Welfare enhancing uncertainty

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Abstract. In this paper it is shown that one of the intrinsic characteristics of the canonical Melitz (2003) type framework is that inclusion of further uncertainties enhances the aggregate welfare of the given economy, though reducing the total varieties available. Here the uncertainty assumed is, payment risk associated with the exchange in international market. The key to such an occurrence is the fact that uncertainty acts like an increased trade costs leading to added exit of low productive firms. Since uncertainty seems to be desirable any efforts to decrease such can be deemed expensive. However, this also proves for a fact that the developing countries with overall lower productivity and higher uncertainty will lose out its import and export share to international competition in the global market.

Keywords: firm heterogeneity, payment uncertainty, welfare.

JEL Classification: F10, F11, F12.
1. Introduction

International trade relations are by and large unpredictable because of constantly fluctuating exchange rates, governmental policies, interests, prices of factor inputs, nature etc. at the macro level, disturbing the decisions of the importers and the exporters at the micro level. Literature that addresses this – both at empirical and theoretical levels – are limited, but expanding. Export uncertainty is directly as well as indirectly implied in some empirical research. For example, Besedes and Prusa (2006) using US import level data observe that median duration of survival for an importing firm is very small, however if it survives in the short run then its probability of continuing exporting for a long period to US increases. Iacovone and Javorcik (2010) using Mexican export data after NAFTA, found that not only a few varieties are able to survive the international market for a long period of time, but the firms begin by selling only a very small amount to the importers. The low survival rate of trade relationships does indicate the presence of international market risk. Furthermore, using trade data of Dutch firms between 2002 and 2008 Cerusen and Lejour (2011) show that not only, just 5% of all Dutch exporters have presently started to export but also the same percentage cease to export with high productive firms more inclined to export and less likely to leave so, among other findings. Bekes et al. (2017) using monthly French customs data find that firms export less in markets with higher demand volatility. Crowley et al. (2018) directly indicate that Chinese firms when subjected to tariff policy uncertainty are less likely to explore new foreign market and also exist from the established foreign markets also if there was no uncertainty Chinese firm entry would have been 2% higher.

From a theoretical perspective, Segura-Cayuela and Vilarrubia (2008) introduced informational externality to counter cost uncertainty using the Melitz (2003) model. In their model, once a firm enters the foreign market, its success/failure reveals information to other domestic firms who decide whether or not to enter the market based on the new information. In this model of learning, it is shown that a sufficient degree of informational externality can overcome the problems that arise from cost uncertainty. Another theoretical model by Handely (2014) considers a ubiquitous but often ignored source of uncertainty, i.e., trade policy uncertainty, in a dynamic heterogeneous firm model, and he observed that risks reduced entry in new destinations and binding agreements did increase entry. Handely (2014) also used Australian product level import data to prove the theoretical observations. Also Carballo, Handley, Limao (2018) using a dynamic heterogeneous firm model show that uncertainty about foreign income, trade protection and their interaction reduce export investment and that it can be eased by the presence of trade agreement and by using US firm level data from 2003-2011 show that uncertainty was the primary reason for trade collapse in 2008 crisis. Now Rauch and Watson (2003) considered buyer’s side of the ambiguity in a theoretical framework, where a developed countries buyer search for a new Less Developed Country (LDC) seller, and its propensity to start small increases with the cost of search but decreases as the probability of the LDC suppliers ability to supply large increases with training.

For this study, the generic Melitz (2003) type framework is extended by incorporating uncertainty in the form of payment defaults by importers. In doing so, it is seen that the
dynamics of this uncertainty percolates into the domestic market and affects the economy’s welfare. Here the firms faced with uncertainty have to bear higher costs than the usual trade costs, so only very efficient firms are able enter the new export market. These firms, because they serve both the domestic and the international market and also charge much lower price earn higher profits. These profits encourage more entrants, and along with the increased cost of uncertainty more pressure builds up on the single factor market, and pushes the real wages higher than in a situation without uncertainty. This causes more firms drawing low productivity exit, as they are unable to survive the increased cost competition which in turn increases the aggregate welfare of the economy, so any policy to reduce the cost by for e.g., reducing credit constraints suggested in Muuls (2008), and Manova (2012) can actually be detrimental. The study may not be fundamentally new, but it has quite a strong implication, it is a well observed fact that south has lower levels of productivity, and higher levels of uncertainty, then not only will its exports be compromised but also will its imports which implies that it will completely loose out to north in the global market and any kind of effort to reduce or buffer the cost may be deemed inefficient. The policy should therefore aim to increase the productivity and transparency levels of the south at par with the north.

In section 2, the model specification is briefly described. In section 3, the pro-competitive effect of incorporating uncertainty is shown, while the intuitive explanation of the findings is provided in section 4. Finally, section 5 consists of the conclusion and the direction for further research.

2. The model

Without loss of generality it is assumed that the world is composed of symmetric economies, home and rest of the world. Where the preference of a representative consumer, is given by a usual C.E.S utility function used in Krugman (1980), Eaton and Kortum (2002), Anderson and Wincoop (2003), Chaney (2008) and primarily Melitz (2003) however there are several theories, for e.g., Feenstra (2003), Kokovin et al. (2010), Behrens and Murata (2012), Arkolakis et al. (2019), Demidova (2017) which move beyond the restrictions of C.E.S utility function but maintain monopolistic competition of the market:

\[
U = \left[ \int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}}
\]  
(1)

the definitions of all the variables in the above equation and the rest if not explicitly expressed are equivalent to Melitz (2003), also the mass of varieties is considered to be an aggregate good, \( Q \equiv U \) Dixit and Stiglitz (1977) which has an aggregate price, associated with it:

\[
P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}
\]  
(2)

Now through the usual maximisation mechanism, the demand for a single variety \( \omega \) is obtained:
\[ q(\omega) = Q \left[ \frac{p(\omega)}{p} \right]^{-\sigma} \]  

(3)

The economies have a single industry, with continuum of firms, each choosing to produce a single distinct variety. The quantities of these varieties \((q)\) are produced using a single homogeneous input; labour \((l)\), and the technology applied is:

\[ l = f + \frac{q}{\theta} \]  

(4)

again, if \(w\) is regarded as the wage rate then total cost of the firm is:

\[ wl = w(f + \frac{q}{\theta}) \]  

(5)

now \(f > 0\) is the fixed cost, identical for all the firms; however, the variable cost differs on the basis of labour productivity, \(\theta > 0\), randomly drawn by the firms and it is further assumed that even if the quantity demanded is higher total cost of high productivity firm is always lower. Labour is inelastically supplied at its aggregate level \(L\) which also indicates the economy’s size.

The market is characterized by monopolistic competition so each firm faces downward sloping a residual demand curve with the constant elasticity \(\sigma\).

3. The pro-competitive effect of export market uncertainty

3.1. Firm behaviour

Since the market structure is that of monopolistic competition, the price charged by the firm in the domestic market is:

\[ p_d(\theta) = \frac{1}{\rho \theta} \]  

(6)

subsequently, the revenue earned is:

\[ r_d(\theta) = \frac{q_d(\theta)}{\theta} + \frac{p_d(\theta)q_d(\theta)}{\sigma} = R \left( \frac{p_d(\theta)}{p} \right)^{1-\sigma} \]  

(7)

and therefore, the profit from the domestic market is:

\[ \pi_d(\theta) = \frac{r_d(\theta)}{\sigma} - f = \frac{R}{\sigma} \left( \frac{p_d(\theta)}{p} \right)^{1-\sigma} - f \]  

(8)

Now, when the economy opens up, the firm, if decides to export, has to bear further costs. One is the usual iceberg type cost, \(\tau > 1\). The other cost is a one-time investment cost of entering the international market, \(f_{ex} > 0\) which is paid every period out of the export profits, i.e., \(f_x = \delta f_{ex}\), where \(\delta\) is the exogenously given probability of death. But apart from the above usual costs, the firms also face the cost of uncertainty, which is intrinsic to the export market, as much of the business is done on credit basis and information on credibility of international buyers are either unobservable or extremely expensive to observe, therefore making it unobservable. Now the price that the firm charges from the export market representing the increased variable cost is:
\[ p_x(\theta) = \frac{\tau}{\rho} = \tau p_d(\theta) \]  

(9)

Since there is an ambiguity regarding the importer’s payment, so the cost of uncertainty enters the model as the expected revenue of a firm (Neumann and Morgenstern, 1944), which is:

\[ E(r_x(\theta)) = (1 - \alpha)\tau^{1 - \sigma} r_d(\theta) \]  

(10)

where \(0 < \alpha < 1\) is the exogenously known probability of default by the importers. Now, the firms expected profit from the export market is:

\[ E(\pi_x(\theta)) = (1 - \alpha) \left( \frac{r_x(\theta)}{\sigma} \right) - \alpha \left( \frac{q_x(\theta)}{\theta} \right) - f_x \]  

(11)

The combined revenue is:

\[ r(\theta) = \begin{cases} r_d(\theta) & \text{if the firm serves only the domestic market} \\ r_d(\theta) + E(r_x(\theta)) & \text{if the firm exports too} \end{cases} \]  

(12)

Again, the combined profit is:

\[ \pi(\theta) = (\pi_d(\theta) + \max\{0, E(\pi_x(\theta))\}) \]  

(13)

Now, there is an abundant pool of identical entrants in the industry. But to enter the market, these prospective entrants must make an initial investment, which is modelled by a sunk entry cost \(f_e > 0\). Only after paying this sunk cost can the firm draw a productivity level, \(\theta\) out of a common ex-ante distribution \(h(\theta)\), and which remains unaffected overtime. \(h(\theta)\) is a general distribution defined over \(R^+\) and has a continuous cumulative distribution \(H(\theta)\). After drawing the productivity, the firms with productivity such that, \(\pi_d(\theta) < 0\) (excluding \(f_e\)) immediately exit the market, while the rest produce. Furthermore, the firms that draw productivity such that \(E(\pi_x(\theta)) \geq 0\) too apart from \(\pi_d(\theta) > 0\) serve both the domestic and the international market, and they continue to do so until faced with the death shock. Thus, present value function of the firms’ expected profit can be given by

\[ v(\theta) = \max\left\{0, \frac{\pi(\theta)}{\delta}\right\} \]  

So, from the above analysis, it is evident that there are two threshold productivity levels; \(\theta^* = \inf\{\theta: v(\theta) \geq 0\}\) is the cut-off productivity above which the firms serve the domestic market, i.e., the firms successfully enters the market, and \(\theta_{x1} = \inf\{\theta: \theta \geq \theta^* \text{and } E(\pi_x(\theta)) \geq 0\}\) is the cut-off of entering the international market and is greater than cut off without uncertainty. The ex-ante distribution of the productivity levels is no longer the ex-post distribution of productivity, which is:

\[ \mu(\theta) = \begin{cases} \frac{h(\theta)}{1 - H(\theta^*)} & \text{if } \theta \geq \theta^* \\ 0 & \text{otherwise} \end{cases} \]  

where \(\mu(\theta)\) is determined by the ex-ante distribution, conditional on \(p_{in} \equiv 1 - H(\theta^*)\), the probability of successful entry in the domestic market. Furthermore, \(p_{x1} = \frac{1 - H(\theta^*)}{1 - H(\theta_{x1})}\) represents the ex-ante as well the ex-post probability that one of the successful firms will enter the export market.
Again, let \( N \) is the equilibrium total number of incumbent firms in both the countries, and \( p_{x1}N \) is the number of firms exporting, so the total number of varieties available to the consumer of any given country is \( N_t = N + p_{x1}N \).

Now the aggregate price in (2) can be written as:

\[
P = N_t^{\frac{1}{1-\sigma}} p(\tilde{\theta}_t),
\]

where \( \tilde{\theta}_t = \left\{ \frac{1}{N_t} \left[ N\theta^{\sigma-1} + N_{x1}\left( t^{-1}\tilde{\theta}_{x1}\right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}} \) is the weighted average of productivity of all of the firms competing in a single country, that reflects the combined market share of all firms as well as the output leakage due to exports, as \( \tilde{\theta} = \tilde{\theta}(\theta^*) \) and \( \tilde{\theta}_{x1} = \tilde{\theta}_{x1}(\theta^*_{x1}) \) alone, which is average the productivity of all firms and only exporting firms, respectively, does not reflect. Since, this study considers the steady state equilibria; the aggregate variables remain unchanged over time. Therefore the other aggregate variables can be expressed as:

\[
Q = N_t^\frac{1}{\sigma} q(\tilde{\theta}_t), R = N_t r(\tilde{\theta}_t), II = N_t \pi(\tilde{\theta}_t)
\]

### 3.2. Open economy equilibrium with payment uncertainty

The overall average of the combined revenue and profit is given by:

\[
\bar{r} = r_d(\tilde{\theta}) + p_{x1}E(R_x(\tilde{\theta}_{x1}))
\]

\[
\bar{\pi} = \pi_d(\tilde{\theta}) + p_{x1}E(\pi_x(\tilde{\theta}_{x1}))
\]

(14)

Now, the zero-cut-off profit condition implies a relationship between the average profits and the cut-off productivity levels:

\[
\pi_d(\theta^*) = 0 \iff \pi_d(\tilde{\theta}) = f k(\theta^*) \text{ and,}
\]

\[
E(\pi_x(\theta^*_{x1})) = 0 \iff E\left(\pi_x(\tilde{\theta}_{x1})\right) = f_x k(\theta^*_{x1}) + \alpha j(\theta^*_{x1})
\]

(15)

where \( k(\theta^*) = \left[ \left( \frac{\theta(\theta^*)}{\theta^*} \right)^{\sigma-1} - 1 \right] k(\theta^*_{x1}) = \left[ \left( \frac{\tilde{\theta}_{x1}(\theta^*_{x1})}{\theta^*_{x1}} \right)^{\sigma-1} - 1 \right], \)

\[
j(\theta^*_{x1}) = \left[ \left( \frac{\tilde{\theta}_{x1}(\theta^*_{x1})}{\theta^*_{x1}} \right)^{\sigma-1} \left\{ \frac{q_{x}(\theta^*_{x1})}{\theta^*_{x1}} - \frac{q_{x}(\tilde{\theta}_{x1}(\theta^*_{x1}))}{\tilde{\theta}_{x1}(\theta^*_{x1})} \right\} \right].
\]

Again, \( \theta^*_{x1} \) can be represented as the function of \( \theta^* \):

\[
\theta^*_{x1} = \theta^* \left[ f_x \frac{f_x}{f(1-\alpha)\tau^{1-\sigma-\alpha}q(x)^{-\sigma}} \right]^{\frac{1}{\sigma-1}}
\]

(16)

So, now \( \bar{\pi} \) can be written as a function of the cut-off of successful entry in the market, i.e., \( \theta^* \) using (14), (15), and (16):

\[
\bar{\pi} = f k(\theta^*) + p_{x1}(f_x k(\theta^*_{x1}) + \alpha j(\theta^*_{x1}))
\]

(17)
The above equation represents the zero-cut-off profit condition of an open economy with payment uncertainty.

All of the firms, apart from the firms that have just successfully entered the market, earn positive profits, so the average profit \( \bar{\pi} \) must be positive. Let \( \bar{\pi} \) denote the present value of the average profit flows, then \( \bar{\pi} = \frac{\pi}{\delta} \). Furthermore, the net value of entry can be defined as:

\[
v_e = p_l n \bar{\pi} - w f_e = \frac{1 - H(\theta^*)}{\delta} \bar{\pi} - f_e
\]

Since the average profit flow of the firm that is able to successfully enter the market is positive, the firms want to invest in the sunk cost of entering the market. Since the market is characterized by monopolistic competition, this implies that there is a free entry and exit. Hence, \( v_e = 0 \) in equilibrium. If \( v_e > 0 \), then the firms will observe a positive value of entry; i.e., the firms will see that the present value of future profits more than covers the sunk cost. Thus, there will be more entries. If \( v_e < 0 \), then the firms will exit. Therefore, the free entry condition is:

\[
\bar{\pi} = \frac{\delta}{1 - H(\theta^*)} f_e
\]

The zero-cut-off profit condition and the free entry condition establish a relationship between the combined average productivity and the entry cut-off, and the unique equilibrium values of the two are obtained:

\[
\bar{\pi} = f k(\theta^*) + p x_1 (f x_1 k(\theta_{x_1}^*) + \alpha j(\theta_{x_1}^*)) \quad \text{(ZCP)}
\]

\[
\bar{\pi} = \frac{\delta}{1 - H(\theta^*)} f_e \quad \text{(FE)}
\]

**Proposition 1:** The equilibrium domestic entry cut-off is greater for an export market with uncertainty than without uncertainty.

Once the equilibrium value of the cut-off productivity for successful entry is obtained, the value of other endogenous variables, such as the cut-off for entering the export market and the average productivity levels, are also obtained. Thus, the mass of incumbent firms can also be determined:

\[
N = \frac{L}{\sigma [\bar{\pi} + f + p x_1 (\frac{\alpha j(\theta_{x_1}^*)}{\delta_{x_1}}) + f_x]}
\]

**Proposition 2:** The mass of firms serving the domestic market (i.e., the total varieties available to the consumers) decreases when there is uncertainty in the export market.

As \( N \) decreases, the number of firms exporting decreases as \( N_{x_1} = p x N \) (i.e., number of imported varieties) decreases. Thus, the total varieties consumed by individuals decrease for trade under uncertain conditions, so the welfare must decrease because consumers prefer variety. The welfare function is:

\[
W = \left( \frac{L}{\sigma} \right) ^{\frac{1}{\rho - 1}} \rho \theta^*
\]
In the above equation, all the variables are equivalent to Melitz, but $\theta^* > \theta_{WU}^*$ where $\theta_{WU}^*$ represents equilibrium domestic entry cut off without uncertainty is less than it is under uncertain conditions, which is already stated in proposition 1. This suggests:

**Proposition 3:** The welfare in trade under uncertain conditions is greater than the welfare in trade under certain conditions.

### 4. A graphical analysis

From the above model, it is clear that uncertainty is welfare enhancing. This phenomenon would further be clarified if the equilibrium conditions in case of closed economy, open economy with certainty, and open economy with uncertainty are compared. The ZCP (Zero-cut-off Profit) condition and the FE (Free Entry) condition for a closed economy are:

$\bar{\pi} = k(\theta^*)f$ (ZCP)

$\bar{\pi} = \frac{\delta f_e}{1-H(\theta^*)}$ (FE)

Both of the above equations provide a relationship between $\bar{\pi}$ and $\theta^*$, and so the equilibrium values of these variables can be obtained by solving them. The equations can also be represented diagrammatically:

![Graphical representation of ZCP and FE conditions](image-url)

The intersection of the two curves provides the equilibrium values of the two variables (the downward sloping schedule represents the ZCP condition while the upward sloping schedule the FE condition). Now considering the equilibrium conditions of an open economy with certainty:

$\bar{\pi} = f_k(\theta^*) + p_x f_p k(\theta^*)$ (ZCP)

$\bar{\pi} = \frac{\delta f_e}{1-H(\theta^*)}$ (FE)
Diagrammatically,

\[\pi = f(\theta^*) + p_{x_1}(f k(\theta_{x_1}^*) + \alpha j(\theta_{x_1}^*)) \] (ZCP)

\[\bar{\pi} = \frac{\delta}{1-\theta^*} f_\theta \] (FE)

From the above diagram, it is observed that the ZCP shifts upwards raising the equilibrium values of both of the variables when the economy moves from autarky to free trade. This occurs because trading involves trade costs, which in turn implies that only the efficient firms enter the export market that charge lower prices and still earn higher profits. Now as explained by Melitz (2003) that there are two potential effects of trade leading to the entry cut-off to increase. One is the fact that trade causes competition to increase in the domestic country due to new foreign firms entering who have high productivities which forces the prices down. But due to CES assumption price elasticity is constant so demand is not affected. But the higher profits of the firms attract more entrants, this increases the demand for labour in the domestic factor market, thereby increasing the real wage. This increased real wage causes the firms with lower productivities not being able to break even and therefore exiting the market. Therefore, increasing the equilibrium market entry cut-off, and since only efficient firms remain in the market, the average profit and so does the level of welfare. Now, the equilibrium conditions of an open economy with uncertainty are:

Diagrammatically,
In the above diagram, the ZCP curve shifts even higher to $ZCP_{TU}$, which increase the equilibrium values of both the domestic entry cut-off and the average productivity than when the shift was to $ZCP_T$. The explanation of this shift is similar to that explained previously. However due to the uncertainty faced by the firms that plan to export, they have to bear further costs apart from the trade costs. This implies that only very highly efficient firms are able to enter the export market. The profits of these firms are higher which attracts higher number of entrants along with the higher costs pushes the real wages higher than that in case of trade under certain conditions. So, more firms in the lower tail exit, which not only pushes up the equilibrium domestic entry cut off but also increase the average profit and the aggregate welfare even though the mass of variety available declines.

5. Conclusion and future research

The literature concerning international trade, heterogeneous firms and export market uncertainty is fast expanding. As the repercussions, of uncertainty in international market cannot be ignored. Furthermore, the literature that exists which considers export market volatility almost always considers it to be the main peril towards the economy and explains ways to eliminate it or at least reduce it because international trade has always been considered to be the engine of growth. In this paper, payment uncertainty in the export market has been incorporated into the standard Melitz (2003) model; however, once uncertainty is introduced, it can be observed that rather than turning out to be welfare diminishing, it enhances it by making the competitive effect of trade even more profound. This occurs because as the market is uncertain, the firms who plan to export products expect higher costs, which is why only highly efficient firms enter the international market. After entering the international market, the firms incur higher profits this results in other firms observing the expanded profits and therefore entering the market. Both increase in costs and entrants leads to an increase in factor demand which drives up the real wages. The higher real wage leads to some more of the less productive firms leaving the export market compared to that in the certain. Since, only the highly productive firms survive, the average profit increases, and so does the aggregate welfare even though the variety available decreases.

While there is extensive theoretical and empirical literature concerning heterogeneous firms and trade, there are still many possible scopes of extensions. It is apparent from these analyses that bigger firms are more likely to enter a risky market, while smaller firms will exploit the safer options. Countries with highly productive firms will be able to export more because they will be able to take more risks, while countries with smaller, less productive firms will be able to export less because they will try to avoid risks and they will lose out to the global competition. This could be verified empirically as well as modelled theoretically by doing away with the symmetric country assumption. Furthermore, in this paper, uncertainty increases efficiency; however, normally it is deemed inefficient, and safeguards are provided to diminish its consequences. Therefore, the theoretical model could be further extended in that direction. Also, the uncertainty considered can be made heterogeneous i.e. the importers probability of default might differ as different firms have different responses to changes in the macroeconomic variables.
Notes

(1) The wage rate is henceforth normalized to unity.
(2) \( R = PQ = \int p(\theta)q(\theta)d\theta = \int r(\theta) \) is the aggregate revenue as well as aggregate expenditure.
(3) For simplicity it is assumed that there is no financial sector, firms bear their own costs.
(4) The model is static hence the level of uncertainty remains unaltered.
(5) The threshold for entering the export market is greater with payment ambiguity compared to than without because if \( 0 < \alpha < 1 \) then \( (1 - \alpha) \left( \frac{\sigma_x(\theta)}{\alpha} \right) - \alpha \left( \frac{\sigma_x(\theta)}{\sigma} \right) - f_x < \left( \frac{\sigma_x(\theta)}{\sigma} \right) - f_x \).
(6) For the derivation of equation (20), refer to appendix B.

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Appendix A

Zero profit cut off condition:

Equation (17) represents the zero cut off profit condition,

\[ \bar{\pi} = f k(\theta^*) + p_{x1}(f_x k(\theta_{x1}^*) + \alpha j(\theta_{x1}^*)) \]

The average revenue of all the firms in the domestic market can be written as,

\[ \bar{r}_d = r(\tilde{\theta}_d) = \frac{R}{M} \]

Now,

\[ \frac{r_d(\tilde{\theta})}{r_d(\theta^*)} = \frac{R(p_d(\tilde{\theta}) - p)}{R(p_d(\theta^*) - p)} \quad \text{(from equation (7) in the paper)} \]

\[ r_d(\tilde{\theta}) = \left( \frac{\tilde{\theta}}{\theta^*} \right)^{\sigma - 1} \]  \hspace{1cm} (A1)

Again, at \( \theta^* \) (cut off of successful entry in the market)

\[ \pi_d(\theta^*) = 0 \]

\[ r_d(\theta^*) - f = 0 \]

\[ r_d(\theta^*) = \sigma f \quad \text{(A2)} \]

The average profit of the all the firms in the source country is,

\[ \pi_d(\tilde{\theta}) = \frac{r_d(\tilde{\theta})}{\sigma} - f \]

\[ \pi_d(\tilde{\theta}) = \left( \frac{\tilde{\theta}}{\theta^*} \right)^{\sigma - 1} \frac{r_d(\theta^*)}{\sigma} - f \quad \text{from (A1)} \]

\[ \pi_d(\tilde{\theta}) = \left( \frac{\tilde{\theta}}{\theta^*} \right)^{\sigma - 1} \frac{\sigma f}{\sigma} - f \quad \text{from (A2)} \]

\[ \pi_d(\tilde{\theta}) = f \left( \left( \frac{\tilde{\theta}}{\theta^*} \right)^{\sigma - 1} - 1 \right) \]

Now, let \( k(\theta^*) = \left( \left( \frac{\tilde{\theta}}{\theta^*} \right)^{\sigma - 1} - 1 \right), \tilde{\theta} = \tilde{\theta}(\theta^*) \) (for the derivation refer to Melitz (2003)

\[ \pi_d(\tilde{\theta}) = f k(\theta^*) \quad \text{(A3)} \]

Again,

\[ \frac{E(r_d(\tilde{\theta}_{x1}))}{E(r_d(\tilde{\theta}_{x1}))} = \frac{(1-\alpha)r_x(\tilde{\theta}_{x1})}{(1-\alpha)r_x(\tilde{\theta}_{x1})} \]

\[ \frac{r_d(\tilde{\theta}_{x1})}{r_d(\tilde{\theta}_{x1})} = \left( \frac{\tilde{\theta}_{x1}}{\theta_{x1}} \right)^{\sigma - 1} \]
\[ r_x(\tilde{\theta}_{x1}) = \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} r_x(\theta_{x1}^*) \quad (A4) \]

Now at \( \theta_{x1}^* \) (the cut off of entering the international market)

\[ E\left(\pi_x(\theta_{x1}^*)\right) = 0 \]

\[ (1 - \alpha) \left(\frac{r_x(\theta_{x1}^*)}{\sigma}\right) = \alpha \left(\frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*}\right) + f_x \]

\[ r_x(\theta_{x1}^*) = \frac{\sigma}{(1 - \alpha)} \left[ \alpha \left(\frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*}\right) + f_x \right] \quad (A5) \]

The average profit of only the exporting firms,

\[ E\left(\pi_x(\tilde{\theta}_{x1})\right) = (1 - \alpha) \left(\frac{r_x(\tilde{\theta}_{x1})}{\sigma}\right) - \alpha \left(\frac{q_x(\tilde{\theta}_{x1})}{\theta_{x1}}\right) - f_x \]

\[ E\left(\pi_x(\theta_{x1})\right) = \frac{(1 - \alpha)}{\sigma} \left[ \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} r_x(\theta_{x1}^*) \right] - \alpha \left(\frac{q_x(\tilde{\theta}_{x1})}{\theta_{x1}}\right) - f_x \text{ from (A4)} \]

\[ E\left(\pi_x(\tilde{\theta}_{x1})\right) = \frac{(1 - \alpha)}{\sigma} \left[ \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} \left(\frac{q_x(\tilde{\theta}_{x1})}{\theta_{x1}^*}\right) - \frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*} \right] - \alpha \left(\frac{q_x(\tilde{\theta}_{x1})}{\theta_{x1}}\right) - f_x \text{ from (A5)} \]

\[ E\left(\pi_x(\tilde{\theta}_{x1})\right) = \alpha \left[ \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} \left(\frac{q_x(\tilde{\theta}_{x1})}{\theta_{x1}^*}\right) - \frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*} \right] + \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} f_x \]

\( \left(\frac{\theta_{x1}^*}{\theta_{x1}^*}\right)^{\sigma - 1} > 0 \) and \( \left(\frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*}\right) > \left(\frac{q_x(\tilde{\theta}_{x1})}{\theta_{x1}}\right) \) as average productivity > cut-off productivity level i.e. \( \tilde{\theta}_{x1} > \theta_{x1}^* \) so the variable labour cost of firms with productivity \( \tilde{\theta}_{x1} \) is less than of the firms with productivity \( \theta_{x1}^* \).

Now, let \( k(\theta_{x1}^*) = \left[ \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} - 1 \right] \) and \( j(\theta_{x1}^*) = \left[ \left(\frac{\tilde{\theta}_{x1}}{\theta_{x1}^*}\right)^{\sigma - 1} \left(\frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*}\right) - \frac{q_x(\theta_{x1}^*)}{\theta_{x1}^*} \right] \)

where, \( \tilde{\theta}_{x1} = \tilde{\theta}_{x1}(\theta_{x1}^*) \) so,

\[ E\left(\pi_x(\tilde{\theta}_{x1})\right) = f_x k(\theta_{x1}^*) + \alpha j(\theta_{x1}^*) \quad (A6) \]

Since only very efficient firms enter the export market as compared to under certain export market the expected average profit is higher,

\[ E\left(\pi_x(\tilde{\theta}_{x1})\right) = f_x k(\theta_{x1}^*) + \alpha j(\theta_{x1}^*) > \pi_x(\tilde{\theta}_{x1}) = f_x k(\theta_{x1}^*) \]

The combined average profit across all domestic firms is (earned from domestic and international sales),

\[ \bar{\pi} = \pi_d(\tilde{\theta}) + p_{x1} E(\pi_x(\tilde{\theta}_{x1})) \]

Using equation (A3) and (A6) combined average profit can be written as,

\[ \bar{\pi} = f k(\theta^*) + p_{x1} (f_x k(\theta_{x1}^*) + \alpha j(\theta_{x1}^*)) \], hence the ZCP.

International market entry cut off as a function of domestic entry cut off:
It is known that,
\[ r_d(\theta^*) = R \left( \frac{\tilde{r}}{\tilde{p} \theta^*} \right)^{1-\sigma} \]  
(A7)

and,
\[ E\left( r_x(\theta^*_x) \right) = (1 - \alpha) R \left( \frac{\tilde{r}}{\tilde{p} \theta^*_x} \right)^{1-\sigma} \]  
(A8)

Now, using (A7) and (A8),
\[ \frac{E(r_x(\theta^*_x))}{r_d(\theta^*)} = (1 - \alpha) \tau^{1-\sigma} \left( \frac{\theta^*_x}{\theta^*} \right)^{\sigma-1} \]  
(A9)

Again at \( \theta^* \),
\[ \pi_d(\theta^*) = 0 \]
\[ r_d(\theta^*) = \sigma f \]  
(A10)

At \( \theta^* \),
\[ E(\pi_x(\theta^*_{x1})) = 0 \]
\[ (1 - \alpha) r_x(\theta^*_{x1}) = \sigma \left[ a \left( \frac{q_x(\theta^*_{x1})}{\theta^*_{x1}} \right) + f_x \right] \]  
(A11)

So, using (A10) and (A11)
\[ \frac{E(r_x(\theta^*_x))}{r_d(\theta^*)} = \frac{\sigma \left( \frac{q_x(\theta^*_{x1})}{\theta^*_{x1}} \right) + f_x}{f} \]  
(A12)

Equating (A9) and (A12),
\[ (1 - \alpha) \tau^{1-\sigma} \left( \frac{\theta^*_x}{\theta^*} \right)^{\sigma-1} = \frac{\sigma \left( \frac{q_x(\theta^*_{x1})}{\theta^*_{x1}} \right) + f_x}{f} \]
\[ \left( \frac{\theta^*_x}{\theta^*} \right)^{\sigma-1} = \frac{\sigma \left( \frac{q_x(\theta^*_{x1})}{\theta^*_{x1}} \right) + f_x}{(1 - \alpha) \tau^{1-\sigma} f} \]
\[ \theta^*_{x1} = \theta^* \left[ \frac{f_x}{f(1 - \alpha) \tau^{1-\sigma} \sigma \left( \frac{\tilde{r}}{\tilde{p} \theta^*_x} \right)^\sigma} \right]^{\frac{1}{\sigma - 1}} \]
Appendix B

Equilibrium number of domestic firms:
The combined average revenue of all the firms is,
\[ \bar{r} = r_d(\bar{\theta}) + p_{x1}E(r_x(\bar{\theta}_{x1})) \]
also,
\[ \bar{r} = \frac{R}{M} \implies N = \frac{R}{\bar{r}} \]
Now, L=R (refer Melitz (2003)).
Again,
\[ \pi_d(\bar{\theta}) = \frac{r_d(\bar{\theta})}{\sigma} - f \]
\[ r_d(\bar{\theta}) = \sigma(\pi_d(\bar{\theta}) + f) \quad (B1) \]
and
\[ E\left(\pi_x(\bar{\theta}_{x1})\right) = (1 - \alpha)\frac{r_x(\bar{\theta}_{x1})}{\sigma} - \alpha \frac{q_x(\bar{\theta}_{x1})}{\sigma_{x1}} - f_x \]
\[ (1 - \alpha)r_x(\bar{\theta}_{x1}) = \sigma\left(E\left(\pi_x(\bar{\theta}_{x1})\right) + \alpha \frac{q_x(\bar{\theta}_{x1})}{\sigma_{x1}} + f_x \right) \quad (B2) \]
So now, mass of firms, \[ N = \frac{R}{\bar{r}} \]
(it is assumed that \[ p_x = p_{x1} \] i.e. the fraction of firms exporting out of total firms exporting remains unaltered in equilibrium in uncertainty). Comparing N in case of export without uncertainty and N with, it is clear that this N is lesser.