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Main aspects on the nature of dynamic models

Constantin ANGHELACHE
Bucharest University of Economic Studies, Romania
"Artifex" University of Bucharest, Romania
actincon@yahoo.com
Dana Luiza GRIGORESCU
Bucharest University of Economic Studies, Romania
danaluiza2004@yahoo.com
Oana BÎRSAN
Bucharest University of Economic Studies, Romania
oana.vilcu@fiscont.com.ro

Abstract. *The dynamic models used in the micro or macro-economic analyzes appeared as a necessity to correlate the statistical variables divided into the resultant variables and the factorial variables, in order to be able to estimate, by the parameters calculated using statistical-econometric methods and to be able to deduce the correlative influence between those two categories of variables.*
Macroeconomic models are static or equilibrium, meaning that the static model refers to the data up to the moment, and the equilibrium model is a special case, especially of a dynamic system. The situation of tomorrow is established by investigating how the result variables considered under the influence of factors (statistical variables considered) evolved. Based on the agreed static or dynamic models, the differential equations are established (this starts from the term of the use of derivatives), so as to justify the way in which the relations between the two variables are expressed and their evolution over time.
Dynamic models are used at the macroeconomic level to determine the National Income, as the net result indicator that is distributed, redistributed and used to meet the needs of those interested.
Dynamic models must be considered as a system in which differential equations refer to two successive periods that can be represented graphically. After all, the dynamic models used in the analyzes must start from the fact that a study was carried out previously based on the statistical data series and then on their graphical representation.
A number of elements are considered important in agreeing on dynamic macroeconomic models, such as the evolution of the anticipated price cycle that changes the value, but not the content, most of the times, the structural content of the macroeconomic aggregates.

Keywords: models, balance, dynamic process, forecast, differential equations, national income.

JEL Classification: C50, C62, E01.

Introduction

In the article the main aspects regarding the nature of dynamic models, the authors went from simple to complex, to define and specify the notion of dynamic models used in macroeconomic studies.

It started from the simple fact that models can be static or equilibrium, and not lastly, forecast models that are based on a future evolution of the indicators (statistical variables) considered.

The system of equations based on which the estimation parameters are determined, represents a determining element in the use of dynamic models in the analysis of macroeconomic evolution.

The authors or referred, one by one, to establishing the differential equations that constitute the system of equations based on which the parameters are determined, then resorted to applying a model to the national income level, taking it as the only macroeconomic indicator of net results that it can contribute both to the allocation for investments and to the allocation to increase the standard of living of the population.

Next, a graphical solution of the dynamic models was used, starting from the fact that this dynamic model must highlight on the basis of the considered data, the way in which the economic evolution can be foreseen in a future period. In fact, the authors summarize the fact that, analyzing the situation of a macroeconomic indicator or the macroeconomic situation as a whole, it can be done after a study has been performed previously on the statistical data series and then on the graphs drawn up based on these data series. Last but not least, the authors express some points of view in relation to the anticipated price cycles, thus aiming to suggest the evolution perspective during the forecast or forecast period considered.

The article is accompanied by graphical representations, data and representative tables that justify the views expressed by this article.

1. Literature review

Anderson, Breedon, Deacon, Derry and Murphy (1992) are concerned with the estimation and interpretation of the yield curve. Anghelache and Anghel (2019) addresses in their work theoretical and practical aspects regarding economic statistics. Anghelache and Anghel (2019) it addresses the problems of economic modeling. Anghelache (2018) makes a thorough analysis of the financial statements of Romania over the last one hundred years. Anghelache, Mitruţ and Voineagu (2013) addresses issues of macroeconomic statistics, focusing in particular on the System of National Accounts. Anghelache and Anghel (2019) analyze the position of Romania in the European context. Anghelache (2019) analyzes in his work the evolution of industrial activity in Romania. Anghelache, Anghelache and Bârsan (2019) analyzes models used in dynamic series analysis. Bollerslev are Wooldridge (1992) are concerned with estimating the probabilities and interference of dynamic models that are time-varying.

Hansen and Lunde (2006) addresses issues related to forecasts for volatile models. Iacob (2019) approaches methods of statistical-econometric analysis of economic phenomena. Ledoit and Wolf (2003) are concerned about estimating stock returns with a portfolio selection request. Ruth and Hannan (2012) turn their attention to their analysis of dynamic economic systems. Scaillet (2004) addresses in this article problems related to non-parametric estimation and analyzes the degree of sensitivity of the predicted deficiency.

2. Research methodology, data, results and discussions

Static or equilibrium models, with only incidental reference to the dynamic properties of these systems, are a particular element. Each equilibrium model is a special case of a dynamic system, but there are many dynamic systems that do not lead to any equilibrium position. All the systems of the real world are dynamic systems and, very rarely, in the social or economic systems we find a real case of equilibrium in which the variables of the system remain at the same level for successive periods of time. Therefore, the equilibrium mode is mainly valuable because if the equilibrium position is stable, we know that within limits, the system will move towards it. Therefore, if we are offered the equilibrium state of a system and its current state, at least we can predict the direction of change by the fact that the near future states will be somewhere between the present state and the equilibrium state. However, we cannot know for sure whether a steady state is stable or not until we have specified the dynamic properties of the system.

We can appreciate that a state of a system consists of its description at one point, and a dynamic process consists of a succession of states, such as successive frames on a drum. If there are stable relationships between moments, which are a constant distance apart in sequence, we have a dynamic system. Thus, if today he has a constant relationship with yesterday, tomorrow he will have the same relationship with today. The forecast can only be made if we know the dynamic system that underlies a process. So, if we know a stable relationship between yesterday and today and at the same time we know the state of the system today, we can predict the situation tomorrow. Knowing the situation of tomorrow allows us to predict the situation of tomorrow and thus we can continue for an unlimited period in the future.

Suppose that S_t represents the state of the system at any time, t. If there is a stable relationship, of the form:

$$S_t = F(S_{t+1}) \quad (1)$$

we have a dynamic system of degree one. If we are specified S_0, $S_1 = F(S_0)$, $S_2 = F(S_1)$ and so on, the whole sequence, S_0, S_1,…, it can be generated at any time S_t. If there is a stable relationship, of the form:

$$S_t = F(S_{t+1}, S_{t+2}) \quad (2)$$

we have a dynamic system of the second degree. S_t depends not only on yesterday, S_{t-1} but also S_{t-2}. If we are given two successive states, we can predict all the other states in the sequence. Similarly, in a ninth-grade dynamic system, S_t is a stable function of n previous states and we must know n successive states before the system can be defined.

A balance system is one in which $S_t = S_{t+1} = S_{t+2} \dots S_e$, S_e being the equilibrium value of the system. Thus, a dynamic system is in balance if today is such that tomorrow is the same as today.

- **Differential equations**

If the state of the system can be described by the value of a unique variable, the dynamic system can be described by a simple difference equation. A difference equation expresses a stable relationship between two or more successive positions of the same variable. Suppose that the value of an interest rate loan doubles over a certain period, say ten years. Then if x_t is the value of the loan at any time, and x_{t+1} is the value ten years after this date, we can write $x_{t+1} = 2x_t$. This is a very simple difference equation. If we are given the value of x at any date, then we can deduce the value at any other date. So if $x_0 = 100$, from the above equation results $x_1 = 200$. Also if $x_1 = 200$, then x_2 must be 400. Similarly x_3 it is 800, x_4 is 1600, x_5 is 3200, and so on, each value being double as compared to the previous value. If the value of the variable is initial so that the same value is repeated period after period, the initial value is said to be the equilibrium value of the system. I mean $x_0 = x_1 = x_2, \dots, x_n$ and so on $x_0 = x_e$, the equilibrium value. The equilibrium value is found by placing $x_{t+1} = x_t = x_e$ in the equation of difference. In the above case, the only balance value is $x_e = 0$. However, this balance is not stable in the above case. A slight variation above zero (or below zero) will set the system off balance. It is considered, by contrast, the system $x_{t+1} = 0.5\ x_t$. If $x_0 = 1$, $x_1 = 0.5$, $x_2 = 0.25$, $x_3 = 0{,}125$ and so on. A disturbance certainly results in an inverse movement at the equilibrium position towards zero.

- **Application to the basic national income model**

These concepts can now be applied immediately to a simple national income system. Suppose that a linear consumption function, of the form: $C_{t+1} = C_0 + cY_t$. We consider consumption in any year dependent on the revenues of the previous year. The linear form of the equation means that the graph of this consumption function is a straight line. The tendency to consume, "c", is a constant and it is assumed that there will be a certain consumption, C_0, even at zero income. Then we assume that the investment will be a constant "A". We know from the identity of the economy-investments that $Y_{t+1} = C_{t+1} + A$. By correlating the consumption function and the identity of the investment-economy, we obtain an equation of difference in national income, of the form:

$$Y_{t+1} = C_0 + A + cY_t \qquad (3)$$

The equilibrium value is determined by considering equality $Y_{t+1} = Y_t = Y_e$. We introduce this equality in equation (3) and thus we obtain:

$$Y_e = C_0 + A + cY_e \ sau \ Y_e = \frac{C_0 + A}{1 - c} \qquad (4)$$

An arithmetic example clarifies the dynamics of this system. To suppose $C_0 = 40$, $A = 20$, $c = 0.8$. The equilibrium value is $Y_e = 60 / (1\text{-}0.8) = 300$. Table 1 shows first the national income rate if we start from a point under equilibrium, suppose 200 and the evolution of the national income rate if we start from a point above the value of the equilibrium, supposed 400.

Table 1. *Model of the dynamics of National Income*

National Income	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
$C_0 + A$		60	60	60	60	60	60	60	60	60
$0.8(Y_t)$		160	176	189	199	207	214	219	223	226
Y_{t+1}	200	220	236	249	259	267	274	279	283	286
$C_0 + A$		60	60	60	60	60	60	60	60	60
$0.8(Y_t)$		320	304	291	281	273	266	261	257	254
Y_{t+1}	400	380	364	351	341	333	326	321	317	314

It will be observed that the equilibrium position is stable. A divergence from equilibrium, either above or below the equilibrium value, is established in moving forces to restore the equilibrium position. It will be observed that there is no cyclical movement. The national income is constantly approaching its equilibrium value at a diminishing rate.

Figure 1. *Balance curve*

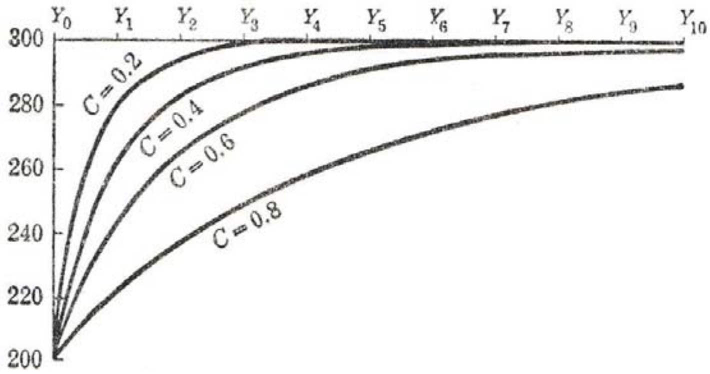
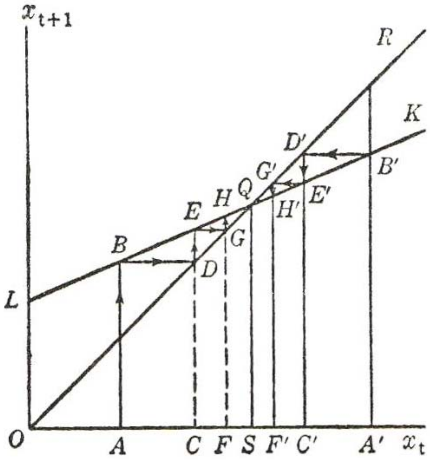


Figure 1 shows the curve towards a balance of 300 with a consumption ratio of 0.8, 0.6, 0.4 and 0.2. It is obvious that the lower the consumption trend, the faster the balance approach.

- **Graphical solution of dynamic models**

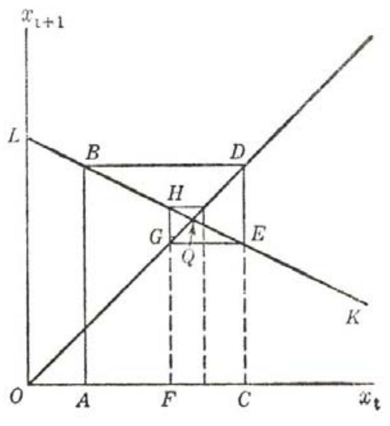
Systems where the difference equations refer to only two successive years can be solved by a simple graphical technique. Suppose we measure x_t horizontal and x_{t+1} on the vertical axis of Figure 2. The difference equation regarding these two quantities can be represented as a line on this graph. Suppose that in Figure 2, LK is such a line.

Figure 2. *The solution of the dynamic model with the slope of the positive line*



We will draw the 45 degree line from the origin, OR, which intersects HK in Q. Q is clearly the equilibrium value, because by the construction OS = SQ, then x_t = x_{t+1}. Now suppose that OA is the initial value x_0. If AB is drawn vertically to intersect HK in B, AB = x_1. If BD is drawn horizontally to intersect OA in D, we have by construction OC = CD = AB = x_1. If CD is drawn up to intersect HK in E, CE = x_2. Similarly, FH – x_2 and so on to the equilibrium at Q. Similarly, starting from a position superior to the equilibrium in A' successive positions x_0, x_1, x_2, x_3 are OA', A'B', C'E', F'H', and so on. The balance in Figure 2 is clearly stable. If the LK curve is steeper than 45 degrees where the right SA intersects, the balance is unstable.

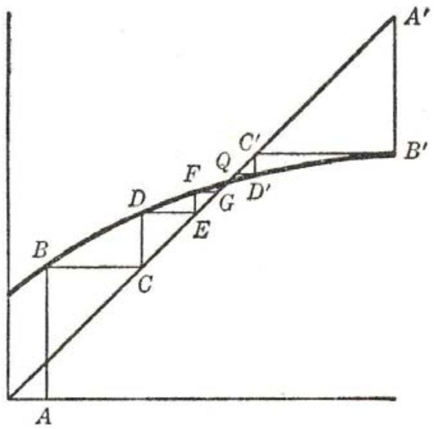
Figure 3. *The solution of the dynamic model with the slope of the negative line*



A curve expressing a differential equation can be called a differential curve. If the slope of the differential curve is negative, as shown in Figure 3, the movement towards or away from equilibrium follows a cyclical path.

Starting from an initial position x_0 = OA, we x_1 = AB (= OC), x_2 = CE (= OF), x_3 = FH and so on in a “cobweb” around the equilibrium point, Q. The famous “cobweb” theorem in price theory is just a special case of this kind of dynamic system. If the inclination of the line LK is 45 degrees, then G it will intersect the right AB, and the cycle will repeat endlessly. If the inclination is higher than 45 degrees, then the cycle will be explosive and the balance will be unstable.

Figure 4. *Nonlinear dynamic model*



The graphical technique has the advantage that it can be applied as easily to differential linear equations. Thus, in Figure 4, we assumed that the differential curve has a downward slope. Regarding our economic model, this would mean that the marginal consumer slope has diminished with the increase of the income, considered an unpleasant hypothesis. Under these conditions, the lower balance approach, following the ABCDEF course, is slower than the higher approach, A'B'C'D. This suggests that the shortcomings of the system would be dynamic and rapid, as the resolution would be slow and prolonged. There is some evidence that business cycles tend to follow this pattern.

Also, from the analysis it turns out that a simple "multiplier" system cannot, by itself, produce a truly cyclical fluctuation about a balance unless the inclination to absorb it is negative, which is also an excessively low hypothesis.

- **Expected price cycles**

Accelerator models that show the evolution of market prices over time can be built along lines similar to those presented above. Suppose we change the market identity p in shape p_t , where $K = M / A$ is constant and $r_t = r_a / r_m$ is an intensity of the demand for net money for the goods. Suppose further that this coefficient of demand, r_t, is equal to a certain normal level of, r_0, plus a factor that is higher if the price is rising, lower if the price is falling. That is, suppose that people are projecting the price trend, so that the price increase will ensure the expectations of further growth, and thus an increase in demand, while the decrease of prices leads to a further expectation of decrease, and thus a demand decline. If these functions are linear, we can formalize the relationship:

$$r_t = r_0 + \alpha(p_t - p_{t-1}) \quad (5)$$

from where we deduce:

$$p_t = Kr_t = Kr_0 + K\alpha(p_t - p_{t-1})$$

or

$$p_t(1 - K\alpha) = Kr_0 + K\alpha p_{t-1} \quad (6)$$

This is a first degree differential linear equation. Therefore, the price course over time can be analyzed by a scheme similar to Figures 5 and 6.

Figure 5. *Price cycles*

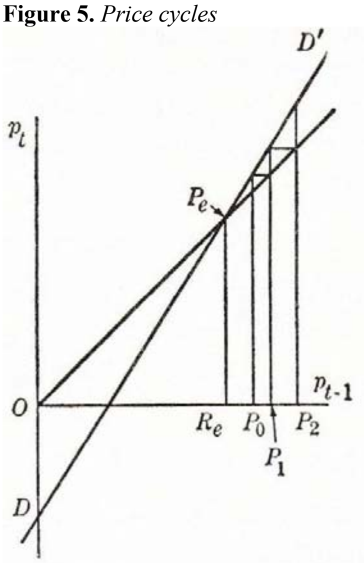
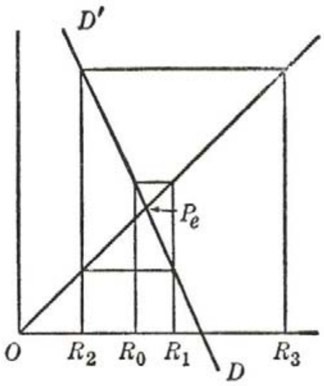


Figure 6. *Price cycles*



Thus in Figures 5 and 6 the line DD' is the curve of the difference corresponding to the equation (2). Kα > 1, the slope of the difference curve DD', -Kα / (1-Kα') will be positive, but less than 1, as in Figure 5. Therefore, the equilibrium at Pe will be unstable. If we start from a price above the equilibrium price, OR_e, to say, OP_0, further prices will be OP_1, OP_2, etc., so the price increases at an accelerated rate. Similarly, if we start from a price below equilibrium, the price will fall at an accelerated rate. As much as Kα is approaching 1, the line DD' becomes steeper, and the movement from equilibrium becomes faster, until Kα = 1 and DD' is vertical, and the balance is unstable. The price influences the change to infinite plus or minus, if it is disturbed at the equilibrium level.

With Kα < 1 a cyclic model will be configured, as in Figure 6. If Ka is between 0.5 and 1, the cycle will be explosive, as in the graph. The course from OR_0 is up to OR_1, beyond OR_0 up to OR_2, beyond OR_1 as far as OR_3 and so on. If Kα = 0.5, the cycle will be perpetual. Kα is between 0 and 0.5, the cycle will be amortized, as in Figure 2. As Kα the smaller, the faster the depreciated cycle. If Kα is zero, the price moves immediately into the equilibrium position, pe = Kr_0. If α is negative, so that rising prices lead to a drop, DD' will have a positive slope of less than 1, i.e. less than 45 degrees, and the price will move steadily to equilibrium, as in Figure 1. The sudden transition from high prices to low prices as in Figure 6, it is not particularly realistic. However, this is only the result of expressing the model in the form of a differential equation of the first degree. If we assume that r_t is a function α (p_{t-1} - p_{t-2}), and we say the relationship:

$$r_t = r_0 + \alpha(p_{t-1} - p_{t-2}) \quad (7)$$

we get a second degree difference equation,

$$p_t = Kr_0 + K\alpha(p_{t-1} - p_{t-2}) \quad (8)$$

One can test that this relationship produces a perpetual cycle if Kα = 1, an amortized cycle if Kα < 1 and an explosive cycle if Kα > 1.

Conclusions

From the study that was the basis of the present article, a number of theoretical and practical conclusions are drawn. From a theoretical point of view, it is suggested that these models are in fact a series of statistical-econometric relations systems, which suggest suggestively the correlation between the macroeconomic variables, divided between the resultant variables and the factorial variables, in a period of time.

The constitution of the model used is particular and it is based on an in-depth study of the data series and their graphical representations regarding the considered statistical variables. Also, from the analysis and the study carried out, it is concluded that these models should be well agreed, so as to provide successive stages of the path to reach a certain objective, but at the same time, allow the micro or macroeconomic management to correct the correlations of the variables so that they are subordinate to the purpose pursued.

From the examples given in this article, it is necessary to take measures to maintain the macroeconomic balance, context in which, the calculated regression parameters, as well as the calculated estimators to be real and applied to the concrete data, in order to be able to forecast for future evolution trends.

In other ideas, the conclusion is drawn that the problem of price cycles for the perspective are those that give additional essence to the statistical-econometric models used in macroeconomic analyzes. Of course, the analysis shows that a single multiplier system cannot produce a cyclical fluctuation, but it must be considered as a hypothesis of probable evolution in the next period.

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