Using the input-output model in macroeconomic analysis and forecasting studies

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Abstract. In this article the authors present the main aspects of a country’s macroeconomic system. It shows that it is a complex structured economic sectors (pure sectors). Also it makes reference to the free market, placing the problem of identifying the interaction of market forces. Inside the national economy there is an intermediate consumption that is showing the amount consumed by branch "c" from all other branches. Input fields are used in this respect in order to show that at a macroeconomic level the model is complex, on the basis of the model calculation and analysis of economic is performed. On the basis of coefficients the existence and intensity of links between branches of national economy is determined. It is important to establish the how aggregation of branches is taking place. The number of branches and the concentration degree are subordinate requirements of study links and proportions that are performed in an economic activity. Aggregation of activities in order to establish the branches used for creating input-output tables is made according to several criteria such as: product identification, common destination of the finished product, similarity raw materials consumed or technological processes and likeness of quantitative structure cost price of production.

The branches for the balance should be homogeneous and represent a grouping of homogeneous production units.

Keywords: forecasting, production, intermediary consumption, technology, coefficient, matrix.

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Introduction

The macro-level analysis can be performed by using the input-output table model. The summary table contains all the data necessary for obtaining the coefficients for direct expenditure ($a_{ij}$) or total ($A_{ij}$). In order to use this model it is necessary to clarify some issues regarding: homogenous branch, system for classification of activities, sizing of the final product or of the final demand, the calculation of the direct costs for material coefficient, determination of gross production, determination of the variables on which the model is based on. The homogeneous branch produces only goods and services specified in the classification and when difficult elements appear we should consider criteria for classification in this system. By using the International Standard Classification of activities and using the system of national accounts and classifications made by the Statistical Office, we find solutions for this input-output model in order to be used not only for analysis but also for analyzing internal and international comparability. Currently, the system used is CAEN 2 which assures a classification of the branches and national economic activities. The input-output system includes a number of tables, but the bottom line is the summary table of this model divided into four quadrants, each with significance that helps economic analysis of production, analysis of economic results, and the unitary calculation of the domestic product per branch and for the total national economy. This model is particularly formalized even though it appears quite complex, offers a perspective to have data and information to ensure effective and efficient management of resources.

In Romania it is also used, by completing this input-output system that gives an opportunity to review and study the national economy. In this article, we will limit ourselves to presenting the main theoretical aspects involved using input-output tables in macroeconomic analysis.

1. Literature review

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Hendrickson, Lave and Matthews (2006) focus on goods and services environmental cycle assessment. Parker (2011) and Romer and Romer (2010) research the results of fiscal policies’ enforcement in particular cases of economic environment conditions, Auerbach and Gorodnichenko (2012) analyze the feedback to fiscal policies. Păunică (2014) focuses on indicators regarding the forecast of infrastructure indicators in a specific region of Romania. The modeling is an important instrument of macroeconomic analysis, the tools of modeling are presented by Anghelache and Anghel (2014) and Anghelache et al. (2016).

2. Research methodology and data

The connections between branches characterize and measure dependencies between various branches of the national economy; for this purpose, using functional dependencies between production volume and production costs.

For it is of great importance, the coefficients of direct material expenses, noted with $a_{ij}$ that show how industry and production is consumed in order to produce one unit of projected output in industry branch $j$. It is calculated as the ratio between the intermediate consumption products and overall consumption or crude intermediate and the branches consuming, the formula is shown below:

$$ a_{ij} = \frac{x_{ij}}{X_j} $$  \hspace{1cm} (1)

It is assumed that part of the sector for production and the productive use in industry $j$ is directly proportional to the output of branch $j$, namely:

$$ x_{ij} = a_{ij} \cdot X_j $$  \hspace{1cm} (2)

The $a_{ij}$ coefficients are identified herein with specific consumptions. Their size depends on the technical level and technological level of production, which is why they are named technological coefficients. The $a_{ij}$ coefficients form a square matrix named A which has an essential role in all the applications of the balance.

Based on the balance model we can calculate the distribution coefficients of production branch $i$ consumed in a productive branch $j$ and the overall product of the production branch, meaning:

$$ h_{ij} = \frac{x_{ij}}{X_j} $$  \hspace{1cm} (3)

Between coefficients $a_{ij}$ coefficients $h_{ij}$ there is a close linked, a mutual dependency as shown below:
The A matrix is characterized the structure of social production, the division of its branches and links between branches. From such considerations the A matrix content is called structural matrix. The elements of A matrix as shown by their contents have only positive or zero values, they cannot have negative values.

The equation:

\[ x_{ji} = a_{ji} \cdot X_j \]  

(5)

shows the between the consumption of a branch and its total there are constant relations.

Equations of sharing production gains following form:

\[ Xi = \sum a_{ji} + X_j + Y_i, \quad j = 1, n \]  

(6)

From this relationship we can determine the expression of the product or of the final demand:

\[ Xi - \sum a_{ji} \cdot X_j = Y_i \]  

(7)

The equations system of production sharing is a static model of the connection balance between the branches; it is a linear system with n equations and 2n unknowns. This model allows to find n variables if we give n values to the final product or the final branches. When required to calculate the overall product it is for the overall product design. To this end it is necessary to determine the coefficients of total material costs noted with \( A_{ij} \), showing the production of each branch must provide the national economy to achieve a final product unit for a certain branch. These rates include all direct material costs and indirect costs. Direct expenses relate the connection between two adjacent products from two adjacent technological branches. Indirect costs express the same kind of occurrence in all sectors that contribute to achieving the concerned product.

The coefficients of total material costs, can only be sized with the help of branches; starting from the static model of the balance in matrix form that can be written as follows:

\[ X = A \cdot X + Y, \]  

(8)

from where we can deduce that:

\[ X = (E - A)^{-1} * Y \]  

(9)

The expression \((E - A)^{-1}\) give value to the coefficients of total material costs. The \( A_{ij} \) coefficients make a square matrix. The model assumes the following form:

\[ X_i = \sum A_{ij} \cdot Y_j \]  

(10)

The static model of the branches reflects the relationship between the overall product or profit of each branch and the end product of all branches, points out that the final product
of a branch can start a change in the chain in all branches of the national economy and not only in the sectors concerned.

One of the main purposes for making the table is to create a basis for improving the methods of macroeconomic analysis.

With the help of statistical data are collected according to the model adopted that allow widening and deepening analyzes on the processes of production, distribution and use of the final product. Developing this tool leads to the discovery of inaccuracies and gaps in the statistics of a country, contributing to the improvement of statistical indicators used and computational techniques or the improvement of statistics as a tool of knowledge.

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The input-output table allowed for the first time, introducing in the economic activity "analysis of inputs and outputs" based on the use of mathematical models and the electronic techniques of calculation.

The amount of data that it provides makes the systematic nature of this information table to have a broad and effective use in economic analysis and forecasting calculations.

The table data are of great importance for the calculation of indicators that allow the study of: economic structure, the proportion of economic development, the interdependence of economic sectors; participation of different sectors in the setting output for final consumption.

The indicators included in the table - gross output, value added of the end product - computed for each branch included in the classification used, analyzed, allow characterization of deeper, more complete structure of the national economy.

For this purpose calculated and analyzed the proportion of the gross production, the final product in terms of GDP and the added value in each branch related to the total.

\[
\frac{P_{bi}}{\sum P_{bi}}, \frac{V_{ai}}{\sum V_{ai}}, \frac{P_{fi}}{\sum P_{fi}}
\]

\[(11)\]

\(P_b\) represents the raw production of a branch;
\(V_a\) the added value of a branch;
\(P_f\) represents the final product of a branch;
\(i\) represents 1,2,..., \(n\).

These sizes which expresses the contribution of each branch to the formation of gross production, the final product (expressed in GDP) or the value added, have a special significance for studying the structure of the economy. It can also calculate the indicators expressing the proportion of various elements of the value added or the final product (household consumption, government consumption, investment, exports) of each branch
to the size of these elements in the national economy. Studying these proportions for a longer period of time makes clearer the changes that have taken place in the economy of the country.

In order to analyze the economic structure, the branches of the classification, as was shown in the previous paragraph, can be concentrated according to certain criteria.

They can be grouped into: the main branches which provides the raw material (mining and agriculture), which is the primary sector; industries that process agricultural products and mining industry, which is the secondary sector; and branches providing services, tertiary sector.

Calculating the ratio of these three sectors Gross final product, or value added, we get a series of indicators that allow to draw conclusions about the structure of the economy and its character.

If the specific gravity of the branches of the primary sector is high, the economy of that country is a rudimentary economy, and if the secondary branches sector have a share in the formation of the indicators listed, the economy of that country is a developed economy. Increasing the share of secondary sector and tertiary gross national product size, final product and added value indicates economic development.

The data from the input-output table also allows to calculate indicators characterizing the distribution of employment, time worked by industry and industry groups, calculating the proportion of the population employed, and the time spent in each branch from the total. To know some aspects of the effectiveness of each branch, these indicators are correlated with the added value of each branch, and its proportion in total.

The indicators listed above, calculated based on table data, compared in time revealed aspects of changes that have taken place in the structure of the national economy and its development compared to other countries reveals similarities or differences between the structures of their economy.

The data from the input-output table underlying the characterization and analysis of interdependencies between economic sectors.

As was shown in the input-output activity each branch is shown under double aspect: as a branch supplying and consuming. In this case, the product flows shown in the table shows dependency output of a branch the size of production at other branches. Thus, entries for each branch power delivery express what products and how much should be produced in other branches to get a certain amount of production in the branch.

Production of a branch does not depend only on its direct relations with suppliers branches, but also the production branches not directly supplying products, but are linked indirectly through the direct relationship they have branches supplying them. These direct and indirect links between branches make some change in the output of a branch to determine changes in many sectors of the economy. Economic dependence on a branch other branches can be measured by expense coefficients.
One of the main results of processing of the input branches is the obtained coefficients - direct costs coefficients \( (a_{ij}) \) and total expenditure coefficients \( (A_{ij}) \).

**Coefficients of direct expenses** – express expenses of a product to obtain other products or spending of a branch to obtain a value of production units from other branches; e.g. coal consumption for electricity production; consumption of metal to obtain tools; the production of metal per 1 000, 10 000 or 100 000 RON or the production of construction equipment for oil extraction etc.

**The coefficients of total expenditure** is calculated by inverting the matrix coefficients of direct expenses and expresses besides direct costs and expenditure on other products involved in obtaining the corresponding production. So coefficients of total expenditure show direct and indirect costs of a product to obtain a unit of another product. For example, to determine the total costs for coal for the production of machine tools have taken into account not only the expenses incurred by the companies that produce machine tools, but also coal consumption in other companies that supply products to obtain machine tools, such as coal consumption in establishments producing metal, electricity etc. So, it is necessary to consider all inputs of carbon held in all its technological links in the production of the machine tools.

In preparing input-output table, firstly it is calculated the table of direct expenses (matrix \( a_{ij} \)).

\[ a_{ij} = \frac{z_{ij}}{s_j} \]  

(12)

Coefficients of direct expenses actually express the branch structure of material consumption per unit output value of a certain branch.

Analyzes of the coefficients of direct costs allows to highlight branches (products) that have a decisive role in the productive consumption of each sector and their impact on various branches of the changes that occur in the output of other branches.

Matrix \( (a_{ij}) \) represents a milestone in the development and analysis of the links between branches. This allows characterization of direct links between branches and is of great importance for economic analysis and planning of the national economy.

On the basis of the matrix \( (a_{ij}) \) it is calculated the coefficients \( A_{ij} \) (total costs coefficients). They are linked to the coefficients of direct expenditure - intermediate consumption and final consumption structure.

Total expenditure coefficients \( A_{ij} \) used to determine the various branches of production required 5

\[ V_j = \sum_{i=1}^{n} N_{ij} + I_{mj} = A_j + I_{nj}^{ind} + VNA_j + VIP_j \]  

(13)
Summing gross value added of all branches to obtain GDP at market prices, according to:
\[ \sum_{j=1}^{n} (VP_j - N_{ij} - l_{mij}) = \sum_{j=1}^{n} (A_j + i^{ind}_{nj} + VNA_j + VIP_j) = PIB_{pp} \] (14)
Since VNA_{ij} + VIP_{ij} represent the income arising from domestic production factors:
\[ \sum_{j=1}^{n} (A_j + VF_j + i^{ind}_{j} + +S_j) = PIB_{pp} \] (15)
Eliminating depreciation of fixed capital from GDP_{pp} results in NDP_{pp}.
If you add up all the components in square II, we obtain GDP at market prices calculated by the method of end-use, namely:
\[ \sum_{i=1}^{n} (CP_i + CST_i + l^b_t + E_{si} + l_{mi}) = PIB \] (16)
Starting from the main objective pursued by constructing input-output tables - highlighting interdependencies between production branches - there are some deviations from the results obtained from macroeconomic calculations. While the determination of indicators in the national accounts does not take account of deliveries between companies (intermediate production), in case of the table these deliveries are taken into account if these deliveries are referring to those that occur between units within the same branch.
To meet the needs of management forecasting of the input branches (BLR) is developed as forecast balance for the period. The starting point is the statistical balance. Development of the balance forecast period involves the following operations:

- **Sizing final product or final demand**

Final demand is determined exogenously off balance model. The national economy, the demand should be designed and structured based on the following elements: analysis of the existing level and structure of final demand based on statistical balance; research and evaluation of the factors influencing the growth of final demand, the correlation between the accumulation fund consumer replacement fund; developing hypotheses prediction on the relationship between the accumulation fund and the consumption fund in the foundation of the hypothesis prediction on the relationship between the accumulation fund and the consumption fund by labor productivity and training in the production resources of labor, changes in accumulation of productive efficiency and the factors conditioning increasing material wealth; development forecasts on the development of each branch of production taking into account the prospects of foreign economic relations.

- **Calculating the coefficients of direct material costs and updating them**

Coefficients of direct material costs is usually calculated based on the static balance of the links between branches; however, these rates cannot be mechanically extrapolated for the forecast period, those coefficients should be updated conditions of the projection period as a result of changes produced in the national economy. Main factors influencing coefficients are direct material costs: mutual substitution of various energy sources; substituting natural raw materials, synthetic materials and products tend to shift towards
more complex, higher processed; general increase in consumption of industrially processed products and service consumption; the downward trend in manufacturing of material consumption.

- **Calculation of the total material cost coefficients**

The coefficients of total material costs are determined based on the updated matrix of technological coefficients. For the calculation of the total material costs coefficients to following operations are followed: calculating the difference matrix \( E - A \) and its determinant; writing and calculation transposed its association; the inverse matrix calculation \( (E - A)^{-1} \).

- **Determination of the gross or overall product of the branches**

This operation is of special importance for establishing proportions and balancing the economy; the balance allows establishing net consumer demand and the gross product of each branch so as to completely cover the requirements of the final product. Final products are coordinated global and balanced each other. The product of the branch is based to the needs of internal and external market and technical and productive potential. Overall product is calculated by the relationship:

\[
X_i = \sum A_{ij} \cdot Y_j
\]  

(17)

- **The link between the necessary volume with the possible volume of the global product and establishing the retained indicator as an objective**

The required volume of global product resulting from the calculations of balance and the possible volume is established outside the balance, depending on production capacities and their use, the possibilities to ensure the necessary raw materials, energy and labor, the financial and currency. The ideal situation would be that amount must be equal to the feasible volume. In reality there are differences between the two variables.

- **Determining the variables from quadrant III:**

Depreciation of fixed capital, total material costs, net value added structured on elements, gross value added and imports.

*Depreciation on branches* \( (Z_i) \) is calculated by multiplying the depreciation coefficients \( (z_i) \) with the overall product:

\[
Z_i = z_i \cdot X_i
\]  

(18)

The depreciation coefficients are calculated based on the synthetic balance of the ties between the \( Z \) branches, as shown:

\[
z_i = \frac{Z_i}{X_i}
\]  

(19)
and are updated for the condition of the period according to the forecast of the factors that have a influences on the action of increase or decrease. On the economy, fund amortization is calculated by adding depreciation branches:

\[ Z = \sum Z_i = Z_1 + Z_2 + \ldots + Z_i + \ldots + Z_n = z_1 \cdot X_1 + z_2 \cdot X_2 + \ldots + z_i \cdot X_i + \ldots + z_n \cdot X_n \]  

(20)

The costs of total material of the branches consumption is calculated by adding the intermediate columns included in quadrant I associated with the corresponding depreciation. Use the following relationships:

\[ CM_i = x_{i1} + x_{i2} + \ldots + x_{i\pi} + \ldots + x_{i\eta} + Z_i = \sum x_{ij} + Z_i \]  

(21)

\[ CM_i = a_{i1} \cdot X_1 + a_{i2} \cdot X_2 + \ldots + a_{ij} \cdot X_j + \ldots + a_{i\eta} \cdot X_\eta + Z_i = \sum a_{ij} \cdot x_j + Z_i \]  

(22)

On the hole of the national economy, expenses for material are is added together by branches:

\[ CM = \sum CM_i = \sum \sum x_{ij} = \sum \sum a_{ij} \cdot x_j + \sum Z_i \]  

(23)

The net value added by industry is calculated as the difference between the overall product and material expenses:

\[ VAN_i = X_i - CM_i \]  

(24)

The gross value added is calculated by adding the fixed capital depreciation:

\[ VAB_i = VAN_i + Z_i \]  

(25)

Imports associated with each branch is established in the preparatory works that serve to determine the main indices of the national economy closely correlated with exports so that the two flows of foreign trade to offset each other.

Total Resources (R) are established by branch and on the national economy. They consist of global product respectively gross global product and import. Balancing of the input branches must ensure equality between total resources and total uses (U) consisting of intermediate consumption and final demand or final consumption.

After going through all these steps to we prepare the total BLR for the forecast period.

Conclusions

From the results presented in this article resulted a number of conclusions highlighting the complexity and usefulness of input-output tables in the calculation of macroeconomic indicators and their use in macroeconomic forecasts. Using the input-output model involves clarifying issues regarding aggregation branches, branches achieve homogeneity and identifying the established connections between them. The synthetic input-output table is accompanied by balance expressing links that are established between these branches of the national economy. We paid attention to building of synthetic model input-output to highlight the need to have reliable data to identify correlations that are
established based on these connections, by establishing the algorithm, then we can proceed to calculate technology coefficients, intermediate consumption, final results, import and export role in the calculation of macroeconomic indicators. Being a complex model of the economy, for example it could use a limited number of branches to present practically the indicators that are calculated and the expressiveness of their planned tests. The tables can be developed for a deep and useful role of macroeconomics. The authors were limited to synthesize as much as possible, the most significant theoretical to suggest the possibility of using this macro model analysis. Of course, the presentation of such a model can be other important elements, but the purpose of this article is to highlight the most commonly used elements.

References