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Long-memory, self-similarity and scaling of the long-term government bond yields: Evidence from Turkey and the USA

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Abstract. *This study investigates the long-memory and self-similarity characteristics of the long-term government bond yields in Turkey and the USA. The Geweke-Porter-Hudak (GPH) log-periodogram regression method has been utilized to measure the long-term persistency in the bond yield returns and volatilities for the period between 28.02.2010 and 28.04.2017. The empirical results show that while bond yield returns have random walk behaviors their volatility dynamics can be represented by a long-memory process. The paper also puts forward that bond yield returns have scale invariant distributional features and fitted with a Levy-stable distribution.*

Keywords: long-memory, bond markets, self-similarity, Levy-stable distribution.

JEL Classification: C14, G01, G10.

1. Introduction

The concept of long-range dependence of time series was introduced by Hurst (1951) and has been an intensely discussed topic in the empirical finance literature. Later, Mandelbrot (1964) adapted the phenomenon of long-range dependence to the financial time series. The seminal paper of Granger and Joyeux (1980) introduced the use of fractional processes in economics and econometrics. The motivation behind the economic literature on fractionally integrated processes stems from empirical evidence that many economic and financial time-series are not characterized as neither $I(0)$ nor $I(1)$ process. There have been numerous studies dealing with analysing the interest rate dynamics by using fractionally integrated models in the literature. The results from the studies are mixed, while some of them revealed existence of long-range dependence in the interest rate series some of them found no evidence of such properties.

Along with the long-range dependence, the scaling property of financial assets was firstly investigated by Mandelbrot (1964). In this pioneering study, he found out that addition to non-Gaussian distributional properties of cotton prices, distributions at different time scales have consistent functional form. Inspired by Mandelbrot, there have been several authors studied the scaling characteristics of financial returns across different asset classes and markets. One of the first study in this field was conducted by Müller et al. (1990) in which they examined the scaling law in intra-day FX prices and found evidence of scaling behaviour in absolute returns. Evertsz (1995a, 1995b) documented distributional self-similarity in USD/DM exchange rate and 30 German stocks. Mantegna and Stanley (1995) empirically suggest that the probability distribution of S&P500 can be identified by a non-normal scaling law. Scaling characteristics of Norwegian stock market have been studied by Skjeltorp (2000). Dacorogna et al. (2001) prove that empirical scaling law of USD/JP and USD/GBP is described by a power law. Gençay et al. (2001a) and Xu and Gençay (2003) also show that return and volatility characteristics of exchange rates follow scaling laws at different time horizons. The other papers that document the scaling law in financial asset prices are; Galluccio et al. (1997), Guillaume et al. (1997), Gopikrishnan et al. (1999), Wang and Hui(2001), Di Matteo et al. (2003, 2005), Sarkar and Barat (2006).

There are handful of studies about the long-memory and scaling dynamics of interest rates in the literature but only a few of them investigated such properties for Turkish rates. Therefore, it is beneficial to delve into this area and shed some light on the long-memory and scaling properties of the interest rates in the Turkish bond market. In this study, we conducted an intensive study on the long-memory, self-similarity and scaling characteristics of the Turkish interest rates vis-a-vis the USA interest rates. The rest of the paper is organized as follows. The notions of long-range dependence and self-similarity are introduced in section 2. Section 3 provides the empirical framework for the detection of the Long-range dependence and self-similairity. Data and empirical results are analysed and discussed in sections 4, and section 5 draws conclusion.

2. Literature review

Backus and Zin (1993) were the first authors who studied the long-term dynamics of the interest rates. They found evidence on long-term dependence in the 3-month zero coupon rates for the US bonds. Also McCarthy et al. (2004) came up with similar findings for US interest rates by using wavelets. Lai (1997), Gil-Alana (2002) and Gil-Alana (2004) also presented existence of Long-range dependence for the US interest rates. However, Caporale and Gil-Alana (2008) showed that the degree of persistence of the US interest rates is very sensitive to the model specification.

Karanasos et al. (2006) investigated the long-range dependence of the monthly US ex-ante and ex-post real interest rates by implementing FARIMA-FIAPARCH model. They state that US real interest rate has Long-range dependence characteristics in both first and second conditional moments. Their findings are in line with the results of Tsay (2000) where he has showed that ex-post interest rates of the US market has Long-range dependence.

Cajueiro and Tabak (2006) presented empirical evidence on the Long-memory properties of US 1, 3, 5 and 10 year maturity bond rates for the period from 1962 to 2005. They split the whole period into five sub-periods according to monetary policy and FED tenure. They argue that interest rate dynamics of the US bonds showed strong Long-memory before the Volcker administration and that after 1982 this characteristic has faded away suggesting a structural break in the dynamics of US interest rates.

Apart from studies on the US market, there have been several papers on investigating the Long-memory characteristics of interest rates world markets. Barkoulas and Baum (1997) have found the Long-memory effects on Euroyen deposit rates and Eureyen term premium for the Japan. Tabak and Cajueiro (2005) also found long-term dependency for the Japanese interest rates. Their study has some interesting features that the Bank of Japan's zero-interest rate policy to boost the economy has caused anti-persistent behaviour for the Japanese short-rate.

Fractional dynamics of the long-term interest rates of the five developed economies (US, Canada, Germany, UK, and Japan) have been examined by Barkoulas et al. (1996). They empirically found out that all interest rate series are integrated of order one I(1) and do not show persistent behaviour. However, their study reveals an important property such that 'even though each interest rate series is best characterized as a unit-root process, they possess a common fractional component'. In other words, interest rate series of five developed markets are fractionally co-integrated.

Iglesias and Phillips (2005) examined the one-month five Euro-market (Denmark, Germany, Netherlands, Portugal, and Spain) and Switzerland interest rates with the FARIMA model. Their study exposes that the one-month interest rate dynamics of the Euro-market countries are best characterized as fractionally integrated processes, whereas interest rates of Swiss market do not show Long-memory behavior and found to be $I(1)$ unit-root process.

In their study, Venetis et al. (2006) investigated the Long-memory behaviour of the ex-post monthly interest rates for the 14 European countries and US. They suggest that the evidence of Long-memory existence in the interest rates might be spurious due to structural breaks and cyclical components. They argue that the European and the USA interest rates are fractionally integrated processes between the 1978 and 1999. Also, Couchman et al. (2006) conducted a similar research by studying the Long-memory behaviour of real interest rates for 16 countries and state that majority of them have fractionally integrated parameter d lie between 0 and 1.

Fractionally integrated dynamics of the developing economies have been also addressed by several authors in the literature too. The study on the Long-memory dynamics of the East Asian countries (Singapore, Thailand, Malaysia, South Korea, Philippines) and Mexico have been proposed in the studies of Gil-Alana (2003) and Candelon and Gil-Alana (2006). According to the results of the papers, while Thai and Singaporean interest rates display mean-reverting characteristics $d \in (0,1)$, the results for the other countries are inconclusive.

Silva and Leme (2011) considered fractionally integrated models for the dynamics of Brazilian inflation rate and real interest rate. Their empirical findings point out a fractionally integrated process with some Long-memory for the Brazilian real interest rate.

A recent study on the Long-memory characteristics of Turkish real interest rates has been carried on by Yurttaguler and Kutlu (2013). They put account GPH estimation method to inspect the fractional differencing parameter for the Turkish monthly interest rates for the period of February 2003 - March 2012. They obtained the fractional differencing parameter as $d = 0.708$.

3. Methodology

3.1. Detection of the long-memory

A covariance stochastic process $X = (X_t, t = 0,1,2,\dots)$ with mean μ and variance σ^2, and autocorrelation function $\rho(k), k \geq 0$ is said to have Long-memory if we assume that X has an autocorrelation function of the form;

$$\boldsymbol{\rho(k) \sim k^{-\beta} L(k), k \to \infty} \tag{1}$$

where $0 < \beta < 1$ and L is slowly varying at infinity. Reversely X has short-memory, if its autocorrelation function has geometric decay rate;

$$\boldsymbol{K > 0, c \in]0,1[, \mid \rho(k) \mid \leq Kc^k} \tag{2}$$

The general behavior of the autocorrelation function cannot solely determine the long-range dependence. Instead, it only specifies the asymptotic behavior when $k \to \infty$. Therefore, we need more robust statistical tests to detect the long-memory in a time series. For this purpose, we resort to the GPH method developed by Geweke-Porter-Hudak.

The spectral density of the fractionally integrated process X_t is expressed as;

$$\boldsymbol{f(\omega)} = \left[\boldsymbol{4sin^2\left(\frac{\omega}{2}\right)}\right]^{\boldsymbol{-d}} \boldsymbol{f_u(\omega)} \tag{3}$$

where ω is the Fourier frequency, and $f_u(\omega)$ is the spectral density corresponding to u_t. The difference parameter d can be estimated by;

$$\ln f(\omega_j) = \beta - d\ln\left[4sin^2\left(\frac{\omega_j}{2}\right)\right] + e_j \text{ for } j = 1,2,\ldots,n_f(T) \tag{4}$$

Geweke and Porter-Hudak showed that using a periodogram estimate of $f(\omega_j)$, the least squares estimate $\hat{d}$ using the regression in Eq.4 is normally distributed in large samples if $n_f(T) = T^\alpha$ with $0 < \alpha < 1$ $\hat{d} \sim N\left(d, \frac{\pi^2}{6\sum_{j=1}^{n_f}(U_j-\overline{U})^2}\right)$ where $U_j = \left[4sin^2\left(\frac{\omega}{2}\right)\right]$ and $\overline{U}$ is the sample mean of U_j. Under the null hypothesis of no long memory ($d = 0$), the t-statistic is $t_{d=0} = \hat{d}.\left(d, \frac{\pi^2}{6\sum_{j=1}^{n_f}(U_j-\overline{U})^2}\right)^{-0.5}$

3.2. Detection of the self-similarity and scaling behaviors

For examining the scale invariance of the distribution of interest rate returns, we follow the methodology proposed by Evertsz (1995a) which is given as:

Let P_t denotes the logarithm of the interest rate series, then the logarithmic changes (returns) of the interest rates are given by

$$X_t = P_{t+i} - P_t \tag{5}$$

By modifying Eq.5 in order to reflect the scale invariance of the rates of return, we will analyze whether the distribution of the absolute returns is scale invariant. The Eq.6 reflects a more relevant financial sense.

$$\frac{P_{t+\Delta t} - P_{\Delta t}}{\Delta t} \sim \left(\frac{\Delta t}{\Delta t(\Delta t)'}\right)^{\frac{1}{\alpha}-1} \frac{P(t + (\Delta t)') - P_t}{(\Delta t)'} \tag{6}$$

Let $F_{\Delta t}(x)dx$ denotes the probability density of interest rate returns in Eq.6 over time periods of size Δt, that is $F(X, \Delta t)$ is the probability density of the left hand side of the equation. Eq.6 hints for the densities, such as

$$\left(\frac{1}{\alpha} - 1\right)\log(\Delta t) + \log F(X, \Delta t) = \log Q\left((\Delta t)^{1-\frac{1}{\alpha}}X\right) \tag{7}$$

As Evertsz (1995a) suggests that, in the case of self-similarity, densities of $F(X, \Delta t)$ collapse onto the -basic distribution- Q by plotting, $\left(\frac{1}{\alpha}\right)\log(\Delta t) + \log F(X, \Delta t)$ vs. $(\Delta t)^{1-\frac{1}{\alpha}}X_t$

Therefore, -fractal dimension of the probability space- α and -basic distribution- Q fully specify the distribution of interest rate returns on all time intervals Δt.

As Mandelbrot (2010) points out there is a number of different methods to determine α. However, these methods generally examine the tails of the distributions, which becomes difficult for larger values of Δt where number of observations decrease making the results unreliable. Thus, we employ the methodology proposed by Mantegna and Stanley (1995) to calculate the α. This method is straightforward but it requires an intensive amount of data manipulation. Let us define the logarithm of interest rates P_t and logarithmic changes $X_{\Delta t}$ over a set of non-overlapping time windows divided by a time interval Δt as:

$$X_{\Delta t} \equiv P_{t+\Delta t} - P_t \tag{8}$$

When we determine the probability distributions of logarithmic changes $F(X, \Delta t)$ on all time scales with traditional methods, investigating the tail behavior of the distribution yields distorted results due to reduced number of observations at larger time scales Δt. In order to address this issue, Mantegna and Stanley (1995) introduced an alternative approach by examining the scaling behavior of the -probability of return to origin- $F(X_{\Delta t} = 0)$, as a function of Δt. By plotting $F(X_{\Delta t} = 0)$ against Δt in a log-log scale, and fitting a least-square regression, the slope coefficient (λ) determines normal or non-normal scaling behavior as the slope of a normal scaling property is equal to $\lambda = -0.5$. The Hurst exponent H can also be determined from the log-log plot of $F(X_{\Delta t} = 0)$ against Δt through following relation, $H = -\alpha$.

Since, Hurst exponent is equal to $H = 1/\alpha$, we can calculate α as $\alpha = \frac{1}{-\lambda}$. If the slope coefficient is $\lambda = -0.5$, the -fractal dimension of the probability space- $\alpha = 2$, the distribution is Gaussian with finite variance. In contrary, if $\lambda \in (-0.5, -1)$, $\alpha > 2$, the variance of the distribution becomes infinite, and distribution is more leptokurtic.

4. Empirical results

4.1. Initial data analysis

The data set we employ for this chapter consists of the daily rates of Turkish Republic Central Bank's and US Federal Reserve Bank's 10-year maturity treasury bonds for the period between 28/01/2010 and 28/04/2017. The data were retrieved from http://tr.investing.com. For statistical tests, we use the logarithmic changes of the interest rate series which is calculated as in Eq.5. Descriptive statistics of the logarithmic changes in the interest rate series are reported in Table 1. The series under examination are stationary as Augmented Dickey-Fuller (ADF) test results suggest the rejecting the null hypothesis of a unit-root at 1% significance levels. The skewness and kurtosis statistics for the series indicate a slightly asymmetric and heavy-tailed distribution rather than the normal distribution. Jarque-Bera test statistics also confirm the deviation from normal law by rejecting the null hypothesis which states normality assumption. Ljung-Box statistics up to 35 lags indicate a presence of serial correlation for the returns series at 5% significance level.

Table 1. *Descriptive statistics of the interest rate returns*

	Mean	St. Dev	Skewness	Ex. Kurtosis	ADF[12]	Jarque-Bera	Ljung-Box[35}
Turkey	-0.005%	1.297%	0.622	6.982	-10.737	3976.680*	38.250**
USA	-0.026%	2.301%	0.116	1.034	-12.336	85.598*	44.096**

4.2. Long-memory

In order to check the degree of long-memory persistency, we applied GPH method for the estimation of the long-memory parameter d. Before we move into empirical estimates, we run a simulation study to check the robustness of the method; since the results of the GPH method are sensitive to the bandwidth parameter which measures the number of periodogram points. We simulated 100 sample paths with 10000 observation points, with different d values ranging from 0.1 to 0.4. Then, we run GPH estimation process on every simulated paths with using different bandwidth parameters $\alpha = 0.5$, $\alpha = 0.6$, and $\alpha = 0.7$.

We present the mean, standard deviation and bias of the d parameters of the simulated series. The results of the simulation study are depicted in Table 2. According to our simulation study, average values for the long-memory parameter d of the simulated series are similar to the nominal values as indicated with the statistically insignificant bias values. Therefore, GPH method is a reliable statistical tool in measuring the long-range dependence.

Table 3 reports the estimated long-memory parameters for the empirical series. The estimation procedure is carried out with using bandwidth values 0.5, 0.6 and 0.7. The long-memory property of the absolute returns of the Turkish interest rates is proven by the GPH test statistic with a statistically significant d parameter. GPH test results also predicate the existence of long-memory in the squared return series of the Turkish interest rates at 1% significance level. The return series of the Turkish bond yields, however, do not show any long-term dependence indicated with statistically insignificant values for the d parameter. The estimation results for the US data have similar implications with the Turkish data. While the interest rate returns show random walk behaviors, the d estimates for the absolute and squared returns are representatives of the strong long-range dependence.

Table 2. *Estimation results for d parameter using 100 independent paths of simulated time series*

			Nominal d		
		d=0.1	d=0.2	d=0.3	d=0.4
$\boldsymbol{\alpha = 0.5}$	Mean	0.1047	0.2089	0.2998	0.4119
	Bias	0.0047	0.0089	-0.0002	0.0119
	p-value	[0.498]	[0.169]	[0.974]	[0.029]**
$\boldsymbol{\alpha = 0.6}$	Mean	-0.0971	0.2036	0.3012	0.4053
	Bias	0.0029	0.0036	0.0012	0.0053
	p-value	[0.502]	[0.356]	[0.735]	[0.391]
$\boldsymbol{\alpha = 0.7}$	Mean	0.1004	0.2025	0.3013	0.405
	Bias	0.0004	0.0025	0.0013	0.005
	p-value	[0.874]	[0.296]	[0.340]	[0.272]

Notes: ** denotes the rejection of null hypothesis (H0: Bias=0) at 5% significance level.

Table 3. *Estimated values for the long-memory parameter d at return, absolute return and squared return levels*

	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$
Panel A: Turkey			
Xt	-0.052	0.077	-0.018
	(0.116)	(0.074)	(0.047)
$\lvert Xt \rvert$	0.418*	0.324*	0.273*
	(0.103)	(0.059)	(0.044)
Xt^2	0.364*	0.474*	0.365*
	(0.059)	(0.053)	(0.045)
Panel B: USA			
Xt	0.081	0.034	-0.061
	(0.115)	(0.081)	(0.048)
$\lvert Xt \rvert$	0.442*	0.325*	0.257*
	(0.095)	(0.063)	(0.046)
Xt^2	0.393*	0.331*	0.249*
	(0.090)	(0.067)	(0.046)

Notes: Corresponding standard errors for the parameter estimates are given in brackets. * denotes the rejection of null hypothesis (H0: d = 0) at 1% level.

4.3. Self-similarity and scaling

We used "probability of zero return" approach (Mantegna and Stanley, 1995) by defining $F(X_{\Delta t} = 0)$ as a function of time intervals Δt. Figure 1 illustrates the plots of $F(X_{\Delta t} = 0)$ versus Δt in a log-log scale. It is evident from the figure that datasets are well fitted by a linear line with slopes $\lambda_{TR} = -0.442$ and $\lambda_{US} = -0.507$ for Turkish and US data respectively. Thus, the scaling power law is observed for Δt for 1-day to 1-month in Turkish and US interest rates. If we make an assumption about the Lèvy-stable distribution(1) of the interest rate returns with scaling index α and scaling parameter γ, we can derive the "'probability of zero return'" from the density function of Lèvy-stable process as:

$$F(X_{\Delta t} = 0) = \frac{\Gamma(1/\alpha)}{\pi\alpha(\gamma\Delta t)^{1/\alpha}} \tag{9}$$

where Γ is the gamma function. We obtain the scaling index values for Turkish and US data as $\alpha_{TR} = 1.58$ and $\alpha_{US} = 1.77$ by using the slope values λ_{TR} and λ_{US} . The estimated values of α_{TR} and α_{US} indicate that the processes deviate from a Gaussian random walk. The fractal dimension parameters $\alpha < 2$ suggest that interest rate returns have a leptokurtic distribution.

Figure 1. *"Probability of return to origin" $F(X_{\Delta t} = 0)$ vs time increments. Δt in a logarithmic plane*

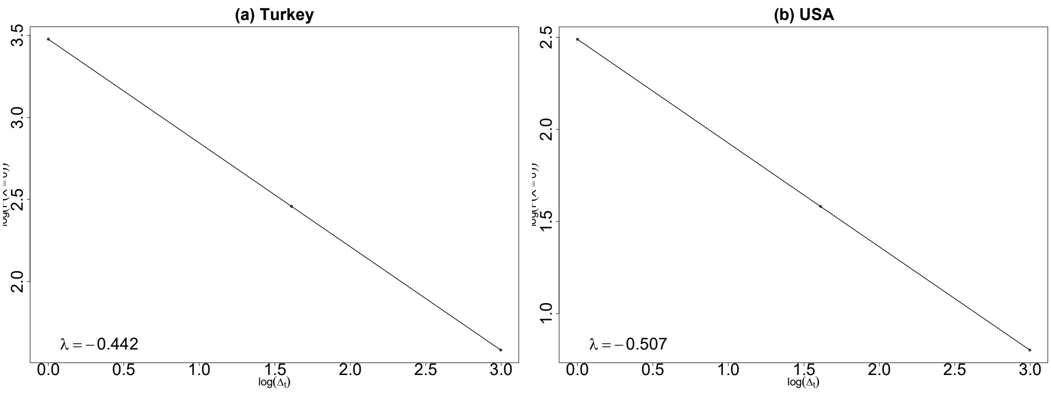
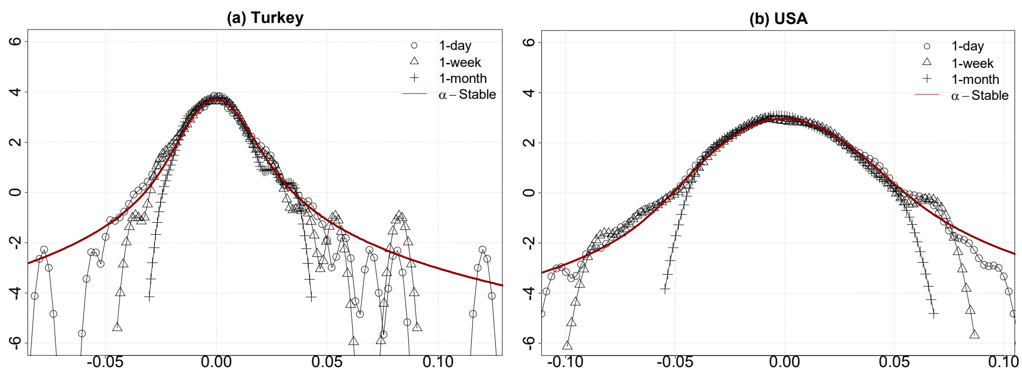


Figure 2 reflects the α-scaled densities of the bond yield returns at different time intervals (Δt = 1, 5, 20) (ranging from 1 day to 1 month) in a logarithmic plane. While all data collapse on the $\Delta t = 1$ day distribution for the Turkish returns, there are some deviations from the basic distribution for the US returns at larger time scales. One can infer from the figure that Turkish and the USA long-term government bond yields show self-similarity characteristics and bond yield returns at different time intervals (daily, weekly and monthly) are best represented with a Lèvy stable distribution.

Figure 2. *Log-density functions of interest rate returns at daily, weekly and monthly scales fitted with Levy-stable distribution*



5. Conclusion

The findings of this have significant economic implications. The long-memory in the first conditional moment of the interest rates helps researchers and practitioners to have more accurate forecasts on the term-structure. Furthermore, the strong persistent behavior found in interest rate volatility could be crucial for risk management and pricing of interest rate derivatives. For a future development, it would be worthwhile to consider long-memory in mean and volatility of interest rates in deriving pricing formulas for the interest rate derivatives. Focusing on the microstructure of the bond markets with taking into account liquidity and term premia can also be an important route for further research.

Another important finding of this study is that, returns of the Turkish and the USA bond markets are scale invariant meaning that distributional characteristics of the interest rate returns do not change at different time scales. The distributional scaling analysis results also suggest that interest rate returns are characterized by a Lèvy stable distribution on all scales. These findings have implications for the assessment of the risks of big losses or gains as a function of time, and thus for the derivative pricing.

In order to provide a new framework to improve new and better economic and financial models, the natural follow up of this study would be the investigation of multifractal characteristics of the interest rates which would take the analysis of dynamic scaling behavior one step further.

Note

(1) Lèvy-stable distribution which is represented by

$$F_\alpha(X, \Delta t) \equiv \frac{1}{\pi}\int_0^\infty \exp\{-\gamma \Delta t u^\alpha\}\cos(uX)du$$

with $0 < \alpha \leq 2$ denoting the characteristic exponent or -fractal dimension of the probability space- X is the log-returns, γ is the scale factor, and Δt denotes the time lag. By using the following transformation, Lèvy-stable distribution can be rescaled as

$F_S \equiv \frac{X_{\Delta t}}{(\Delta t)^{1/\alpha}}$ and $F_\alpha(X_S, 1) \equiv \frac{F_\alpha(X,\Delta t)}{(\Delta t)^{-1/\alpha}}$

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