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The main theoretical aspects regarding the capital adequacy models

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Abstract. *The credit institutions shall apply the measurement approach to a lesser extent to determine capital adequacy. The Basel Accords pay attention to this very aspect of capital sizing in order to be able to finance economic activities, but especially to reduce the effect of the risks caused by capital inadequacy. This study can be performed using statistical-econometric models, based on which to estimate the establishment of the necessary capital. If capital is not adequate (brought to the size of market demand), a number of risks arise that disrupt credit-based financing. Although many credit institutions have chosen the simplest method for determining the capital requirement for operational risk, efforts must be made to use the "standard approach", thus ensuring the premises for the transition to the "advanced approach", considered the effective form of operational risk monitoring.*

Keywords: credit institutions, capital, market, risk, standard approach, advanced approach, statistical-econometric models and methods.

JEL Classification: C10, G10.

Introduction

In this article, the authors considered that, starting from the proposed objectives, a number of credit institutions should be tested in connection with the concrete ways of identifying and measuring operational risk. For this purpose, a statistical research was carried out, practically, on a sample of subjects, respectively financial-banking intermediaries.

In the study, a database was created for a credit institution chosen as a reference. Thus, the database was built for a representative banking entity, considering all the events of the nature of the operational risk occurred during a period of time. These events were grouped by activity lines and event types. This group was finally tested for robustness by breaking down the composite group of low-observation groups into subgroups.

The study underlying this article, although predominantly theoretical, the models considered can be used without great difficulty. The financial activity includes the operational risk that takes into account the size of the capital of the considered entity. The results of some researchers in this field were presented, proposing certain models of capital adequacy.

Literature review

Anghel, Anghelache and Dumitrescu (2016) conducted an analysis of the main financial ways to support innovative SMEs. Anghelache, Anghel and Iacob (2020) highlighted the statistical-econometric instruments used in economic analyzes. Anghelache, Sfetcu and Bodo (2017), precm and Currie (2004) presented the evolution of the regulatory framework provided by the standards developed by the Basel Committee. Anghelache, Anghel, Diaconu and Lilea (2017) studied the influence of operational risk on the profitability of the bank or financial-banking agent. Belhaj (2010) addressed a number of issues regarding capital requirements for operational risk. Chaudhury (2010) identified the main problems in modeling operational risk capital. Cope (2012) turned his research to methods of quantifying operational risks. Cummins and Phillips (2009) investigated issues related to capital adequacy. Dickinson and Sommers (2008) referred to the importance of competitive advantage over the future profitability of the firm. Kohn (2006) studied the role of the central bank in financial crises. Muermann and Oktem (2002), Peters et al. (2009), but also Scott and Jackson (2002) studied fundamental elements of operational risk, focusing on the combination of different sources of information and treatment under the new Basel agreement.

Methodology, data, discussions, results

The Basel Accord, analyzing issues related to the capital of the economic entity, proposes some methodologies for measuring the own funds requirements for operational risk.

- The Basic Indicator Approach is the simplest of the methods for calculating the capital requirement. Only one risk indicator is used at the level of the credit institution (gross operating income). The credit institutions using this approach must have their own funds to cover operational risk, corresponding to a fixed percentage of the average operating income recorded in the last three years. This share, expressed as a capital adequacy ratio (denoted α), was set by the Basel Committee at 15%. The second component of the capital requirement is the exposure indicator, represented by the average of the last three years only of positive annual gross income.

$$K_{or\_BIA} = \alpha \cdot \frac{\sum_{i=1}^{3} \max(VO_i;0)}{3-n} \quad (1)$$

where:

K_{or_BIA} – capital requirement for operational risk according to the basic indicator approach;

VO_i – gross operating income at the level of the credit institution, for each of the three years;

α – coefficient set by the Basel Accord at 15%;

n – number of years (out of the last three) for which the institution recorded negative gross operating income.

This option can be easily implemented by credit institutions without the need to meet special conditions, especially recommended for small institutions that have a relatively simple portfolio of activities. The basic indicator approach is a method of calculating the capital requirement for operational risk, not a way of measuring operational risk.

- The Standardized Approach is a redefinition of the basic approach – the same method is applied for determining capital requirements – and gross operating income is detailed on eight lines of activity (corporate finance, trading and sales, payments and settlements, commercial banking, agent services, retail banking, retail brokerage, asset management). Gross income for each line of business is considered to be an indicator of operational risk exposure.

The own funds requirement is determined separately for each category, by applying a specific coefficient on gross income. The coefficients between 12% and 18% approximate the intensity of the relationship between the volume of activity at the level of an operational line and the losses generated by the manifestation of the risk. Thus, the manifestation of the operational risk generated by the first three lines (corporate finance, trading and sales, payments and settlements) is considered to produce the highest losses on the credit institution, for which they were assigned a coefficient of 18%.

The next two categories (commercial banking, agent services) received a coefficient of 15%, and the last three categories mentioned (retail banking, retail brokerage, asset management), for which the related operational risk is significantly lower, li a coefficient of 12% was assigned.

In this approach, the bank capital for the minimum required operational risk is given by the relation:

$$K_{or\_SA} = \frac{\sum_{j=1}^{3} \max(\sum_{l=1}^{8} \beta_l \cdot VO_{j,l};0)}{3} \quad (2)$$

where:

K_{or_SA} – the capital requirement for operational risk according to the standard approach;

$VO_{j,l}$ – gross operating income for each of the three years, corresponding to each of the eight basic banking activities;

β_l – capital adequacy ratio set by the Basel II Accord for each of the eight types of banking activities.

- The third method is the advanced approach and allows credit institutions to develop their own capital requirement calculation model based on their internal estimates. In order to use this approach, credit institutions must have the approval of the supervisory institution and meet a number of conditions to demonstrate that their methodology is sophisticated enough to capture severe events, which can occur with a probability of 0.99%.

The level of capitalization is determined by credit institutions in Romania in accordance with the requirements of the Basel Accord. The agreement leaves to the credit institutions the way to determine the required capital. In Romania, the method of determining and reporting the capital requirement for operational risk at credit institutions was as follows: 27 credit institutions apply the basic approach (AIB), 4 credit institutions apply the standard approach (SA) and 2 credit institutions apply approach to advanced measurements (WADA).

The capital requirement calculated according to the Basic Approach to cover operational risk has the disadvantage of appearing to underestimate the specific characteristics and requirements of the credit institution and its actual risk profile.

The standard approach better reflects the differences in the risk profile of credit institutions and represents an intermediate step towards the approach of advanced measurements. The advanced method leads to a different capital requirement, which leads to a capital saving compared to the basic and standard indicator method.

Under these conditions, the inclusion of moral hazard in determining the level of capitalization of the credit institution is an important and necessary step. The motivation for including the moral hazard in the evaluation is based on the fact that the optimization of the capital requirements of financial institutions can generate a lower level than the current one, which can have the effect of engaging credit institutions in risky activities, due to loss of motivation to keep its own funds unaltered.

- Unlike credit risk and market risk, where approaches based on internal models refer only to unpredictable losses, for operational risk institutions must include both unexpected and projected losses. Information on losses caused by operational risk must be structured by lines of activity and by categories of operational risk events. The most used advanced approaches are the scorecard approach and loss distribution modeling.

Scorecard approaches are tools by which credit institutions identify vulnerable elements that may have the effect of producing operational risk. The indicators used by this type of approach can be both financial and non-financial indicators.

A representative financial indicator under this approach is the cost / income ratio, which measures the costs borne by the credit institution for each monetary unit of income. The reduction of this indicator is a favorable sign for the credit institution, it shows an increase in efficiency, but the credit institution must take into account a threshold from which this link between costs and revenues can only be sustained by assuming operational risks. high. For example, in order to achieve a low cost / income ratio, credit institutions may reduce costs for audit and control, for monitoring systems or for the development of the IT system, all of which have a significant adverse impact on operational risk.

Non-financial indicators help credit institutions to develop their risk management system as they go through operational risk calculation methodologies. One such indicator is the ratio of back-office and front-office staff. A low value of this report shows an increase in the probability of errors in the processing of back-office transactions. On the other hand, a high level of this ratio, correlated with the lack of a clear distribution of responsibilities, favors the occurrence of operational risk.

Another non-financial indicator is represented by the expenses with the training of each employee. Credit institutions must ensure good training for all their employees, who must know the activity carried out within the credit institution as well as the procedures and risks related to the activity carried out. An increase in staff training costs is not enough to reduce the risk caused by staff inexperience. Credit institutions must establish the initial level of knowledge of employees and organize continuous training courses. Other key risk indicators that need to be constantly monitored by financial institutions are: staff turnover, the volume of transactions, the number of employees who do not have ten consecutive days off, the number of complaints received from customers.

Tracking these indicators helps credit institutions analyze their internal processes and better manage risk, which will lead to a reduction in operational risk losses and even a reduction in the minimum capital requirement to cover it.

The approach to modeling the distribution of losses involves estimating the probability for each line of activity and for each type of event, the losses and the frequency of occurrence. Using these two probabilities one can calculate the aggregate probability of operational risk losses. Estimates are made on the basis of historical data on losses caused by operational risk for a period of at least one year.

The probability distribution shapes the occurrence of losses caused by the operational risk within the credit institution. Being a discrete distribution, for short periods of time, it is modeled as a Poisson or binomial distribution.

The distribution of losses is difficult to model as a classical one and therefore it is recommended to divide it into an ordinary distribution, which models small losses with high frequency, and into an extreme distribution that takes into account high impact and low frequency losses. The ordinary distribution will include losses starting with the minimum limit imposed by the credit institution up to the threshold for which the losses are considered exceptional, and the extreme distribution will include losses exceeding this threshold. The distribution of ordinary losses can be modeled as a positive and continuous distribution such as Exponential distributions.

The capital requirement calculated using this method is similar to the value at risk method. The capital requirements for each line of activity are calculated, for each type of event and then the total requirement for the respective institution.

For a line of business and a type of operational risk event, expected loss (EL) and unexpected loss (UL) are defined by the relation:

$$EL(i,j) = E[\nu(i,j)] \int_0^\infty x dG_{i,j}(x) \quad (3)$$

$$UL(i,j;\alpha) = G_{i,j}^{-1}(\alpha) - E[\nu(i,j)] = inf\{x/G_{i,j}(x) \geq \alpha\} - \int_0^\infty x dG_{i,j}(x) \quad (4)$$

where:
i – a line of activity and j a type of event;
ν(i,j) – a random variable that represents the loss of an operational risk event for activity line i and event type j;
α – the confidence level;
$G_{i,j}$ – the composite distribution of losses and frequencies.

We consider that the number of events between t and t+τ is random; the corresponding variable N (i, j) has a probability function $P_{i,j}$. The frequency distribution of losses is:

$$P(i,j) = \sum_{k=0}^{n} p_{i,j}(k) \quad (5)$$

The loss suffered by the credit institution for the line of activity i and the type of event j between t and t+ τ is:

$$\nu(i,j) = \sum_{n=0}^{N(i,j)} \varsigma_n(i,j) \quad (6)$$

The expected loss corresponds to the expected value of the random variable ν(i,j), and the unexpected loss is given by the percentile of α minus the mean.

Although the Basel Committee proposes that the capital requirement be calculated only on the basis of unexpected losses, credit institutions also include expected losses for operational risk calculations.

$$CaR(i,j;\alpha) = EL(i,j) + UL(i,j;\alpha) = G_{i,j}^{-1}(\alpha) \quad (7)$$

The expected loss can be calculated using the following relation:

$$E[\nu(i,j)] = E\{E[\nu(i,j)/N(i,j)]\} = E[N(i,j)] \cdot E[\varsigma(i,j)] \quad (8)$$

Determining the unexpected loss with a high level of accuracy is more difficult to achieve. Errors that occur in the aggregation process of the two distributions can lead to a result that is far from what happens in reality, especially when the distribution has thick queues. That is why it is very important that the distributions are modeled as accurately as possible.

The main problem facing financial institutions in adopting this approach is the lack of a complete and coherent database. The data used should be collected for a period of at least one year, but would preferably be between 3 and 5 years. The data must include all activities and exposures in all geographical subdivisions and locations. In addition to the gross amount of the loss, information must also be collected on the date of the event, recoveries from the loss and a description of the event. However, the data may be incomplete for certain lines of activity or types of events. Also, only losses that exceed a certain set threshold are recorded, with credit institutions having to find a balance between the cost of recording small losses and the accuracy of losses in the event of too high a limit. The approach must also take into account external databases. Their harmonization with the credit institution's internal data is difficult to achieve given that, in general, these are events with large losses and different circumstances.

For a more accurate analysis of operational risk events, the two databases, the internal and the external, credit institutions may include in their analysis the opinions of experts in risk analysis. They analyze events that have occurred and have resulted in significant losses, identify strengths and weaknesses, all in combination with existing loss data.

Once the data problem is solved, the next step is to choose the distribution that best describes them. Being positive values, the distribution of losses can be modeled as an exponential distribution, gamma, hi-square, pareto, logistic or weibull. In order to be able to determine in which type of distribution the distribution of losses can be included, statistical tests are used that compare the empirical distribution and the reference one. The most used tests are Kolmogorov-Smirnov, Anderson-Darling, Cramer-von Mises and Watson.

The Kolmogorov-Smirnov test compares the observed distribution function for a variable with a specified theoretical distribution and calculates the largest difference, in absolute value, between the empirical and theoretical cumulative distribution functions, thus verifying whether the observations come from the specified distributions.

The Anderson-Darling test is an adaptation of the Kolmogorov-Smirnov test and pays more attention to the distribution tails. Unlike the Kolmogorov-Smirnov test, the critical values differ for each type of distribution tested.

Another way of choosing the distribution that best models the distribution of losses is the graphical representation of the quantiles of the empirical and theoretical distribution, in case the quantiles of the theoretical distribution are close to those of the actual distribution we can say that the two distributions match.

The analysis of these graphs must be correlated with the results of the Kolmogorv-Smirnov and Anderson-Darling tests because they provide a better picture of the tails of the empirical distributions. In order to be able to test to what extent the actual distribution matches the theoretical distributions, it is necessary to calculate the parameters of each distribution.

The generalized exponential distribution requires the estimation of two parameters μ (position parameter) and λ (scale parameter), both positive, and has the following probability density function:

$$f(x;\mu,\hat{\lambda}) = \begin{cases} \lambda e^{-\lambda(x-\mu)}, x \geq 0 \\ 0, x < 0 \end{cases} \quad (9)$$

In the case of the standard exponential distribution (μ = 0) to determine $\hat{\lambda}$ (estimated value of λ) the probability function for the parameter λ is calculated on a sample $x = (x_1, \dots, x_n)$ of the variable. This function has the following form:

$$L(\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i} = \lambda^n e^{-\lambda n \overline{x}}, unde \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (10)$$

The derivative of the logarithm of the likelihood function is:

$$\frac{d}{dy}\ln L(\lambda) = \frac{d}{dy}(n\ln\lambda - \lambda n\overline{x}) = \frac{n}{\lambda} - n\overline{x}\begin{cases} > 0, 0 < \lambda < \frac{1}{\overline{x}} \\ = 0, \lambda = \frac{1}{\overline{x}} \\ < 0, \lambda > \frac{1}{\overline{x}} \end{cases} => \hat{\lambda} = \frac{1}{\overline{x}} \tag{11}$$

The Gamma distribution is a distribution of the same family of exponential distributions, having a density function expressed by three parameters: a shape parameter, k, a scale parameter, Θ and a position parameter m. The parameter k and Θ have values positive.

The probability density function for a gamma distribution is:

$$f(x;k;\Theta) = \frac{\left(\frac{x-m}{\Theta}\right)^{k-1} e^{-\frac{x-m}{\Theta}}}{\Theta\Gamma(k)}\ , for\ x > 0 \tag{12}$$

Γ is a gamma function with the following formula:

$$\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt \tag{13}$$

The estimated parameters for this distribution are:

$k = (\frac{\overline{x}}{s})^2$ and $\Theta = \frac{s^2}{\overline{x}}$, where $\overline{x}$ and s represent the mean and the standard deviation of the analyzed distribution.

The Pareto distribution is part of the family of exponential distributions and is expressed using two parameters. The density function is:

$$f(x|a,k) = \frac{ak^a}{x^{a+1}}\ , k \leq x < \infty\ ; a,k > 0 \tag{14}$$

The parameter k shows the smallest value that the random variable can reach. To determine k you can use the likelihood function using the relation:

$$L(k,a|x) = \prod_{i=1}^{n} \frac{ak^a}{x_i^{a+1}}; 0 < k < min\{x_i\}, a > 0$$

$$=> \hat{k} = min\{x_i\}\ and\ \hat{a} = \frac{n}{\sum_{i=1}^{n} \log\frac{x_i}{\hat{k}}} \tag{15}$$

The Weibull distribution is a distribution used to model phenomena that can present extreme values. The parameters of this distribution are positive, respectively: a is the shape parameter; s the scale parameter; and the probability density function is as follows:

$$f(x|s,a) = \begin{cases} \frac{a}{s}\left(\frac{x-m}{s}\right)^{a-1} e^{-\left(\frac{x-m}{s}\right)^a}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{16}$$

For a = 1 the Weibull distribution is even the exponential distribution. The probability function (for m = 0) is:

$$L(x_1, \dots, x_n | a, s) = \prod_{i=1}^{n} \left(\frac{a}{s}\right) \left(\frac{x_i}{s}\right)^{a-1} e^{-\left(\frac{x_i}{s}\right)^a}$$

$$\begin{cases} \frac{d \ln L}{da} = \frac{n}{a} + \sum_{i=1}^{n} \ln x_i - \frac{1}{s} \sum_{i=1}^{n} x_i^a \ln x_i = 0 \\ \frac{d \ln L}{da} = -\frac{n}{s} + \frac{1}{s^2} \sum_{i=1}^{n} x_i^a = 0 \end{cases} =>$$

$$=> \frac{\sum_{i=1}^{n} x_i^a \ln x_i}{\sum_{i=1}^{n} x_i^a} - \frac{1}{a} - \frac{1}{n} \sum_{i=1}^{n} \ln x_i = 0 => s = \frac{\sum_{i=1}^{n} x_i^a}{n} \quad (17)$$

The normal distribution is the simplest and most used distribution, its shape depending on two parameters: μ and σ, the first representing the average and the second the variant (the size of the distribution distribution). The probability density function is as follows:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (18)$$

Estimating the distribution parameters using the maximum likelihood function leads to the optimal values for |a and a:

$$\ln L(\mu, \sigma^2) = \sum_{i=1}^{n} \ln f(x_i|\mu, \sigma^2) = -\frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \quad (19)$$

Deriving the maximum likelihood function depending on the distribution parameters we obtain:

$$\begin{cases} \frac{d \ln L(\mu,\sigma^2)}{d\mu} = 0 \\ \frac{d \ln L(\mu,\sigma^2)}{d\sigma^2} = 0 \end{cases} => \begin{cases} \hat{\mu} = \overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \\ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^2 \end{cases} \quad (20)$$

The Poisson distribution used to model the frequency of occurrence of the studied phenomena has the following form of the probability density function:

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (21)$$

where k is the number of events for which the probability density is to be calculated, λ is the average number of occurrences of the phenomenon in the time interval for which the calculation is made.

Conclusions

From the analysis undertaken in the article “The main theoretical aspects regarding the capital adequacy models” a series of theoretical and practical conclusions can be drawn. Thus, a first conclusion is that in order to avoid systemic risk, the authorities should consider limiting the moral hazard by strengthening prudential supervision and should manifest its status as a lender of last resort only in exceptional cases, when the financial system is deeply affected.

Monetary authorities need to manifest this quality only by analyzing the relationship between the advantages of avoiding systemic risk and the disadvantage of increasing moral hazard, which in turn may be reflected in systemic risk. Not only must the authorities act in order to limit the moral hazard, but each credit institution must also improve its supervisory techniques.

Future research must also take into account the aspect according to which a reduced equity determined as a result of applying the capital adequacy method to operational risk can generate moral hazard, the credit institution not being tempted to improve its internal control processes. Capital adequacy models can be easily adapted and used for the intended purpose.

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