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Economic value of portfolio diversification: Evidence from international multi-asset portfolios

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Abstract. *We examine alternative approaches of measuring portfolio diversification, and test the empirical relation between diversification and the future risk-adjusted performance in a cross-section of international multi-asset portfolios. We use the Woerheide and Persson measure as a weight-based diversification measure, the conditional diversification measure as a risk-based diversification measure, and the effective number of bets (ENB) as a factor based diversification measure. We find that only the ENB measure is a significant predictor of the future Sharpe ratios. The economic gains of diversification, as measured by the ENB measure, are large and robust to the investor's risk aversion and investment horizon.*

Keywords: portfolio, diversification, effective number of bets, unsystematic risk.

JEL Classification: G10, G11, G12.

1. Introduction

The benefits of portfolio diversification are well established in the investment theory. Diversification of investment portfolios reduces the unpriced idiosyncratic risks without affecting the expectation of future returns. Consequently, diversification improves the expectation of future risk-adjusted returns. Surprisingly, there is no unique and broadly accepted quantitative measure of portfolio diversification (Meucci, 2009). The earliest works on diversification focused on identifying the minimum number of assets that would make a portfolio reasonably diversified (Beck, Perfect and Peterson, 1996; Elton and Gruber, 1977; Evans and Archer, 1968; Statman, 1987). This naïve approach to diversification ignores the distribution of portfolio weights and the correlations between asset returns. More sophisticated measures of portfolio diversification attempt to address these issues, and the these measures can be classified in three categories– measures based on portfolio weights, measures based on the risk structure of the asset returns, and the measures based on the allocation across the underlying risk factors.

The first category of diversification measures are the based on the distribution of portfolio weights. The portfolios in which a considerable proportion of the total capital is allocated to relatively few assets are considered poorly diversified. Conversely, the portfolios that allocate the capital uniformly across a large number of assets are considered well diversified. The concertation of portfolio weights is usually measured using either the Shannon entropy measure (Bera and Park, 2008; Vermorken, Medda and Schroder, 2012) or the Herfindahl-index (Hamza et al., 2006; Kacperczyk, Sialm and Zheng, 2005; King, 2008; Kumar, 2007; Woerheide and Persson, 1992). Maximizing diversification under such a definition leads to an equally weighted portfolio allocation. However, the limitations of the equally weighted portfolio are obvious. First, it is possible that the portfolio assets may be strongly correlated. In this case, investing across large number of strongly correlated assets yields little diversification benefit. Second, the composition of the investment universe may induce bias in the equally weighted portfolio allocation. For instance, in most equity markets, small cap stocks far outnumber the large cap stocks. Consequently an equally weighted portfolio of all stocks would be tilted towards small-cap stocks. The second category of diversification measures attempt to address these issues by incorporating the information about the weight concentration, volatility and correlation structure of portfolio assets. Notable examples include the Goetzmann-Li-Rouwenhorst measure (Goetzmann, Lingfeng Li and Rouwenhorst, 2005), the diversification ratio (Choueifaty and Coignard, 2008), and the conditional diversification benefits measure (Christoffersen et al., 2012). In general, these measures suggest that a portfolio is poorly diversified if the portfolio weights are concentrated over a few assets and/or if the portfolio constituents are highly correlated with each other.

Roll (2013) notes that in the presence of multiple underlying risk factors, the empirical correlation between two assets can be low even when their returns are driven by the same risk factors. Therefore, in order to achieve effective portfolio diversification, investors must seek to distribute their portfolio exposures uniformly across large number of uncorrelated risk factors, rather than diversifying across a large number of stocks or asset classes. Several studies employ principal component analysis to extract the underlying risk factors (Frahm and Wiechers, 2013; Lohre et al., 2012). The problem with approach is that regardless of the portfolio allocation most of variance is explained by the first few principal component factors, and therefore there is little variation in portfolio diversification values. In addition, these factors are linear combinations of the original assets, and generally bear no resemblance to the original assets. To overcome these problems, Meucci (2015) recommends the minimum linear torsion procedure to extract risk factors for any given investment universe. The minimum linear torsion procedure generates risk factors which are the closest orthogonal representation of original assets. In addition, the volatility of the risk factors is constrained to be same as the volatility of the original assets. This ensures that the first few factors do not dominate the rest of the factors, as is the case with principal component based factors.

In this analysis we compare the three types of diversification measures and examine their relation with future risk-adjusted performance of international multi-asset portfolios. We use the Woerheide and Persson measure (Woerheide and Persson, 1992) as a weight based measure of portfolio diversification; the conditional diversification measure of Christoffersen et al. (2012) as a risk-based measure of portfolio diversification and the effective number of bets measure as a factor-based measure of portfolio diversification. In addition, we measure the economic value of superior diversification for different levels of relative risk-aversion and for different investment horizons.

2. Data

We use weekly returns of thirty assets belonging to multiple asset classes, namely, equities, currencies and bonds. The sample period of the study extends from April 2, 1999 to May 9, 2014. The sample period ranges from 2 April 1999 to 9 May 2014. Table 1 lists the sample assets and provides some descriptive statistics. The U.S. three month Treasury bill rate is used as the risk-free rate. All data are sourced from the Bloomberg database.

3. Diversification measures

3.1. The Woerheide and Persson measure

The Woerheide and Persson (1992) measure (WPM) is based on the Herfindahl Index of portfolio weights. It is calculated as one minus the sum of squared portfolio weights.

$$\mathrm{WPM} = 1 - \sum_{i=1}^{N} {w_i}^2 \tag{1}$$

where w_i is the portfolio weight of the i^{th} portfolio constituent and N is the number of portfolio constituents. The WPM is minimized to a value of zero when the entire portfolio capital is allocated to a single asset, and it is maximized to a value of $(N-1)\,/\,N$ when all assets have the same portfolio weight, i.e. $w_i = 1/N, \forall i$.

3.2. The conditional diversification measure (Christoffersen et al., 2012)

Under the assumption of normally distributed asset returns, the conditional diversification measure (CDM) is defined as

$$CDM = 1 - \frac{\sqrt{w^T \Sigma w}}{w^T \sigma} \quad (2)$$

where: w the column vector of portfolio is weights $\{w_1, w_2, .., w_N\}$ and σ is the column vector of asset volatilities $\{\sigma_1, \sigma_2, .., \sigma_N\}$

Table 1. *Dataset and descriptive statistics*

Description	Country	Asset Class	Mean Return	Std. Deviation
Euro USD Spot Exchange Rate	EU	CURRENCY	1.65	10.23
Yen USD Spot Exchange Rate	Japan	CURRENCY	1.08	10.37
Pound USD Spot Exchange Rate	United Kingdom	CURRENCY	0.25	9.36
Australian dollar USD Spot Exchange Rate	Australia	CURRENCY	2.5	13.36
Swiss franc USD Spot Exchange Rate	Switzerland	CURRENCY	3.44	11.01
Canadian dollar USD Spot Exchange Rate	Canada	CURRENCY	2.12	8.92
Mexican Peso USD Spot Exchange Rate	Mexico	CURRENCY	-2.02	10.05
New Zealand dollar USD Spot Exchange Rate	New Zealand	CURRENCY	3.18	13.61
Swedish Krona USD Spot Exchange Rate	Sweden	CURRENCY	1.61	12.15
Russian Ruble USD Spot Exchange Rate	Russia	CURRENCY	-1.89	8.34
S&P 500 Index	United States	EQUITY	2.53	18.83
S&P TSX Composite Index	Canada	EQUITY	7.4	23.6
FTSE 100 Index	United Kingdom	EQUITY	0.95	21.35
Nikkei 225 Index	Japan	EQUITY	0.4	21.89
CAC 40 Index	France	EQUITY	2.18	25.21
S&P ASX 200 Index	Australia	EQUITY	6.68	24.54
Hong Kong Hang Seng Index	Hong Kong	EQUITY	4.77	23.62
Swiss Market Index	Switzerland	EQUITY	4.67	20.51
Deutscher Aktien Index	Germany	EQUITY	6.23	26.82
Bovespa Index	Brazil	EQUITY	8.96	40.06
Bloomberg/EFFAS US Govt. Bond Index	United States	BONDS	5.03	4.66
Bloomberg/EFFAS Germany Govt. Bond Index	Germany	BONDS	6.34	10.76
Bloomberg/EFFAS Austria Govt. Bond Index	Austria	BONDS	6.66	11.17
Bloomberg/EFFAS UK Govt. Bond Index	United Kingdom	BONDS	5.35	10.67
Bloomberg/EFFAS Australia Govt. Bond Index	Australia	BONDS	8.14	12.59
Bloomberg/EFFAS Canada Govt. Bond Index	Canada	BONDS	7.29	9.09
Bloomberg/EFFAS Japan Govt. Bond Index	Japan	BONDS	2.86	10.84
Bloomberg/EFFAS Denmark Govt. Bond Index	Denmark	BONDS	6.74	11.9
Bloomberg/EFFAS France Govt. Bond Index	France	BONDS	6.48	11.06
Bloomberg/EFFAS New Zealand Govt. Bond Index	New Zealand	BONDS	9.25	13.22

Notes: The mean return and standard deviation are reported as annualized percentage. The sample period is April 2, 1999 to May 9, 2014.

3.3. The effective number of bets

We extract orthogonal risk factors from the original asset returns using the minimum linear torsion (MLT) procedure of Meucci et al. (2015). Let η represent a $M \times T$ matrix of original assets returns, where M is the number of assets in the investment universe and T is the number of periods. Then, the returns of orthogonal risk factors can be represented by a matrix $\eta_F = \mathrm{A}'\eta$, where A is a transformation matrix obtained by applying MLT on the original asset returns. We refer the reader to Meucci et al. (2015) for the details of implementation of the MLT procedure.

The MLT procedure ensures that the new factors represent the closest uncorrelated representation of original assets, and the volatilities of the new factors is same as the volatilities of the original assets. Therefore, the covariance matrix of factor returns, Σ_F can be represented as a diagonal matrix $D^2 = \mathrm{diag}(\Sigma)$, where Σ is the covariance matrix of original assets. The smallest linear transformation is derived by minimizing the squared tracking errors between the new factor returns and the original asset returns. Formally the optimization problem can be stated as

$$A^* = \mathrm{argmin}_A \left(TE\{r_{F_1}, r_1\}^2 + \ldots + TE\{r_{F_N}, r_N\}^2\right) \tag{3}$$

where: N is the number of returns and $TE(\cdot)$ denotes the tracking error function. Solving for the sum of squared tracking error we get

$$\sum_{i=1}^{N} TE\{r_{F_k}, r_k\}^2 = \sum_{i=1}^{N} Var\left(r_{F_k} - r_k\right) = \sum_{i=1}^{N} Var(a_k' r - e_k' r)$$

$$= \sum_{i=1}^{N} Var([a_k - e_k]' r) = \sum_{i=1}^{N} [a_k - e_k]' \, \Sigma \, [a_k - e_k]$$

$$= tr([A - I_N]' \, \Sigma \, [A - I_N]) = tr(A'\Sigma A - A'\Sigma - \Sigma A + \Sigma)$$

$$= tr(A'\Sigma A - A'\Sigma - \Sigma A + \Sigma) = tr(D^2) + tr(\Sigma) - 2tr(A'\Sigma)$$

where: a_k is the kth column of matrix A and e_k is the kth elementary vector. Since D^2 and Σ, to minimize the sum of squared tracking error, we need to maximize $tr(A'\Sigma)$. We use the principal component decomposition of $\Sigma = P\Lambda^2P'$, P is a matrix of eigenvectors and Λ^2 is matrix of eigenvalues. Then, we can expand $tr(A'\Sigma)$ as

$$tr(A'\Sigma) = tr(DD^{-1}A'P\Lambda \, \Lambda P') \tag{4}$$

Let $Q' = D^{-1}A'P\Lambda$, then Equation (4) can be rewritten as

$$tr(A'\Sigma) = tr(Q'\Lambda P'D) \tag{5}$$

where: Q satisfies the property $QQ' = I_N$. Next, we compute singular value decomposition of $\Lambda P'D$ as

$$\Lambda P'D = USV' \tag{6}$$

where: U and V are orthogonal to each other. S is the diagonal matrix containing singular values of $\Lambda P'D$. Substituting the value of $\Lambda P'D$ in Equation (5), we obtain

$$tr(Q'\Lambda P'D) = tr(Q'USV') = tr(V'Q'US) \tag{7}$$

where: $Z = V'Q'U$ satisfies $ZZ' = I_N$. Substituting $Z = V'Q'U$ in Equation (7), we obtain

$$tr(V'Q'US) = tr(ZS) = \textstyle\sum_{k=1}^{N} z_{kk}s_{kk} \le \sum_{k=1}^{N} s_{kk} \tag{8}$$

Clearly, Equation (8) is maximized when $z_{kk} = 1, \forall k$ or $Z = I_N$. Solving for Q' we get

$$Z = V'Q'U = I_N$$

$Q' = UV'$

Since $Q' = D^{-1}A'P\Lambda$, we can solve for the transformation matrix A as follows

$Q' = D^{-1}A'P\Lambda$

$UV' = D^{-1}A'P\Lambda$

$A = P\Lambda^{-1}UV'D$

Therefore the optimal transformation matrix A is

$$A = P\Lambda^{-1}UV'D \tag{9}$$

Any portfolio of the original assets, represented by a vector of portfolio weights w, can now be represented as an equivalent portfolio of the MLT risk factors with a transformed weight vector $w_F = A'w$.

Since the risk factors are orthogonal to each other, the portfolio variance, σ_P^2, can be calculated as the sum of the squares of weighted volatilities of the individual risk factors.

$$\sigma_P^2 = \sum_{i=1}^{N} w_{F_i}^2 \sigma_{F_i}^2 \tag{10}$$

where: $w_{F_i}$and σ_{F_i} are the weight and the volatility of the i^{th} risk factor. The percentage risk contribution of the i^{th} risk factor, RC_{F_i}, can be calculated as

$$RC_{F_i} = \frac{w_{F_i}^2 \sigma_{F_i}^2}{\sigma_P^2} \tag{11}$$

The effective number of bets (ENB) measure indicates how the portfolio risk is distributed across the various risk factors, and it is calculated as

$$\text{ENB} = \exp\left(-\sum_{i=1}^{N} RC_{F_i}\, ln\left(RC_{F_i}\right)\right) \tag{12}$$

If the entire portfolio risk is contributed by a single factor, the ENB is minimized to a value of one (least diversified portfolio). Conversely, when all risk factors contribute equally to the overall portfolio variance, the ENB is maximized to a value of N (most diversified portfolio).

4. Empirical analysis

We simulate a cross-section of portfolios by generating 10,000 unique randomly generated portfolio weight vectors. The portfolio dimension is allowed to vary randomly from 1 to 30. The calculations of the CDM and ENB measures require an estimate of the covariance matrix of asset returns. We use the shrinkage estimator of Ledoit and Wolf (2004) with a rolling window of 104 weekly returns (approximately two years) to estimate the covariance matrix of asset returns. The initial 104 weeks of data in our sample period comprises the burn in sample for initializing the first covariance matrix estimate. In the remaining period (688 weeks), for each week we calculate the portfolio diversification measure for each of the 10,000 portfolios using all three diversification measures.

Next, we test the relation between the diversification level and future risk-adjusted performance by using Fama-Macbeth regressions. We carry out a series of cross-sectional regressions, wherein, for each week t, the Sharpe ratios for the period t to $t+h$ are regressed on the diversification measures calculated at end of week t. Five future horizons are considered: one month (h=4), one quarter (h=13), one year (h=52), two years (h=104) and five years (h=260). For each future horizon, the time-series average of the regression slopes is calculated, and we test whether it is statistically different from zero by using the standard t-test with the Newey-West (NW) standard errors (Newey and West, 1987). The selection of lag length for the computation of NW standard errors is based on the automatic bandwidth selection procedure of Newey and West (1994). Table 2 reports the result of the Fama-Macbeth regressions. Regardless of the choice of the diversification measure the average regression slopes are positive for all future horizons. This suggests that, on an average, improving portfolio diversification does tend to improve the future risk-adjusted performance. However, for the WPM and CDM measures the relation between portfolio diversification and future Sharpe ratios is not statistically significant. In contrast, when portfolio diversification is measured using the ENB measure, the relation between diversification and future Sharpe ratios is statistically significant and robust across all investment horizons. The Average R^2 values for ENB regressions are also higher than those obtained in regressions using the WPM and CDM measures. This suggests that ENB performs better than the WPM and CDM measures in explaining the cross-sectional variations in future Sharpe ratios. Notably, across all diversification measures the explanatory power tends to decline with an increase in the investment horizon as indicated by a monotonic decline in the t-statistics.

To measure the economic value of portfolio diversification, we use a two stage procedure. In the first stage, we construct diversification decile portfolios to sort and classify the cross-section of portfolios into different levels of diversification. At the end of each week, we sort the portfolios into deciles based on their diversification as measured by the WPM, CDM or ENB measures. Then, we construct diversification decile portfolios, $\delta_d, d = 1,2,..10,$ to represent each decile. For any diversification decile, d, δ_d is the equally weighted portfolio all portfolios in the d^{th} decile, with $d = 1(10)$ denoting the lowest (highest) level of diversification. Note that the portfolios falling in the d^{th} decile can vary for different periods. In the second stage, we use the utility equivalence procedure of Fleming et al. (2003) to measure the economic gains of switching from a lower diversification decile portfolio to a higher diversification decile portfolio. Consider an investor with an initial wealth W_0 and a coefficient of relative risk aversion γ. Further, assume that this investor can choose to buy-and-hold any one of the ten diversification decile portfolios, δ_d. Following Fleming et al. (2003), we assume a quadratic utility function, $U(r_{d,t})$, for weekly returns defined as

$$U(r_{d,t}) = W_0\left((1 + r_{d,t}) - \frac{\gamma}{2(1+\gamma)}(1 + r_{d,t})^2\right) \tag{13}$$

where: $r_{d,t}$ is the return of the δ_d portfolio for week t and $U(r_{d,t})$ is the utility realized from the return $r_{d,t}$. Given the economic utility function of Equation (13), we estimate a constant Δ_γ such that when a constant fee Δ_γ is subtracted from the weekly returns of the diversification decile portfolio $\delta_d, d = 2, 3, .., 10$, the utility derived from the δ_1 and δ_d portfolios are equalized. This is represented formally as

$$\sum_{t=1}^{T} U(r_{1,t}) = \sum_{t=1}^{T} U(r_{d,t} - \Delta_\gamma) \quad d = 2, 3, \dots, 10 \qquad (14)$$

where: T is the number of weekly periods. The constant Δ_γ can be interpreted as the maximum weekly premium that a risk-averse investor is willing to sacrifice to switch from the lowest diversification portfolio, δ_1, to one of the more diversified portfolios, $\delta_d, d = 2, 3, .., 10$.

Table 2. *Regression of future Sharpe ratios with the diversification measures*

	Following Month	Following Quarter	Following Year	Following 2-Years	Following 5-Years
Panel A: Woerheide and Persson measure (WPM)					
Average R^2	1.25%	1.87%	3.03%	3.53%	2.40%
Coefficient	5.92	1.87	1.75	1.89	1.03
t-statistic	1.58	1.11	1.19	1.06	0.36
Panel B: Conditional diversification measure (CDM)					
Average R^2	1.27%	2.08%	3.37%	4.25%	3.49%
Coefficient	51.64	19.94	16.45	16.07	11.24
t-statistic	1.61	1.54	1.49	1.21	0.89
Panel C: Effective number of bets (ENB)					
Average R^2	3.14%	4.63%	4.88%	7.12%	7.95%
Coefficient	26.08	16.43	12.96	11.73	8.16
t-statistic	2.71**	2.50**	2.13**	2.01**	1.88*

Note: Coefficient denotes the average slope of the regressions. The *t*-statistics are calculated using the Newey-West standard errors. * / ** denote the significance at the 10% / 5% levels.

Table 3 reports the annualized values of Δ_γ for different diversification measures and for different levels of the relative risk aversion parameter, γ. Following Fleming et al. (2003), we use $\gamma = 1(10)$ to represent investors with low (high) level of relative risk aversion.

Since the Δ_γ premiums are always positive, a risk-averse investor would always prefer a higher diversification portfolio $\delta_d, d = 2, 3, \dots, 10$ over the least diversified δ_1 portfolio. If there is a positive relation between the level of portfolio diversification and the economic utility derived by the investor, one would expect that the premium for switching from δ_1 *to* δ_m would be lower than the premium for switching from δ_1 *to* δ_n if $m < n$. In other words, if δ_n has a better diversification than δ_m, the investor should prefer δ_n over δ_m. Clearly such a relation does not hold when the portfolio diversification is measured using the either the WPM or the CDM measure, as the Δ_γ values do not increase monotonically with the level of diversification. For instance, when portfolio diversification is measured using the WPM, an investor with low risk aversion (Δ_1) is willing to pay an annualized fee of 38.6 basis points to switch from δ_1 *to* δ_3, but only 3.6 basis points to switch from δ_1 *to* δ_4. Similar observations can be made when the portfolio diversification is measured using the CDM. However, when the portfolio diversification is measured using the ENB measure, the Δ_γ premiums increase monotonically with the level of diversification.

Regardless of the level of relative risk-aversion, in all comparisons, the premium for switching from δ_1 *to* δ_m is lower than the premium for switching from δ_1 *to* δ_n if $m < n$.

In addition, the magnitude of diversification premium is the largest when portfolio diversification is measured using the ENB measure. For instance, an investor with low (high) risk aversion would be willing to pay 93 (429) basis points per annum to switch from the least diversified portfolio to the most diversified portfolio (δ_1 to δ_{10}) when portfolio diversification is measured using the ENB measure.

Table 3. *Annualized diversification premium*

	Annualized Premium to switch from δ_1 to $\delta_d, d = 2, 3, .., 10$					
	WPM		*CDM*		*ENB*	
Risk Aversion	Δ_1	Δ_{10}	Δ_1	Δ_{10}	Δ_1	Δ_{10}
δ_1 to δ_2	0.090	0.150	0.104	0.156	0.120	0.798
δ_1 to δ_3	0.386	0.799	0.073	0.857	0.455	1.407
δ_1 to δ_4	0.036	0.331	0.131	0.343	0.490	2.073
δ_1 to δ_5	0.121	0.526	0.136	0.573	0.608	2.534
δ_1 to δ_6	0.070	0.228	0.158	0.243	0.617	2.684
δ_1 to δ_7	0.108	0.269	0.036	0.284	0.641	2.891
δ_1 to δ_8	0.329	0.301	0.115	0.307	0.668	2.993
δ_1 to δ_9	0.156	0.171	0.120	0.187	0.712	3.175
δ_1 to δ_{10}	0.265	0.339	0.159	0.342	0.931	4.293

Note: This table reports annualized return, Δ_γ, that a risk-averse investor would be willing to pay to switch from the δ_1 portfolio to one of the $\delta_d, d = 2, 3, .., 10$ portfolios. γ is the degree of relative risk aversion, with γ =1 (10) representing an investor with low (high) risk aversion.

5. Conclusion

Diversification of investment portfolios reduces the unpriced idiosyncratic risks without affecting the expectation of future returns. Therefore, on an average, better portfolio diversification should lead to better risk-adjusted performance. We test the empirical relation between portfolio diversification and the future risk-adjusted performance. The absence of a unique quantitative measure of diversification has motivated diverse approaches to achieve portfolio diversification. We use three alternative approaches of measuring portfolio diversification in our tests – the Woerheide and Persson measure as a portfolio weight-based diversification measure, the conditional diversification measure as a risk-based diversification measure, and the effective number of bets (ENB) as a factor based diversification measure. There is evidence that the ENB measure is a statistically significantly predictor of the future Sharpe ratios. The economic gains of diversification, as measured by the ENB measure, are large and robust to the investor's risk aversion and choice of investment horizon. When the portfolio diversification is measured by the ENB measure, we estimate that an investor with low (high) risk aversion would be willing to pay 12 to 93 (79 to 429) basis points per annum to capture the benefits of diversification.

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