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Dynamic models used in analysis capital and population

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Abstract. *From the production function of Cobb-Douglas, we know that production is based on three essential factors. These are the population, capital and financial-material resources. In this article, the authors are concerned about the possibility of establishing a dynamic macroeconomic model in order to analyze the evolution and especially the role of capital and population in the development of the economy. When we speak of development of the economy we apply the general principle that it is a goal of the whole economic activity regardless of countries, geographical areas, development level of each country, the objectives of international bodies and so on.*
The model used in the study of capital is one that has to be established taking into account the trend up to this moment of the analysis, which according to the factors considered to ensure an increase consequently for the next period. This model must take into account the effect of capital, how it is constituted and especially the harmonized way in which capital is distributed within a national economy, based on the strategy of sustainable or complex economic development in each case.
The study of capital is a basic element that ensures an increase in correlation with the other two factors that we talked about future development conditions.
As far as the population problem is concerned, this is a defining one in terms of securing the labor force reserve, securing the reserves for the active or employed population. The population increases or decreases according to the birth rate. Natality in turn increases or decreases depending on the existing socio-economic conditions or other aspects to be considered.
In the dynamic models used we have to take into account how the population of a country, continent or world population has evolved in order to be able to predict what will happen in the next period. Of course, the change of population is made through inputs (births) and exits (deaths or emigration in the case of a study located in a country or continent).
We assume that in order to ensure a proper balance and evolution of the population, it is necessary to take a series of stimulatory measures to ensure the direction of this future evolution. Of course, population growth is determined by a series of statistical variables that taken and included in the analysis model give the perspective of identifying the future evolution trends of this very important indicator at the level of a nation.

Keywords: dynamic models, capital, production, goods, cycle, oscillations, birth, death.

JEL Classification: C52, J10, J13.

Introduction

In the case of this article, the authors have departed from the general principles for substantiating the dynamic models and their use at the macroeconomic level. Of course, by saying at the macroeconomic level, this means that dynamic models can be used in the study of capital and population to establish the evolution strategy at the country level.

The article progressively went by identifying the resultant variable that is capital and trying to suggest certain variables, measures that can lead to the increase of the resultant variable. At the same time, short series of data, easy-to-understand and interpreted examples were used to suggest how dynamic models can be used in capital analysis.

As we said before, capital must be correlated with resources and labor. We realize that if the forecasts are made randomly, it results either the insufficiency of a factor of the three mentioned ones, or an excess of one of them that by non-use bring any decreases or negative elements of evolution.

The dynamic model of capital analysis must be correlated with the dynamic analysis model of labor (population) and material and financial resources. Permanently in the models constituted and analyzed on the basis of the parameters (regressive coefficients) of evolution, we will ensure a balanced growth of the economy.

We paid attention to this factor, the capital, because even in the case of Romania, the problem arises that some distortions within the national economy are based on the fact that we do not have investment capital that ensures the increase in the number of jobs, the quality of jobs, the quality of the personnel used, the increase of wages, the increase of the tax base on the basis of the national wage fund and many others.

In the article on the dynamic models used in the analysis of capital and population, the authors started the study regarding the population starting from its genesis at some point, the pyramid of ages represented by the population at any time, from a static analysis and a dynamic analysis.

The population study is a very important one that should not be lacking in any macroeconomic forecast, because by the way the population evolves new contingents (cohorts) of population are created. It depends first and foremost on female birth. It is clear that the analysis can be directed on the basis of a model that takes into account the resultant variables population and factors including female fertility in a country, an organized structure, a continent or worldwide.

There are in the evolution of the population and other factorial variables that the authors have studied concretely and have highlighted the way in which this problem is put.

After all, the article looks at how these dynamic models are constructed and then how these dynamic models are used in population study. As factorial variables besides birth, which is the most important, we also consider others, income, health, climatic conditions and all other causes, including female psychology, couples psychology in the sense of wanting to increase or decrease the population.

From the article it is very clear that the dynamic model used in the population analysis must be constructed after a precise study has been done on the previous evolution of the resultant variable, the population, according to the determining factors. This, together with graphical representations, gave the possibility of a quicker understanding of the correlation that exists between the variables considered, the resultant one and the factorial one, so that the forecast is carried out in the future to cover and ensure a population growth in the considered terms.

This study can be used for static analysis of population structure but also for forecasting analysis in which to identify trends of population evolution in a country, on a continent or worldwide.

In this article, the authors considered some short examples that validate the considered considerations and constitute a basis for the dynamic models that can be used in the population study.

1. Literature review

Andersen, Borgan, Gill and Keiding (1993) approache the analysis of statistical models that are based on counting processes. Anghelache and Anghel (2018) approaches in their work some methods and models for analyzing the quality of life in Romania. Anghelache and Anghel (2019) addresses issues related to economic modeling. Anghelache, Anghel, M.G. et al (2018) are studying the population with indicators of residence and residence. Anghelache and Anghel (2018) approaches the econometric modeling of various economic phenomena. Anghelache, Anghelache and Bârsan (2019) analyzes models used in dynamic series analysis. Anghel, Anghelache, Avram, Burea and Marinescu (2018) addresses some aspects regarding the natural circulation of the population, the labor force and the vacancies in the economy. Barndorff-Neilsen, Hansen, Lunde and Shephard (2008) analyzes in their work the ex-post variation of equity prices in the presence of noise. Black (1972) analyzes the balance of the capital market under the conditions of restricted loans. Iacob (2019) approaches various methods and models of statistical-econometric analysis of economic phenomena. Linton (2016) is concerned in his work with probabilistic statistical-econometric analyzes. Stambaugh and Yuan (2017) raises the issue of factors not taken into account that influence the various economic phenomena.

2. Research methodology, data, results and discussions

The analysis of capital and population is important not only for the perspective of the evolution of human populations, but because it can be applied to all kinds of populations. In economics, the analysis is of particular importance in the theory of capital, because all capital goods of all types can be considered populations. This means that it must be reasonable to assume stable, or moderately, stable relationships between the age of the goods and their rates of consumption or survival. In most cases, this assumption is by no means unjustified. Thus, cars form a population, in which the age of each car can be determined with some precision and in which “life tables” or “survival tables” can be constructed and show what proportion of cars produced in any year will survive in others. years or will “die” in other years.

- **The functions of capital creation**

Commodity populations differ from biological populations, especially in the totally different nature of their birth functions. In biological populations, as we have shown, it is reasonable to assume that the number of births in any given year is a function of the numbers in the different age groups of the population.

Simple birth functions can be considered in the case of physical capital, producing patterns of population movement that throw away much of the processes of the real world. Suppose, for example, that the production of any good fulfills two functions: one to replace those items that were used during the period, and the other to add the total stock in response to increased demand. The first can be called “replacement production”, the second “expansion production”. The total production in any year will be the sum of these two quantities, the second of them, of course, will be negative in case of a diminished demand. If the demand for the good in question is constantly assumed, so that the total stock is kept at a constant level, the number of births in any year will be equal to the number of disposals in that year.

Table 1. *Cycle of goods*

Year	Age (Years) 0–1	1–2	2–3	3–4	4–5	5–6	Total	Production
0								100
1	100						100	100
2	100	100					200	100
3	100	100	100				300	100
4	100	100	100	100			400	100
5	100	100	100	100	100		500	100
6	100	100	100	100	100	0	500	200
7	200	100	100	100	100	0	600	100
8	100	200	100	100	100	0	600	100
9	100	100	200	100	100	0	600	100
10	100	100	100	200	100	0	600	100
11	100	100	100	100	200	0	600	200
12	200	100	100	100	100	0	600	100
13	100	200	100	100	100	0	600	100

Table 1 presents an interesting model of a population of goods, i.e. goods that are used up to a given lifespan (five years in the table) and then classified. In terms of analysis, the function of use is given by $k_1 = k_2 = k_3 = k_4 = k_5 = 1$, $k_6 = 0$. Suppose the production is 100 units per year for the first six years. It is obvious that until the fifth year a population of equilibrium has been reached and that, if the total population is maintained at 500 units, this equilibrium will continue endlessly, with 100 units to be used and 100 to be produced in each year. We will assume an increase in demand for this item in year 6, which leads to an increase of the total population from 500 to 600. This can only be obtained by increasing the production in that year from 100 to 200. This cohort of 200 passes gradually through the age distribution, being 0-1 years in year 7, 1-2 years in year 8 and so on, until it is finally used in year 11. Every year, between 7 and 10, 100 units are classifies and, since the population is only maintained as a number, there must be 100. In the year 11, however, 200 are worn out and 200 must be produced. every fifth year and 100 in the intervening years. Similar cycles will be created wherever there is a distortion in the age distribution, regardless of the cause.

- **Distortions follow a too rapid increase**

Table 2. *The cycle of goods with a constant growth*

Year	0–1	1–2	Age (Years) 2–3	3–4	4–5	5–6	Total	Production
6	100	100	100	100	100	0	500	120
7	120	100	100	100	100	0	520	120
8	120	120	100	100	100	0	540	120
9	120	120	120	100	100	0	560	120
10	120	120	120	120	100	0	580	120
11	120	120	120	120	120	0	600	120
12	120	120	120	120	120	0	600	120

It should be noted that the distortion results from the attempt to increase the total stock too quickly. If instead of producing 200 in year 6, every year 120 would have been produced, in year 11 a new equilibrium position would be reached which would be permanent and there would be no cycle of establishment. This is illustrated in Table 2. The number in each age group is increased by an equal amount, and the relative age distribution (the proportion of total stock in each age group) remains the same.

- **Damaged oscillations**

The conclusions are substantially modified if we relax the assumption that the commodity is a certain product that lasts a certain number of years and then disappears immediately from the spot. Just as in human life, not everyone lives within the allotted range and some go beyond it, as in the case of some goods, some are out of existence after a short time, while others last longer.

Cars, like people, have a life of use, some have worn out in the first year of their existence, others are still on the road after 20 years. It is not difficult to show that the result of this modification of the initial assumption is to give us not a perpetual cycle, but a damped oscillation, that is, a cycle in which the amplitude decreases continuously as time passes. If there were no other distortions, then the use would gradually change back into balance. Probably, there are new distortions, causing new cycles.

- **The principle of acceleration**

The effect of distortions on the age distribution of goods is accentuated by another principle that underlies the "acceleration principle". If a commodity, B, is necessary for the production of another commodity, A, then the fluctuations of production A will be reflected by the intensified fluctuations of the production of B. Let us refer again to the previous example. Suppose that to produce 10 units of this commodity we must have a machine, B.

Table 3. *Accelerator*

	Age (Years)								
Year	0–1	1–2	2–3	3–4	4–5	5–6	Total	(Idle)	Production
4	2	2	2	2	2	0	10	(0)	2
5	2	2	2	2	2	0	10	(0)	12
6	12	2	2	2	2	0	20	(0)	0
7	0	12	2	2	2	0	18	(8)	0
8	0	0	12	2	2	0	16	(6)	0
9	0	0	0	12	2	0	14	(4)	0
10	0	0	0	0	12	0	12	(2)	20
11	20	0	0	0	0	0	20	(0)	0
12	0	20	0	0	0	0	20	(10)	0

According to Table 1, when 100 units of product A are produced, a stock of 10 machines is required. However, when 200 are produced in year 6, 20 cars are required. If we assume that the machine also has a life of 5 years and that it had an age distribution in equilibrium, it is easy to see in Table 3 that there will be a very large distortion in the age distribution and a consistent recurring cycle. In year 4, everything is in balance. Then, in year 5, the number of cars must increase from 10 to 20, waiting for the increase of the production of goods A from 100 to 200 units in year 6. The production in that year must therefore be 12 cars - 2 for substitutes and 10 to extend the number from 10 to 20. Then, in year 6, the number of machines is 20, but no production is needed at all, because in year 7 only 10 machines will be needed to produce those 100 units of freight A Therefore, there will be no production of cars in years 7-10. In year 7 there will be 18 cars. In year 8 there will be 16 machines, 2 being classified, with 6 inactive, and so on until year 10, when 20 cars have to be manufactured, as the 12 products in year 6 disappear, the stock being zero. In year 11, 20 machines must produce the 200 units of freight A. Here again, a perpetual cycle is configured with 20 machines produced every five years, not produced at all in the intermediate years and on average 4 cars.

If we assume that the production of the goods A is distributed as in Table 2, so that it does not result in the distortion of its age distribution, however, there will be a distortion in the age distribution of the machine, B, as shown in Table 4.

Table 4. *Second accelerator*

Year	Age (Years) 0–1	1–2	2–3	3–4	4–5	Total Stock	Pro-duction
4	2	2	2	2	2	10	2
5	2	2	2	2	2	10	4
6	4	2	2	2	2	12	2
7	2	4	2	2	2	12	2
8	2	2	4	2	2	12	2
9	2	2	2	4	2	12	2
10	2	2	2	2	4	12	4
11	4	2	2	2	2	12	2
12	2	4	2	2	2	12	2

Now a stock of 12 machines is required each year to produce the 120 units of freight A. In year 5, therefore, the production of cars will increase from 2 to 4 units, and this will be repeated every five years. In this case, the fluctuation is not as intense as in Table 3. However, if we now assume that a machine tool, C, is required to produce machine B, the fluctuations at the output of the machine tool will be much greater than fluctuations at the output of the machine.

The smoother life tables of reality work to cushion and smooth these oscillations. It is true, even in the general case, that machine tool output fluctuations are greater than machine output fluctuations, and machine output fluctuations are greater than the production fluctuations of the goods they perform. This principle is called the acceleration principle, because the demand for cars depends not only on the demand for their product, but also on the rate of change in demand for the product. It is clear that the problem of adjusting the growth rates of stocks of various types of commodities, is necessary in order to avoid changing the age distributions. In this sense, a certain fluctuation of economic activity is a necessary cost of economic progress, which will be greater as we try to move faster.

- **Age distribution distortions**

In the case of both human populations and goods populations, distorting the age distribution for any cause can take a long time. The sudden drop in infant mortality in many tropical countries around the 1950s creates a youth problem in the 1960s; even in the United States, rising births in the 1940s creates a problem of unemployment among young people in the 1960s, as an unusual number of young people reach a labor market that is adjusted to a smaller number of participants. The fact that virtually no cars were produced in the United States in the years 1942-45 due to the war created difficulties in Detroit in 1958-60, when there were no cars in the age groups with the highest rate. disposal.

Distribution distortions over time sometimes produce what is called the “shadow effect” in the second generation; thus, high birth rates in the 1940s in the United States produced high rates of marriage and high birth rates in the 1960s-70s, when children in their 40s were fertile. At present, the United States has balanced in terms of birth, but also of production, as well as of international trade.

In Europe, the evolution of production as well as population growth raises a number of problems. The European Union aims at increasing production and population. In the last decade of the 21st century, the fertility rate decreased. Europe is facing a high wave of immigration that seems to be changing the population structure on the old continent.

Models of population analysis and production of goods are topical despite the distribution distortions.

- **The Austrian theory of capital**

The analysis of capital as a population of goods has importance in interpreting the Austrian theory of capital, mainly associated with the name Bohm-Bawerk. According to this conception, capital is created when the “original factors of production” (labor and land) are incorporated into goods and is destroyed when the services of these factors are finally realized. Therefore, capital is considered as a value population, where values are created when the factors of production are employed and disappear when the services are provided. The average production period is then the age of disappearance of the values, that is, the average time that elapses between the employment of a production factor and the final consumption as a utility. Thus, the bread I eat today represents the services of delivery people this morning, yesterday's bakers, last month's millers, farmers last year and so on. The average age of the services incorporated in bread is the average period of production.

In a population with a stable equilibrium, the average age at the exit is equal to the ratio of the total population to the annual number of inputs or outputs. Thus, if 100 units appear and disappear from a population every year, and the average age at death is 30 years, the total population will be 3000. In the steady state, therefore, the ratio between total capital (total population of value) at the annual income (the income of the factors or the consumption of the product) is equal to the average period of production (the average life of the considered values).

If the average production period is extended, this must mean either an increase of the total capital, if the income is constant, or a decrease of the income, if the capital is constant. It has been tried to interpret the depressions with regard to a decrease of the income required by an over-expansion of the production period with an inadequate capital stock. The possibility of such difficulties cannot be ruled out. However, the normal depressions of the business cycle seem to be quite different. The basic difficulty regarding the analysis of the production period is that it is valid only in the steady state and is an analytical device too crude to cope with the complex movements of the dynamic systems.

- **Dynamic models and risk factor**

The models of differential equations are useful to draw attention to certain possible patterns in the dynamic systems of the economy. However, we should be careful not to place too much importance on them, as predictors, because the parameters of social systems can change in quite unpredictable ways. Thus, many years ago, many predictions were made about the future population that proved to be inaccurate, mainly due to a rather unexpected increase in fertility. In addition, purely mechanical dynamic systems, which are very successful in describing and predicting the solar system, where second- or third-degree differential equations are necessary, but inadequate to describe the complex processes of social systems, where current behavior may depend of the memories and recordings that go back many years in the past.

Human history is a dynamic system of almost infinite degree and, for this reason, it has a great element of unpredictability. However, it is not entirely unpredictable, and the analysis can at least give us some conditional predictions of the form. In addition, the overall dynamic process of the company can usually be interpreted in terms of stress adjustments. Here we see that mechanical dynamics does not shift the analysis of equilibrium to the extent that precipitation can be interpreted as a divergence between the current state of a system and its equilibrium state. Until a moment, the precipitation cannot produce any change, the resistance to a change is too high for the precipitation to be exceeded. At one point, precipitation exceeds resistance and change occurs. Such systems are sometimes called resistive systems, because the voltage must reach a certain stage before any change occurs. After it has been achieved a certain adjustment of the system must be made, and the principle here is that what is adjusted is adjustable, that is, those parts of the system that have the least influence will adjust the first.

For example, in a competitive market, prices are adjustable and any concern in the form of a deficit or surplus will change prices. In monopoly or organized markets, prices are not easy to adjust and can produce adjustments of preferences or from a technological point of view. A deflation of income will lead to a fall in prices and wages if they are adjustable and a decrease in output and employment if prices and wages are not adjustable. Pressure for change sometimes results in increased resistance to change, and sometimes low resistance. Given these complexities and uncertainties, it is clear that mechanical dynamic models should be treated as solutions rather than predictors.

A very important area of dynamic analysis covers the theory of movements of a population. A population can be defined as any aggregate that can be measured in a common unit and in which the age of each unit can be identified or at least defined. The age of a unit is the length of time that has elapsed between a certain "current" date and the "birth" date or entry of the unit into the population. In all ordinary populations, it is generally defined that each unit has a finite age of "death" or exit from the population. Birth and death in this sense does not necessarily mean the creation or destruction of the unit in question, but refers to the dates on which the unit begins to conform or ceases to conform to the definition of the aggregate that constitutes the population. Thus, if our aggregate is the human population from a finite area, the entry of an immigrant is a "birth" and the exit of an emigrant a "death" in that population, as much as the physiological births and deaths, with one important difference: births naturally result in individuals of zero age, while immigrants can add members of any human age to the population.

The dynamic analysis of population movements is based on the postulate of certain stable equations of difference. We can determine a stable "survival function", which shows how many units survive to each age from a certain number of births. Thus, out of 100 births, 90 can survive for up to one year, 85 for two years, 81 for three years and so on until the last survivors. The survival function is thus constructed from a series of differential form equations:

$$a_1 = k_1 a_0, a_2 = k_2 a_0, \dots, a_n = k_n a_0 \quad (1)$$

where a_n is the number that survives until the year n from a total number of births, B_0, in year 0. In the simplest form of the function, the survival coefficients, k_1, k_2, ..., k_n they are constant. The number that dies or leaves the population, outside the group that is r in year t, between the year t and $t+1$ it is at $a_r - a_{r+1}$ or $k_r a_0 - kr_{+1} a_0$ or $(k_r - k_{r+1} / k_r)\, a_r$.

- **Dynamics of a stable population**

If we consider a "birth function" regarding the number of births in a given year with the number or structure of the population, the whole evolution of the population can be sketched. The process can be illustrated by mathematical examples.

We will consider the situation presented in Table 5. Each row in this table represents the numbers in a population divided into five age groups. We consider that in year 1 there are 100 units between 0 and 1 year, 96 between 1 and 2.66 between 2 and 3.56 between 3 and 4, 30 between 4 and 5, and none older than 5 years. The arrows marked by the arrows show the survival distributions of each cohort or group in the population that has the same year of birth. Thus, suppose that out of 100 people aged 0 to 1 at the end of the year, 1.80 survive to the end of the year 2, 60 to the end of the year 3, 40 to the end of 4, 20 to the end of year 5, and none until the end of year 6. If the "net births", B, are defined every year between 0 and 1 year at the end of the year, then suppose that the survival function in the above table is given by $k_1 = 1$, $k_2 = 0.8$, $k_3 = 0.6$, $k_4 = 0.4$, $k_5 = 0.2$, $k_6 = 0$. The figures in parentheses at the top of the table show the net births from which age groups from year 1 are descending, if the survival function is assumed to be constant. Thus, if in year 1 there are 96 units between 1 and 2 years, they must be the survivors of a cohort of 120 births in the previous year: 96 = 120 (0.8). In year 2, then, the numbers between 2 and 3 years must be 72 [= 120 (0.6)], and in year 3, the numbers between 3 and 4 must be 48 [= 120 (0.4)] and so on.

Table 5. *Stable population*

	Age (Years)						
Year	0–1	1–2 (120)	2–3 (110)	3–4 (140)	4–5 (150)	5+	Total
1	100	96	66	56	30	0	348
2	100	80	72	44	28	0	324
3	100	80	60	48	22	0	310
4	100	80	60	40	24	0	304
5	100	80	60	40	20	0	300
6	100	80	60	40	20	0	300

The same process can also be described using a death function. Thus, if a number (0.6) 8 ages 2-3 becomes (0, a) B by the end of the following year, (0.2) B died during that year – that is, one third of the number 2 – 2 years 3. Thus, of the 66 2-3 years in year 1, 22 will die in the following year, leaving 44 survivors aged 3-4 years in year 2. In Table 6 we assumed that the number of net births after the year 1 is constant at 100 per year. In this hypothesis it is evident that the population is soon established at a balance value of 300, in which the same age group is repeated year by year. A population of equilibrium can be defined as one in which the number of births and deaths per year is equal and constant and in which the numbers in each age group are also constant.

- **Population dynamics: population growth**

Suppose now that the number of births each year is not constant, but is a function of the numbers of different age groups. As long as this function is known and stable, it is still possible to track the course of any population over time, so suppose (Table 5) that after year 6, the number of births in each year was equal to the number total of units in the age groups 2-3 and 3-4 years. In year 7, the number of births would still be 100 [= 60 + 40], and the population will be in balance. So the number of births was equal to the number of units in the age groups 1-2 and 2-3. The population would follow the course in Table 6.

In this table, the first figure in each row (after the first) is equal to the sum of the age groups 1-2 and 2-3 of the previous year. Thus 140 = 80+ 60, 172 = 112 + 60 and so on.

Table 6. *Population expansion*

Year	Age (Years) 0–1	1–2	2–3	3–4	4–5	5+	Total
6	100	80	60	40	20	0	300
7	140	80	60	40	20	0	340
8	140	112	60	40	20	0	372
9	172	112	84	40	20	0	428
10	196	138	84	56	20	0	494
11	222	157	103	56	28	0	566

It is assumed that all those born during one year are between 0 and 1 in the following year. The survival function is assumed to be the same as in Table 6. It is obvious that, based on these assumptions, the population will grow endlessly. Each birth increase offers a larger age group as the basis for births and more.

- **Decreasing population**

Suppose that the number of births in each year is equal to the numbers in the 3-4 and 4-5 age groups. The population will follow the course in Table 7. In this case, each birth cohort produces less than its own number to replace it, and the population will decrease continuously.

Table 7. *Decreasing population*

Year	Age (Years)						Total
	0–1	1–2	2–3	3–4	4–5	5+	
6	100	80	60	40	20	0	300
7	60	80	60	40	20	0	260
8	60	48	60	40	20	0	228
9	60	48	36	40	20	0	204
10	60	48	36	24	20	0	188
11	44	48	36	24	12	0	164
12	36	35	36	24	12	0	143

- **Algebraic solution**

It is not difficult to formulate the conditions in which the populations will expand, contract or stay under stable functions of birth or survival. May it be a_1, a_2,..., a_n, be the numbers in the nine age groups of a population. Then the birth function can be described by a series of constants, b_1, b_2,..., b_n, the relationship results:

$$B = b_1 a_1 + b_2 a_2 + \cdots + b_n a_n \qquad (2)$$

This is the simplest useful form of the function. Some of the variables may be zero, and in a biological population they will be the largest in the most fertile age groups. The complication introduced by the necessity of the cooperation of two sexes is neglected here. In these conditions we define a survival function, as a set of constants, k_1, k_2,..., k_n. Therefore, we have the relationship:

$$a_1 = k_1 B_{t-1}, a_2 = k_2 B_{t-2}, \dots a_n = k_n B_{t-n} \qquad (3)$$

It follows, combining equations (2) and (3), that

$$B_t = b_1 k_1 B_{t-1} + b_2 k_2 B_{t-2} + \cdots + b_n k_n B_{t-n} \qquad (4)$$

If the population is stable, it results:

$$R = b_1 k_1 + b_2 k_2 + \cdots + b_n k_n = 1 \qquad (5)$$

- **The net reproduction report**

The quantity, R, is a measure of the net reproduction ratio. In a balanced population, the survival distribution of each birth group is the same as the age distribution in any year. We see this in Table 5, where it is clear that the distribution of survival, once the population has a balance (the series on the marked diagonals), is the same as the age distribution: 100, 80, 60, 40, 20, 0. In a population of balance, R measures the number of net births at which each cohort of the population is born. If R = 1, the population reproduces only: each generation as it dies leaves a new generation of equal size. If the net reproduction ratio is higher, the population will grow continuously, as in Table 6, for each generation, as it dies, leaving behind a cohort of larger dimensions than itself. Similarly, if R < 1, the population will eventually decrease, for each generation it leaves behind a smaller cohort than itself.

A dynamic equilibrium of the population at a constant growth rate is possible only under predetermined conditions.

The basic difference equations behind this table are derived from the survival distribution coefficients, 1, 0.8 and 0.4, and the birth function coefficients, $B_t = 3a_2 + 8a_3$. Thus, in year 1 the number of births is 40 x 3 + 10 x 8 or 200; of these 200, 200 survive to be the first age group in year 2, 160 survive to be the second age group in year 3, and 80 survive to be the third age group in year 4.

Table 8. *Balance with a constant growth rate*

Year	Age (Years) 0–1	1–2	2–3	Total
1	100	40	10	150
2	200	80	20	300
3	400	160	40	600
4	800	320	80	1200

It is observed in Table 8 that the population grows at a constant rate of 2 per year and that the number of births and the number of each age group increase at the same rate.

Conclusions

Some theoretical and practical conclusions are drawn from the presentation and the data used. Of these, the first would be that when making the macroeconomic forecasts, the study of the evolution of the capital up to a moment and of the prospect of its growth is essential. Therefore, dynamic models are those that show the tendency of the evolution of capital in a country and based on it the measures to be taken for the next period according to the strategy used, so that the growth will take place progressively.

On the other hand, it is clear that models of dynamic analysis of all three capital factors, labor resources and financial-material resources must be used in an economic forecast, so that the established parameters allow the possibility of harmonizing the three factors so that the effect is maximum.

Also, the article highlights the fact that when building a dynamic capital analysis cell, he has to do a very careful study of the principles underlying the growth of capital within a national economy and in order to identify statistical variables, in other words, extending the term slightly, the measures that must be taken for this social capital with national capital to evolve at a correlated rate to give the effect that it has to have on the macroeconomic evolution.

At the same time, it is necessary that when constructing a model used in the dynamic analysis of the population, one must carefully study the dynamic evolution until the static moment of the moment when we propose a forecast of population growth or change in the future.

The variables considered must be carefully analyzed in order to identify the evolutionary trend until the moment of the analysis and in this way by calculating the parameters of the coefficients and estimators it can be made an extension, a real forecast on the evolution of the population from the geographical area under analysis.

The final conclusion is that in any macroeconomic forecast at the state level, at the continental level or on a wider, global level, the factors (the factor variables) that can influence the population change must be carefully considered. These dynamic models can be successfully analyzed in the analysis of the forecast of the future evolution of the population in the considered geographical area.

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