Unknown Costs, *Econometrica*, 50, pp. 911-930.
- Dasgupta, P.S., Hammond, P.J. and Maskin, E.S., 1979. The Implementation of Social Choice Rules: Some Results on Incentive Compatibility, *Review of Economic Studies*, 46, pp. 185-216.
- Dupuit, J., 1844. De la Mesure de l'Utilité des Travaux Publics. *Annales des Ponts et Chaussées*, 2nd Series, 8 (translated by R.H. Barback as On the Measurement of the Utility of Public Works. *International Economics Papers*, 2, pp. 83-110, 1952).
- Gradstein, M., 1995. Implementation of Social Optimum in Oligopoly, *Economic Design*, 1, pp. 319-326.
- Harris, M. and Townsend, R.M., 1981. Resource Allocation under Asymmetric Information, *Econometrica*, 49, pp. 33-64.
- Laffont, J.J. and Tirole, J., 1986. Using Cost Observation to Regulate Firms, *Journal of Political Economy*, 94, pp. 614-641.
- Lewis, T. and Sappington, D., 1988. Regulating a Monopolist with Unknown Demand, *American Economic Review*, 78, pp. 986-998.
- Myerson, R.B., 1979. Incentive Compatibility and the Bargaining Problem, *Econometrica*, 47, pp. 61-74.
- Riordian, M., 1984. On Delegating Price Authority to a Regulated Firm, *RAND Journal of Economics*, 15, pp. 108-115.
- Rochet, J.C., 1984. Monopoly Regulation with Two-Dimensional Uncertainty, *CEREMADE Discussion Paper*, August.
- Sappington, D., 1983. Optimal Regulation of a Multiproduct Monopolist with Unknown Technological Capabilities, *Bell Journal of Economics*, 14, pp. 453-463.

## Appendix

**Proof of Proposition 1.** The problem of firm  $i$ , given that the rest of the firms choose quantities in  $q_{-i}$ , is given by

$$\max_{q_i \geq 0} \pi_i(q_i, q_{-i}; \theta) = \max_{q_i \geq 0} P(Q)q_i - \theta q_i - K. \quad (35)$$

The first-order condition for (35) is given by

$$q_i = \frac{a - \theta - \sum_{j \neq i} q_j}{2}. \quad (36)$$

The symmetry of the firms implies  $q_j = q_i$  for all  $j \neq i$ , further implying  $\sum_{j \neq i} q_j = (N - 1)q_i$ . Inserting this into (36) yields (6). The second-order condition for maximization is satisfied since  $\partial \pi_i(q_i, q_{-i}; \theta) / \partial q_i = -2$  at all  $(q_i, q_{-i})$ . *Q.E.D.*

**Proof of Proposition 2.** In our model the inverse demand function is a polynomial of degree 1 and  $N$  is assumed to be at least 2. Proposition 1 of Gradstein (1995, p. 324) shows that if the inverse demand function is a polynomial of at most  $(N - 1)$ th degree, then socially optimal outcome can be Nash implemented by a mechanism  $T_i: [0, a] \rightarrow \mathfrak{R}$ ,  $i = 1, \dots, N$  such that  $\sum_i T_i(q) = 0$  for all feasible profiles  $q$ . Let  $(T_i)_i^N$  be such a mechanism, and consider the modified mechanism  $T_i^*: [0, a] \rightarrow \mathfrak{R}$ ,  $i = 1, \dots, N$  such that  $T_i^*(q) = K + T_i(q)$  for all feasible profiles  $q$ . It is clear that the modified mechanism Nash implements the socially optimal outcome and also satisfies the participation constraint (15). *Q.E.D.*

**Proof of Proposition 3.** Due to Assumption 1,  $\bar{p}(\theta)$  is nondecreasing in  $\theta$  and therefore  $\bar{q}(\theta)$  and  $\bar{r}(\theta)$  are nonincreasing in  $\theta$ . Thus, the proposition is a direct corollary to Proposition 1 of BM (1982, pp. 920-921). *Q.E.D.*

**Proof of Corollary 1.** Note that Assumption 2 can be rewritten as  $a > \sqrt{2K} + \theta_1 + (1 - \alpha)F(\theta_1)/f(\theta_1)$ , since  $F(\theta_1) = 1$ . Then Assumptions 1 and 2 together would imply that for all  $\theta \in \Theta$  we have  $a > \sqrt{2K} + \theta + (1 - \alpha)F(\theta)/f(\theta) = \sqrt{2K} + \bar{p}(\theta)$ , further implying  $\bar{q}(\theta) = a - \bar{p}(\theta) > \sqrt{2K}$  or  $\bar{q}(\theta)^2/2 > K$ . Since  $V(\bar{q}(\theta)) - \bar{p}(\theta)\bar{q}(\theta) = \bar{q}(\theta)^2/2$ , equation (29) implies that  $\bar{r}(\theta) = 1$  for all  $\theta \in \Theta$ . *Q.E.D.*

**Proof of Proposition 4.** Since Assumption 1 holds, the optimal mechanism regulating a monopoly is given by (27)-(30). Now let  $\alpha = 1$  and consider any  $K > 0$ ,  $\theta \in \Theta$  and integer  $N \geq 2$ . Equation (10) implies

$$SW^C(\theta, 1, K, N) = \frac{(a - \theta)^2 (N^2 + 2N)}{2(N + 1)^2} - NK, \quad (37)$$

whereas equation (17) implies

$$SW^G(\theta, 1, K, N) = \frac{(a - \theta)^2}{2} - NK. \quad (38)$$

On the other hand, it follows from equation (33) that

$$SW^{BM}(\theta, 1, K) = \frac{(a - \theta)^2}{2} - K, \quad (39)$$

since  $\alpha = 1$  implies  $\bar{q}(\theta) = a - \theta$  and Corollary 1 implies (thanks to Assumptions 1 and 2)  $\bar{r}(\theta) = 1$  for all  $\theta \in \Theta$ . Apparently,  $SW^{BM}(\theta, 1, K) > SW^G(\theta, 1, K, N) > SW^C(\theta, 1, K, N)$ . Since  $SW^{BM}(\theta, \alpha, K)$ ,  $SW^G(\theta, \alpha, K, N)$ , and  $SW^C(\theta, \alpha, K, N)$  are all continuous w.r.t.  $\alpha$ , there exists  $\bar{\alpha}(K, N) \in [0, 1)$  such that for all  $\alpha \in [\bar{\alpha}(K, N), 1]$ , we have  $SW^{BM}(\theta, \alpha, K) > SW^G(\theta, \alpha, K, N) > SW^C(\theta, \alpha, K, N)$  for all  $\theta \in \Theta$ . *Q.E.D.*

**Proof of Proposition 5.** Since Assumption 1 holds, the optimal mechanism regulating a monopoly is given by (27)-(30). Let  $K = 0$ , and pick any  $\theta \in \Theta$  and integer  $N \geq 2$ . Also let  $\alpha = 1$ . Equation (10) implies

$$SW^C(\theta, 1, 0, N) = \frac{(a - \theta)^2}{2} \frac{(N^2 + 2N)}{(N + 1)^2}, \quad (40)$$

whereas equation (17) implies

$$SW^G(\theta, 1, 0, N) = \frac{(a - \theta)^2}{2}. \quad (41)$$

On the other hand, it follows from equation (33) that

$$SW^{BM}(\theta, 1, 0) = \frac{(a - \theta)^2}{2}, \quad (42)$$

since  $\alpha = 1$  implies  $\bar{q}(\theta) = a - \theta$  and Corollary 1 implies (thanks to Assumptions 1 and 2)  $\bar{r}(\theta) = 1$  for all  $\theta \in \Theta$ . Apparently,  $SW^{BM}(\theta, 1, 0) = SW^G(\theta, 1, 0, N) > SW^C(\theta, 1, 0, N)$ . Since  $SW^{BM}(\theta, \alpha, 0)$ ,  $SW^G(\theta, \alpha, 0, N)$ , and  $SW^C(\theta, \alpha, 0, N)$  are all continuous w.r.t.  $\alpha$ , there exists  $\bar{\alpha}(N) \in [0, 1)$  such that for all  $\alpha \in [\bar{\alpha}(N), 1]$ , we have  $SW^{BM}(\theta, \alpha, 0) = SW^G(\theta, \alpha, 0, N) > SW^C(\theta, \alpha, 0, N)$  for all  $\theta \in \Theta$ . *Q.E.D.*

**Proof of Proposition 6.** Since Assumption 1 holds, the optimal mechanism regulating a monopoly is given by (27)-(30). Now, pick any  $\alpha \in [0, 1)$  and  $K > 0$ . Equation (10) implies

$$\lim_{N \rightarrow \infty} SW^C(\theta, \alpha, K, N) = \lim_{N \rightarrow \infty} \frac{(a - \theta)^2}{2} \frac{(N^2 + 2\alpha N)}{(N + 1)^2} - \alpha NK = -\infty. \quad (43)$$

On the other hand,  $SW^{BM}(\theta, \alpha, K)$  is always finite. Thus, there exists an integer  $\bar{N}_1(\alpha, K) \geq 2$  such that for all integers  $N \geq \bar{N}_1(\alpha, K)$ , we have  $SW^{BM}(\theta, \alpha, K) > SW^C(\theta, \alpha, K, N)$  for all  $\theta \in \Theta$ . Also note that subtracting (10) from (17) yields

$$\begin{aligned} SW^G(\theta, \alpha, K, N) - SW^C(\theta, \alpha, K, N) \\ = \frac{(a - \theta)^2}{2} \frac{[1 + 2(1 - \alpha)N]}{(N + 1)^2} - (1 - \alpha)NK. \end{aligned} \quad (44)$$

It follows that

$$\lim_{N \rightarrow \infty} [SW^G(\theta, \alpha, K, N) - SW^C(\theta, \alpha, K, N)] = -\infty, \quad (45)$$

implying that there exists an integer  $\bar{N}_2(\alpha, K) \geq 2$  such that for all integers  $N \geq \bar{N}_2(\alpha, K)$ , we have  $SW^C(\theta, \alpha, K, N) > SW^G(\theta, \alpha, K, N)$  for all  $\theta \in \Theta$ . Let  $N(\alpha, K) = \max\{\bar{N}_1(\alpha, K), \bar{N}_2(\alpha, K)\}$ . So,

$SW^{BM}(\theta, \alpha, K) > SW^C(\theta, \alpha, K, N) > SW^G(\theta, \alpha, K, N)$  for all  $\theta \in \Theta$  if  $N \geq \bar{N}(\alpha, K)$ . *Q. E. D.*

**Proof of Proposition 7.** Consider any  $\alpha \in [0, 1)$ .

*Part (i):* Inserting  $K = 0$  into (10) and (17), we obtain

$$SW^C(\theta, \alpha, 0, N) = \frac{(a - \theta)^2}{2} \frac{N^2 + 2\alpha N}{(N + 1)^2}, \quad (46)$$

and

$$SW^G(\theta, \alpha, 0, N) = \frac{(a - \theta)^2}{2}, \quad (47)$$

respectively. Evidently,  $SW^G(\theta, \alpha, 0, N) > SW^C(\theta, \alpha, 0, N)$  for all integers  $N \geq 2$  and  $\theta \in \Theta$ . Since  $SW^C(\theta, \alpha, K, N)$  and  $SW^G(\theta, \alpha, K, N)$  are continuous w.r.t.  $K$ , we conclude that for all  $N \geq 2$  there exists  $K(\alpha, N) > 0$  such that  $SW^G(\theta, \alpha, K, N) > SW^C(\theta, \alpha, K, N)$  for all  $K \in [0, K(\alpha, N)]$  and  $\theta \in \Theta$ .

*Part (ii):* Since Assumption 1 holds, the optimal mechanism regulating a monopoly is given by (27)-(30). We insert  $K = 0$  to (33) to get

$$SW^{BM}(\theta, \alpha, 0) = \frac{(a - \theta)\bar{q}(\theta)}{2} - (1 - \alpha)\pi^{BM}(\theta), \quad (48)$$

since  $K = 0$  implies  $\bar{r}(\theta) = 1$  for all  $\theta \in \Theta$ . Note also that

$$\lim_{N \rightarrow \infty} SW^C(\theta, \alpha, 0, N) = \frac{(a - \theta)^2}{2}, \quad (49)$$

which is higher than  $SW^{BM}(\theta, \alpha, 0)$ , for all  $\theta \in \Theta$  and  $\alpha \in [0, 1)$ , since  $\bar{q}(\theta_0) = a - \theta_0$  and  $\bar{q}(\theta) < a - \theta$  for all  $\theta \in (\theta_0, \theta_1]$ , and  $\pi^{BM}(\theta) > 0$  for all  $\theta \in [\theta_0, \theta_1)$  and  $\pi^{BM}(\theta_1) = 0$ . Therefore, there exists an integer  $\bar{N}(\alpha) \geq 2$  such that for all integers  $N \geq \bar{N}(\alpha)$ , we have  $\lim_{N \rightarrow \infty} SW^C(\theta, \alpha, 0, N) > SW^{BM}(\theta, \alpha, 0)$  for all  $\theta \in \Theta$ . Because  $SW^C(\theta, \alpha, K, N)$  and  $SW^{BM}(\theta, \alpha, K, N)$  are continuous w.r.t.  $K$ , there exist  $K(\alpha) > 0$  and an integer  $N(\alpha) \geq 2$  such that  $SW^C(\theta, \alpha, K, N) > SW^{BM}(\theta, \alpha, K)$  for all  $K \in [0, K(\alpha)]$ , integers  $N \geq N(\alpha)$ , and  $\theta \in \Theta$ . *Q. E. D.*