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Wavelets based multiscale analysis of select global equity returns

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Abstract. *This paper examines the relationship between Indian equity prices other developed markets, in the time-scale domain, using wavelets based multiscale analysis and cross wavelet analysis. Stock markets are analyzed at different levels of resolution which makes it possible to perform a scale by scale analysis enabling us to detect the correlation and cross-correlation structures at time periods with high frequency oscillations and also the relatively low frequency structures. There seems to be a weak integration between BSE and other developed markets at almost all levels of time-scale resolution and a strong relationship between French and German Markets. Analyzing the stock returns at different multiscale resolution makes it easier for agents dealing with different trading horizons.*

Keywords: wavelets, wavelet cross-correlation, multiresolution analysis, Daubechies filter.

JEL Classification: C40, G15, F15, F30.

Introduction

Frequency domain techniques like spectral analysis and Fourier based methods are more suited to study economic and financial datasets that exhibit a cyclical behavior. Fourier methods allow us to analyze the frequency components of the time-series by enabling us to quantify the importance of various frequency components of the time-series under investigation. This provides the researcher access to particular frequency information about the time-series, which makes it easier to infer certain information like the length of a business cycle and the phase lag behavior of the time-series, Masset (2008). But, spectral methods require the data to be stationary, and very often, in the case of economic and financial data, there is a presence of strong non-stationary pattern; e.g. long-memory, jumps etc. in case of the presence of volatility. Also, the time information is completely lost and the assumption of natural periods and stationarity are problematic since economic time-series is characterized by variation in frequencies and non-natural periods. These drawbacks of spectral methods are easily mitigated by the use of time-scale decomposition techniques using wavelet base multiresolution analysis, as wavelet analysis possesses the ability to separate the dynamics in a time-series over different time scales and horizons. A time-series signal, at first observation, might look stationary but a deeper analysis of the signal with excellent time localization, made possible by the use of windowed Fourier transforms of wavelet filters, might help detect the presence of discontinuities. Hence, at a finer and detailed level of signal analysis, presence of non-stationarity could be detected, Capobianco (2000).

Therefore, multiresolution analysis, which by allowing us to analyze the data at different scales of resolution, is definitely a good choice for economic and financial time-series analysis as it gives us the information about both time-space and frequency-varying components of the signal. The information extracted using highly time-localized wavelet windows, from non-stationary financial time-series, can be very useful due to the importance of the information available from the local features of the signal. Wavelet methods, therefore, are most suitable for the analysis of non-stationary financial and economic time-series due to its capability of breaking down the information into different layers of resolution and its time-scale localization properties. Moreover, wavelets are very handy in spotting the exact location in time of regime shifts, discontinuities, and isolated shocks to the dynamical system, Ramsey (1998). The capability of wavelet analysis to decompose a time-series on different time scales and at the same time preserve time localization is one of the main reasons for its induction into economic and financial research. One possesses a better understanding about the time-series and the dynamic market mechanisms behind the time-series by analyzing the time-series at different levels of resolution. This framework of analysis also allows us to isolate many interesting structures and other features of economic and financial time-series, which previously would not have been possible by the use of traditional time domain and Fourier based methods.

The increasing interest, in wavelet analysis, by economic researchers, and its applicability in areas like time-scale decomposition, forecasting, density estimation etc. have led to the emergence of various wavelet based techniques for the analysis of non-stationary financial time-series, Crowley (2005). Wavelet based multiresolution analysis is ideal for the analysis of high frequency data generated by financial markets, providing valuable information for trading decisions, as the analyst can focus on a particular time scale where trading patterns are considered important. Therefore, wavelet analysis has tremendous potential in economics and finance, as relationships between different variables can be analyzed in time-frequency space, allowing one to analyze the relationships between variables at different frequencies and, simultaneously, the corresponding information about the evolution of a variable in time.

The application of wavelet theory was limited to the analysis of deterministic functions, as most of it was applied in the areas of engineering and the natural sciences. The application of wavelet analysis to study the behavior of stochastic processes, which characterize the underlying system in economics and finance, is relatively new. In the next section we review some of the important contribution of wavelet based methods to the field of economic and financial research.

Literature review

The Nineties saw the introduction of wavelet based approaches in statistics. Nason and Silverman (1994) introduced discrete wavelet transforms for statistical applications. Percival and Walden (2000), provides a detailed introduction to wavelets methods for time-series analysis. The maximal overlap discrete wavelet transform (MODWT), Percival and Walden (2000), is particularly suitable in analyzing economic and financial data. This method is a modification of the discrete wavelet transform where the transform loses the property of orthogonality, but since it has the ability to analyze non-dyadic processes; it is very much suited for the analysis of financial time-series. Correlation analysis in state-space is made possible by wavelet coherence analysis, Grinsted et al. (2004).

The application of wavelet methods, particularly in the field of economics and finance, is described by Gencay et al. (2001). High frequency foreign exchange rates were analyzed by Ramsey and Zhang (1995) using waveform dictionaries and a matching pursuit algorithm. Ramsey and Lampart (1998) found that the relationship between money and income varies according to scale. At higher scale levels, money supply Granger caused income and at lower scale, income granger caused money supply. The multiresolution analysis of high frequency Nikkei stock market data, using the matching pursuit algorithm of Mallat and Zhang (1993), is carried out by Capobianco (2004). Hidden periodic components are unearthed using the algorithm.

Maximal overlap discrete wavelet transform is applied by Crowley and Lee (2005) to analyze the frequency components of European business cycles. Date from countries with lesser degree of integration exhibited non-similar frequency components. The lead-lag relationship between the Dow Jones Industrial Average stock price series and the index of industrial production series of the US is analyzed by Gallegati (2008), using wavelet correlation and cross-correlation methods. At lower frequencies, stock market returns lead economic activity as reflected in IIP series. Since increase in timescale is associated with lower frequency bands, the leads in stock market returns increases with the increase in scale.

Conraria and Soares (2011) study business cycle synchronization across the European union-15 and Euro-12 countries using wavelet analysis. France and Germany are found to be highly synchronized with other European countries and French business cycle leads German business cycle as well as the business cycles from the rest of the European countries.

The comovements between the stock markets of the US, Germany, UK and Japan were analyzed by Rua and Nunes (2009) using wavelet coherence analysis. Market interdependencies were found to change across frequencies and along the time horizon. Strongest comovements were observed between the markets of US and Europe, and the coherence between US-Germany and UK-Germany increased in time.

Barunik et al. (2011) used wavelet coherence analysis to study the time-scale dynamics of local correlations between Central European and Western European stock markets. The interdependencies between major European markets were found to change significantly in time and across scales.

The study of correlation structure between S&P 500 and other international markets was carried out by Benhmad (2013) using wavelet analysis. The co-movements of stock market were found to be a function of scale, apart from its dependence on time dynamics. S&P 500 and European stock markets were found to exhibit strong interdependencies, which changed according to changes in time-scale. The next section gives a brief review of the methodology used in this analysis which will be followed by its use in analyzing equity prices, some empirical evidences and conclusions.

Methodology

A wavelet is a function $\psi(.)$ defined on the real line $\mathbb{R}$, such that $\int_{\mathbb{R}} \psi(t)dt = 0$ and $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$. A signal can be decomposed into its finer detail and smoother components by projecting the signal onto mother and father wavelets given by ψ and ϕ respectively. Dilation and translation operation is performed on both mother and father wavelets to form a basis for the space of squared integrable function, $L^2(\mathbb{R})$. Therefore, any function $x(t)$ in $L^2(\mathbb{R})$ can be represented as linear combinations of these basis functions. The dilated and translated versions of mother and father wavelets are denoted by $\psi_{b,s}(t)$ and $\phi_{b,s}(t)$ respectively, where

$$\psi_{b,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-b}{s}\right) \quad (1)$$

$$\phi_{b,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-b}{s}\right) \quad (2)$$

s and b represents the scaling (dilation) and translation parameter, respectively. Here s = *1,…, S* controls the number of multiresolution elements. Formally, a function $x(t)$ can be represented in the wavelet space as

$$x(t) = \sum_b a_{S,b}\phi_{S,b}(t) + \sum_b d_{S,b}\psi_{S,b}(t) + \sum_b d_{S-1,b}\psi_{S-1,b}(t) + \ldots + \sum_b d_{1,b}\psi_{1,b}(t) \quad (3)$$

where $a_{S,b}$ are coefficients describing coarser features of $x(t)$, and $d_{s,b}$ are detail coefficients that captures information from multiple resolutions or time-horizons.

Wavelet based correlation and cross-correlation

Let $X_t = (x_{1,t}, x_{2,t})$ be a "bivariate stochastic process with univariate spectra" (autospectra) $S_1(f)$ and $S_2(f)$ respectively, and let $W_{s,b} = (w_{1,s,b}, w_{2,s,b})$ be the scale s wavelet coefficients computed from X_t. These wavelet coefficients are obtained by applying the wavelet transform to all elements of X_t. The obtained wavelet coefficient contains both $a_{S,b}$ (coarser approximations) and $d_{s,b}$ (wavelet details). For a given scale s, the wavelet covariance between $x_{1,t}$ and $x_{2,t}$ is given by

$$\gamma_X(s) = \frac{1}{2\pi} Cov(w_{1,s,b}, w_{2,s,b}) \quad (4)$$

The wavelet covariance "decomposes the covariance of a bivariate process on a scale-by-scale basis", i.e.

$$\sum_{s=1}^{\infty} \gamma_X(s) = Cov(x_{1,t}, x_{2,t}) \quad (5)$$

By introducing an integer lag τ between $w_{1,s,b}$ and $w_{2,s,b}$, the notion of wavelet cross-covariance can be introduced, and is given by

$$\gamma_{X,\tau}(s) = \frac{1}{2\pi} Cov(w_{1,s,b}, w_{2,s,b+\tau}) \quad (6)$$

In some situations it may be beneficial to normalize the wavelet covariance by wavelet variance, which gives us wavelet correlation

$$\rho_X(s) = \frac{\gamma_X(s)}{\sigma_1(s)\sigma_2(s)} \quad (7)$$

where $\sigma_1^2(s)$ and $\sigma_2^2(s)$ are the wavelet variances of $x_{1,t}$ and $x_{2,t}$ (at scale s), respectively. Just like the usual correlation coefficient between two random variables, $|\rho_X(s)| < 1$. However, wavelet correlation gives correlation among variables from a multiscale dimension Also, by allowing the two processes $x_{1,t}$ and $x_{2,t}$ to differ by an integer lag τ , we can define wavelet cross-correlation, which gives us the lead-lag relationship between two processes, on a scale-by scale basis. The approximate confidence bands for the estimates of wavelet correlation and cross-correlation is given in Percival and Walden (2000) and Gencay et al. (2002). Moreover, the reader is referred to Fernandez-Macho (2012) for the technique of wavelet multiple correlation (WMC) and multiple cross-correlation (WMCC).

Empirical data

The data used for this study comprises of BSE sensitive index and four other indices from the developed markets. BSE sensitive index constitutes 30 Clue Chip securities traded in Bombay stock exchange, and represents major portion of BSE in terms of market capitalization. We use BSE 30 series against Shanghai stock exchange (SSE) and four developed stock market indices, viz., FTSE 100 index of the UK, Nikkei 225 of Tokyo stock exchange, CAC40 of France and DAX index of Germany. The series used for our study is the closing price level series. The study period spans over a period of January 2000 to March 2013, thus involving around 3330 data points, which provides a fairly rich data set for our analysis.

Empirical results

The daily stock market returns are decomposed applying the MODWT with the D(4) Daubechies wavelet filter, with decomposition up to six levels of resolution.

Figure 1 shows the wavelet correlation between SSE and BSE. The wavelet correlations between SSE and BSE are all low, across all six scales, with negative correlations for the first two and a half scales which correspond to a period of 8-16 days. The correlations tend to increase as scale increases, but since the lower confidence band lie below zero across all scales, there is no significant correlation between SSE and BSE. See Table 1 for the values of wavelet correlation across all the six scales.

Figure 2 shows the wavelet cross correlation between SSE and BSE. Wavelet cross correlation is performed between SSE and BSE, with leads and lags up to 36 months. The variable that maximizes the correlation, as against the other variable, is shown in the upper-left portion of all figures. The cross correlations are all near zero, across all six scales for all lags, which signals a very weak cross correlation between SSE and BSE.

Figure 1. *Wavelet correlation between SSE and BSE*

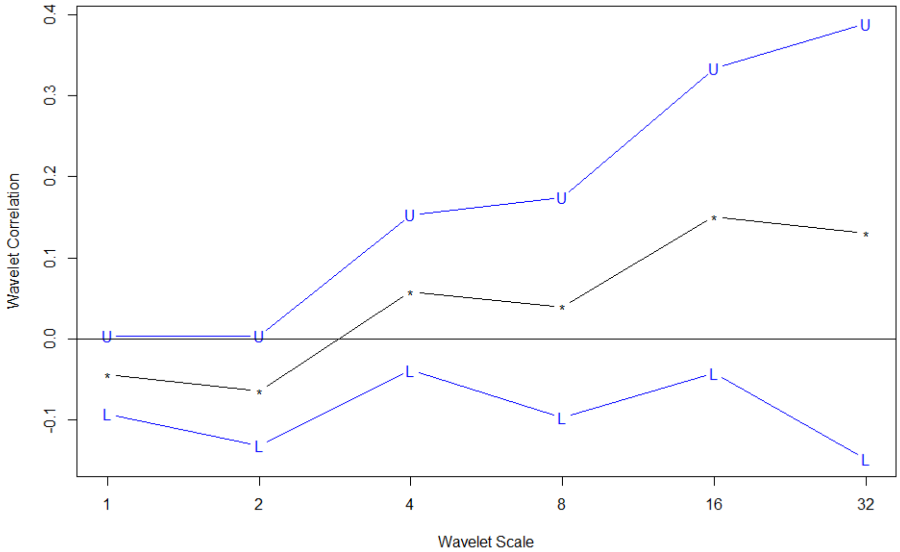
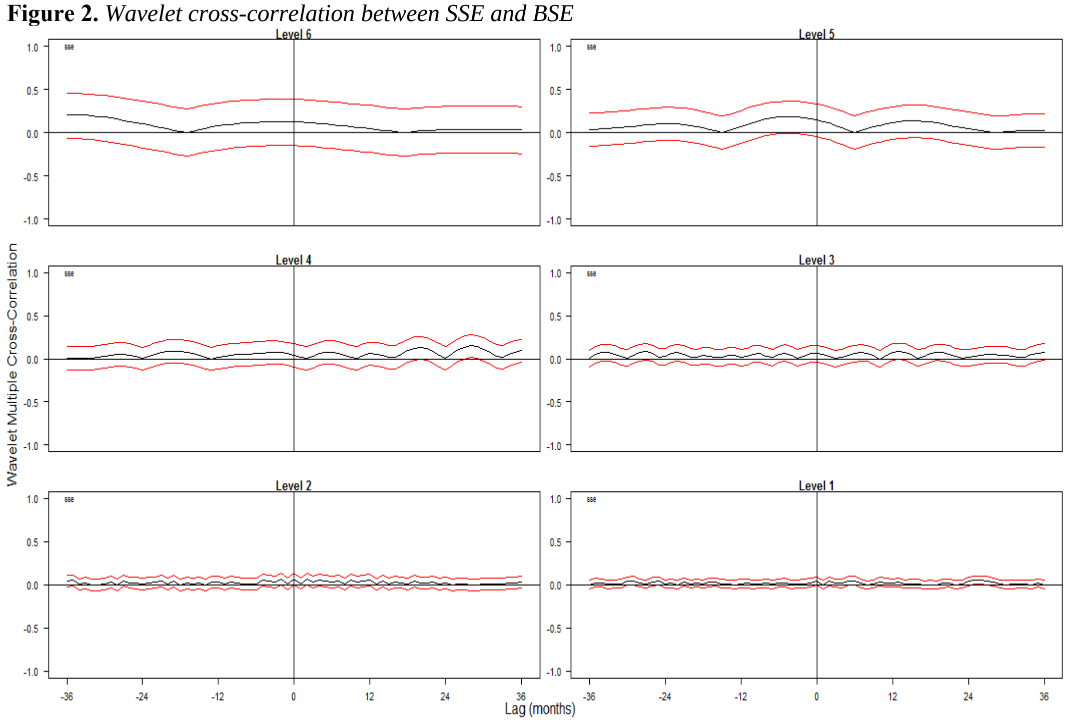


Figure 2. *Wavelet cross-correlation between SSE and BSE*



Figures 3 and 4 shows the wavelet correlation and wavelet cross-correlations between FTSE and BSE respectively. There is no significant correlation between FTSE and BSE and no significant cross correlations too as the cross correlations are all near zero, across all scales for all lags. This indicates a weak integration between FTSE and BSE. Similar results hold for wavelet cross correlations between CAC40 and BSE.

Figure 3. *Wavelet correlation between FTSE and BSE*

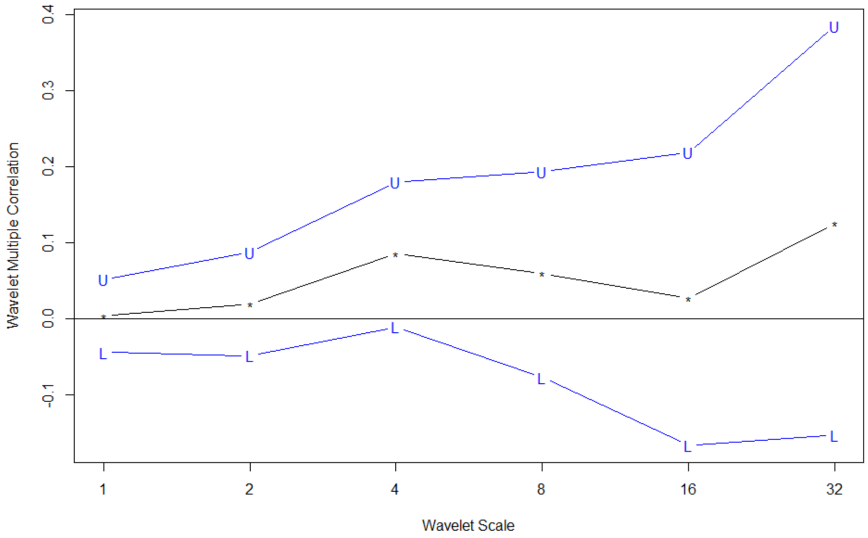


Figure 4. *Wavelet cross-correlation between FTSE and BSE*

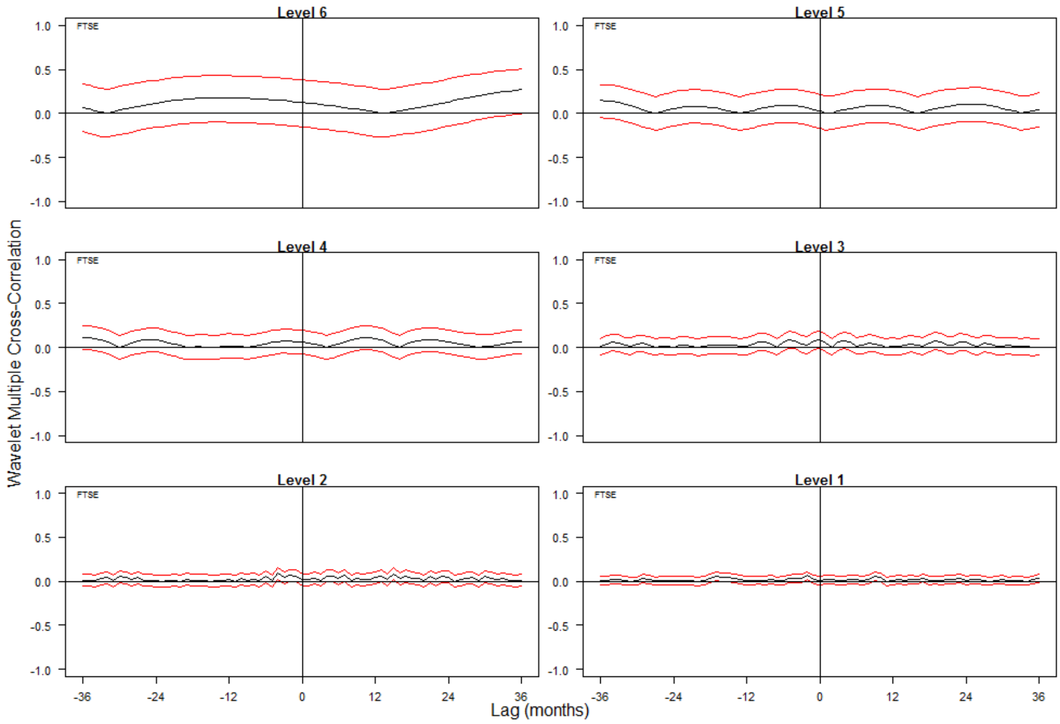


Figure 5 displays the wavelet cross correlations between NIKKEI225 series and the BSE series.

The first two levels, associated with periods of 2-4 and 4-8 days (intra-week and weekly scales), shows many lags converging towards zero and many lags different from zero but all positive. The lags where wavelet correlations are positive, and different from zero, lie around the zero axes. However, the correlations are not statistically significant as the plot of lower confidence band lie below the zero axes. We see no significant correlations at level three, except a slight correlation at around lag -20. There is a slight correlation between the BSE series and the NIKKEI series at lag 24 at level four, but no correlation at other lags. As we move towards higher levels of resolution we see increasing correlations between BSE and NIKKEI. Some positive cross correlations around lag 24 is observed at level six which correspond to a period of 64-128 days(quarterly to biannual scales).

Figure 5. *Wavelet correlation between NIKKEI225 and BSE*

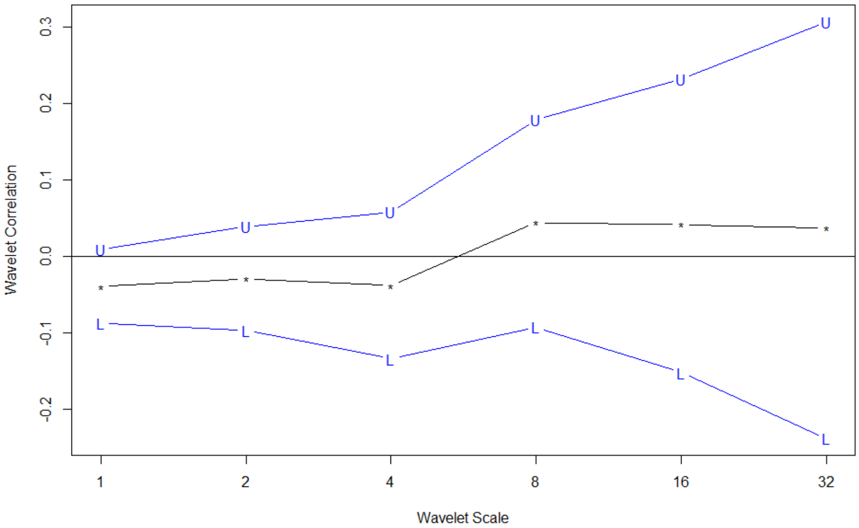


Figure 6 shows the wavelet correlation between DAX and CAC. The wavelet correlations between DAX and CAX are positive but low, across the first and second level of resolution which corresponds to intra-week and weekly scales. There is an increase in wavelet correlation at the third level which corresponds to a period of 8-16 days. At level four, which covers the monthly scale, we see a sharp drop in correlation. However, we see a significant rise in correlations at the next two levels of decomposition which roughly corresponds to a period of 64-128 days (quarterly to biannual) and 128-256 days (biannual scale). The correlations tend to increase as we look at higher levels of decomposition, except a decrease in correlations at monthly scale analysis. This shows the strong integration between these two markets, with good market integration at quarterly to annual scales.

Figure 7 shows the wavelet cross correlation between DAX and CAC. Wavelet cross correlation is performed between DAX and CAC, with leads and lags up to 36 months. The variable that maximizes the correlation, as against the other variable, is shown in the upper-left portion of all figures. Wavelet cross correlation analysis at level 1 and level 2 reveal some significant correlation at lags 10, 11 and 12. As we increase the resolution level, there seems to be a slight increase in cross correlations with correlations oscillating between zero lag up to one year in the future. Statistically significant cross correlations are observed after increasing the level of decomposition, with strong cross correlations at level 5 and level 6, for almost all leads and lags. This suggests weak but positive correlations between DAX and CAC when the analyses are performed at smaller levels of resolution (intra –week and weekly period), but higher cross correlations when time scales are increased for analyzing the data at quarterly, biannual and annual periods. This further gives us some evidence about good market integration between DAX and CAC.

Figure 6. *Wavelet Correlation between DAX and CAC*

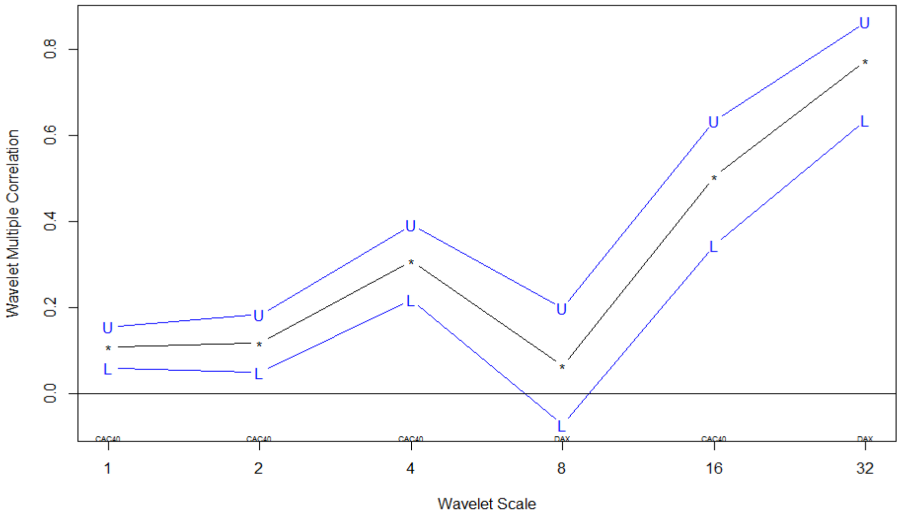
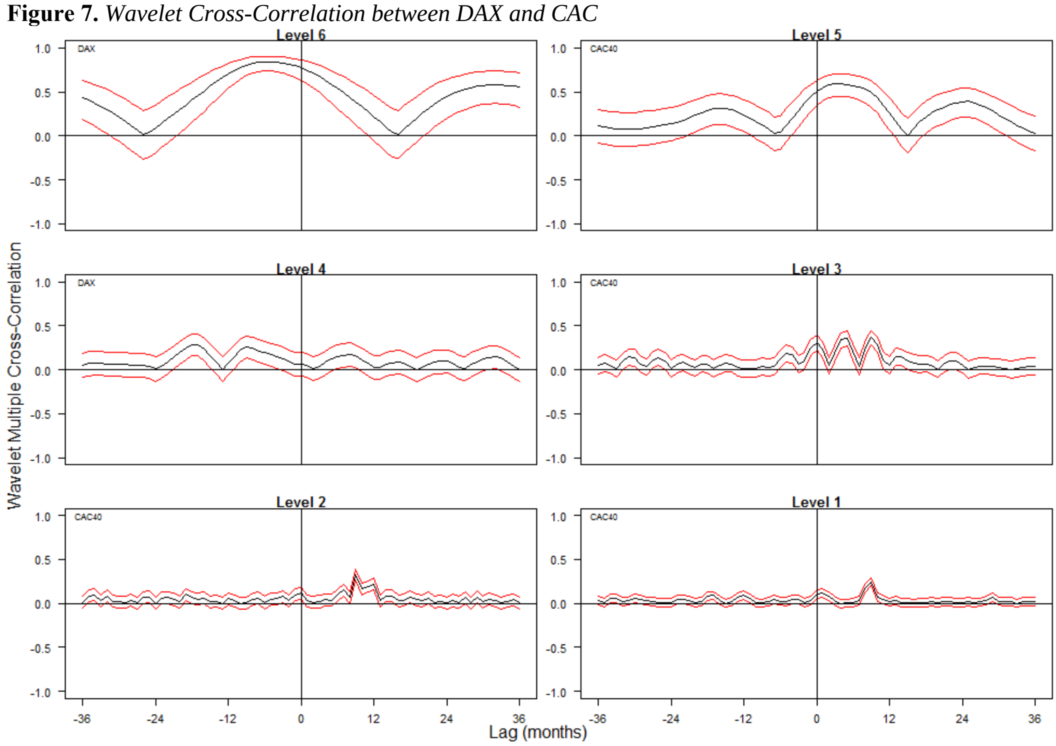


Figure 7. *Wavelet Cross-Correlation between DAX and CAC*



Conclusion

Traditional methods like spectral analysis works by projecting the time series data into a set of sines and cosines, wherein the output signal is a function of frequency only. As a result the time information is completely lost. This problem is resolved by using wavelets based decomposition where the output signal is a function of both time and scale, providing us with simultaneous information from both time and frequency domains. This approach helps us to decompose the one dimensional time series data into a time-frequency plane by projecting the time signal into a set of orthogonal wavelet basis functions.

Therefore, a Time-scale decomposition of Bombay stock exchange (BSE) stock returns and returns from other select international markets, using wavelet based multiresolution analysis, is performed. Wavelet correlation analysis is used to analyze the correlation structure between BSE and stock exchanges from China, France, Germany and Japan. Very weak correlation between BSE and Chinese stock returns (SSE) are recorded at all levels of decomposition. The correlations tend to increase as we increase the timescale of analysis, but no statistically significant correlations are recorded. Wavelet cross-correlation between BSE and SSE, at all leads and lags, are very weak as all correlations lie near zero at all levels of resolution. This indicates a weak integration between BSE and SSE stock markets. Same holds true for analyses of BSE performed with FTSE (London stock exchange) and CAC40 (French stock market index) stock returns. Some positive cross correlations around lag 24 is observed at level six which correspond to a period of 64-128 days(quarterly to biannual scales). BSE seems to be slightly correlated with NIKKEI (Tokyo stock exchange) at a resolution of level six, which corresponds to quarterly-biannual scales. On the other hand, French and Markets are highly integrated as wavelet correlation and wavelet cross correlations (at most of the leads and lags) are positive and increases significantly with the increase in the level of resolution.

This approach allows us to detect changes in stock market behavior from a time-scale perspective where the data can be analyzed at different time horizons. The dynamics of stock returns can be studied by decomposing the stock returns into several layers of time-scale resolution (i.e. Short time period analysis to long period analysis), which can provide useful insights for investors with different trading horizons in mind.

Tables showing wavelet correlations between pairs of select markets

Table 1. *Wavelet Multiple correlation between SSE and BSE*

Levels	WMC	LowerCI	UpperCI
1	0.04459197	0.003469	0.0924478
2	0.06445618	0.003527	0.131846
3	0.05794457	0.038415	0.1532365
4	0.03965149	0.096912	0.17475
5	0.15121177	0.042618	0.3340713
6	0.13002267	0.148134	0.3891159

Table 2. *Wavelet Multiple correlation between FTSE and BSE*

Levels	WMC	LowerCI	UpperCI
1	0.003968619	0.044094	0.0520126
2	0.019467548	0.048564	0.0873194
3	0.085205468	0.011031	0.1798775
4	0.059260635	0.077405	0.1937399
5	0.026907131	0.166544	0.2183638
6	0.125241818	0.152884	0.384984

Table 3. *Wavelet Multiple correlation between CAC40 and BSE*

	WMC	LowerCI	UpperCI
1	0.0135397	0.03454	0.061554
2	0.00318541	0.0648	0.071137
3	0.04259046	0.05378	0.13817
4	0.09077961	0.04583	0.224054
5	0.13911413	0.05495	0.323047
6	0.04646006	0.22935	0.315362

Table 4. *Wavelet Multiple correlation between DAX and NIKKEI*

Levels	WMC	LowerCI	UpperCI
1	0.003132742	-0.044928	0.0511789
2	0.04663081	-0.021404	0.1142362
3	0.018538062	-0.077746	0.1144796
4	0.054547953	-0.082102	0.1891852
5	0.060086397	-0.134053	0.2497841
6	0.193326789	-0.084006	0.4428622

Table 5. *Wavelet Multiple correlation between FTSE and NYSE*

Levels	WMC	LowerCI	UpperCI
1	0.002823504	-0.04524	0.050871
2	0.00777433	-0.06022	0.075702
3	0.048070075	-0.0483	0.143552
4	0.185210195	0.05044	0.313356
5	0.254215066	0.064799	0.425949
6	0.741953405	0.588139	0.843968

Table 6. *Wavelet Multiple correlation between NIKKEI225 and BSE*

Level	WMC	LowerCI	UpperCI
1	0.03916004	0.008911	0.0870501
2	0.02897762	0.039067	0.0967545
3	0.03802189	0.058337	0.1336788
4	0.04423065	0.092366	0.1791935
5	0.04195577	0.15186	0.232664
6	0.03748039	0.237853	0.3072382

Table 7. *Wavelet Multiple correlation between CAC and DAX*

Levels	WMC	LowerCI	UpperCI
1	0.10778593	0.060043	0.1550368
2	0.11706433	0.0494907	0.1835711
3	0.30788209	0.2181954	0.3924117
4	0.06419309	-0.072481	0.1985007
5	0.50388835	0.3447563	0.6348672
6	0.77502356	0.6368115	0.8649775

Table 8. *Wavelet Multiple correlation between FTSE and CAC*

Levels	WMC	LowerCI	UpperCI
1	0.023040084	-0.025042	0.0710154
2	0.008664079	-0.059338	0.0765864
3	0.094352935	-0.001809	0.188786
4	0.149253908	0.0134866	0.2796175
5	0.145565312	-0.048379	0.3289322
6	0.082871054	-0.194426	0.3479034

Table 9. *Wavelet Multiple correlation between FTSE and DAX*

Levels	WMC	LowerCI	UpperCI
1	0.02509043	-0.022991	0.0730563
2	0.06944789	0.0014875	0.1367697
3	0.12349066	0.0276736	0.2170591
4	0.04023634	-0.096332	0.1753178
5	0.01581551	-0.177313	0.2077709
6	0.34130332	0.0754285	0.5618701

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