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Export supporting strategy, distance to frontier and economic growth

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Abstract. *This paper analyses the influence of an export supporting policy on the technical progress of a country. The model is closely related to the model by Acemoglu et al. (2006) which constitutes the context between growth and the distance to the world technology frontier but modified considering import and export sector of a small economy. An export supporting strategy is introduced which increases the project size in the export sector. It shows that the project size is an influencing factor for the outcome and the growth of the technological knowledge. An economy benefits from trade in every stage of development. Furthermore, technological small and big countries are distinguished and analyzed, which are confronted with the exogenous world technology frontier or rather to the endogenous world technology frontier. In both cases the economy benefits by trade.*

Keywords: growth, trade, distance to frontier, developing countries.

JEL Classification: O24, O33.

1. Motivation and related literature

In January 2018 the World Bank published their Global Economic Prospects with an interesting statement: The motor of global growth are the developing countries. The economic crisis concerns mainly the high-income countries. The GDP growth of the world was at 3.0 percent in 2017 and will rise by 0.1 percent to 3.1 percent in 2018. But the growth rate of the high-income countries will remain constant between 2.2 and 2.3 percent. The forecast for the group of developing countries is that the GDP will increase to 4.7 percent in 2018 from 4.5 percent in 2017. According to the World Bank Prospects the driving force for the global GDP growth will be the group of developing countries in 2018.

The developing countries also play a prominent role in the course of trade. The world trade volume is expected to expand by 4.3 and 4.0 percent in 2017 and 2018, starting from 7 percent in 2015. This leap is due to the developing countries, where the intrasectoral trade still takes place. The impact of development disparities is not merely relevant for trade and trade policy, but rather for the technological development of a country. Different initial conditions lead to different growth rates.

This paper deals from the interplay between globalization and technical progress. On the one hand there is a strong alignment with the model of Acemoglu, Aghion and Zillibotti (2006) where the focus is on the distance to the world technology frontier. But on the other hand, the theory of Krugman (1991) about trade will be considered. Therefore, a growth model by Acemoglu, Aghion and Zillibotti from 2006 provides the basic framework and is combined with the trade model by Krugman (1991). The fundamental result message of the paper 'Distance to Frontier, Selection and Economic Growth' by Acemoglu, Aghion and Zillibotti from 2006 is that countries can benefit from an active government. It shows that countries can catch up on technological knowledge with more developed economies and may be able to constitute the world technology frontier in the end. A good example for this process is China. They entered into agreements with European companies like Airbus. But the government bought a high quantity of products under the permission that the main part of the production process will take place in its own country. Thereby employment was stimulated and it additionally enlarges the technological knowledge about aircraft manufacture of the country. That is just one example of China´s strategies to catch up the distance to the technological frontier.

But not only China gains from trade policy. In the following it is shown, that an export support policy benefits also economically small countries. Especially in relatively less developed countries it is more advisable to start imitation with smaller projects to accumulate knowledge. Depending on the development stage different strategies are expedient. Some rising up questions are: What happens to the technological development stage of an open country? Which influence has an export support for the choice between an imitation- or innovation-based strategy? More generally: Which effects result from the export support?

Related literature is the seminal paper by Nelson and Phelps (1966) which emphasizes the importance of the skills and human capital for the choice of a proper strategy for technological progress. Similar thoughts had Galor and Tsiddon (1997). They focus on the human capital endowment of entrepreneurs during times of structural change. The main aim of Hassler and Rodriguez (2000) work is the impact of human capital on the success of innovations. Concerning growth and convergence, the papers of Barro and Sala-i-Martin (1997) and Howitt and Meyer (2002) should be mentioned. Barro and Sala-i-Martin (1997) show that technologically less developed countries converge towards the leading technology. They further examine the selection of the technologically leading firm as well as the distinctions between the innovation and imitation strategy. Howitt and Meyer (2002) demonstrate the convergence of growth by considering different groups of countries. Aghion and Howitt (1992) study growth effects due to resulting innovations caused by the accumulation of knowledge. They use the setting of Schumpeter (1992) and describe the process of creative destruction, where innovations replace former innovations. The standard models concerning trade include Krugman (1979,1991) and Samuelson (1948). The paper of Krugman (1979) justifies the new trade theories and neglects serious differences between the countries. Regarding the equalization of factor prices the important work of Samuelson (1948) is consulted.

The paper is organized as follows. It starts with a detailed description of the modified model in section 2. Section 3 characterizes the macroeconomic equilibrium. The role of trade and export support policies are analyzed in section 4. Section 5 concludes.

2. The model

The basic framework by Acemoglu, Aghion and Zillibotti (2006) deals with a two-sector model composed of a final good sector and an intermediate good sector. The final good sector is perfectly competitive, whereas in the intermediate sector this is not the case.

In contrast to Acemoglu, Aghion and Zillibotti (2006) the following modification has two perfect competitive final goods in a small economy. These are needed due to the trade modification. These final goods are made by using labor and intermediate goods as production factors. The aggregate production functions of both sectors is shown with $\alpha_{\mathrm{I}} > \alpha_{\mathrm{II}}$ and

$$y_j = \frac{1}{\alpha_j} N_{tj}^{\,1-\alpha_j} \left(\int_0^1 \left(A_t(\upsilon)\right)^{1-\alpha_j} x_{tj}(\upsilon)^{\alpha_j} d\upsilon \right) \qquad \text{with } j = \mathrm{I}; \mathrm{II} \qquad (1)$$

The common determinants of the production functions y_{I} and y_{II} are the productivity in intermediate sector υ at time t, $A_t(\upsilon)$, and the flow of intermediate good υ used in final good production at time t, $x_{tj}(\upsilon)$. Both sectors use the same technology and thus have the same productivity. Since the elasticity of productivity of the intermediate goods in sector two is higher than that of sector one, it makes the production of the intermediate good for sector one more important than for sector two. Whereby labor is more important for sector two. The total labor force is divided into the two final product sectors.

$$N_t = \sum_{j=\mathrm{I}}^{\mathrm{II}} N_{tj} \qquad (2)$$

Here N_t is the amount of labor at time t.

The intermediates, $x_t(\upsilon)$, are used in both sectors. Each of them is produced by a monopolist who is using the leading technology. The leading monopolistic company is able to transform the final good back into an intermediate good, conditioned by the access to the most productive technology. Therefore, the price is not only a measure of demand, but also a measure of the amount of final goods needed to regress an intermediate good.

$$p_{tj}(\upsilon) = \chi_j \tag{3}$$

Parameter χ_j is the limit price for the monopolist.

$$\chi_j = \left(\frac{A_t(\upsilon)N_{tj}}{x_{tj}(\upsilon)}\right)^{1-\alpha_j} \tag{4}$$

The market is less competitive the higher the limit price, because it is a considerable expenditure to imitate. So, there is only one firm, υ, producing an intermediate good.

$$\frac{1}{\alpha_j} \geq \chi_j > 1 \tag{5}$$

Equation (4) together with the current demand (3) gives the equilibrium profit for sector one.

$$\pi_{tj}(\upsilon) = [\chi_j - 1]x_{tj} \tag{6}$$

$$\pi_{tj}(\upsilon) = [\chi_j - 1]A_t(\upsilon)N_{tj}\ \chi_j^{-\frac{1}{1-\alpha_j}} \tag{7}$$

$$\pi_{tj}(\upsilon) = \delta_j A_t(\upsilon)N_{tj} \tag{8}$$

The average level of technology in the regarded economy at time t is given by the following equation.

$$A_t \equiv \int_0^1 A_t(\upsilon)\, d\upsilon \tag{9}$$

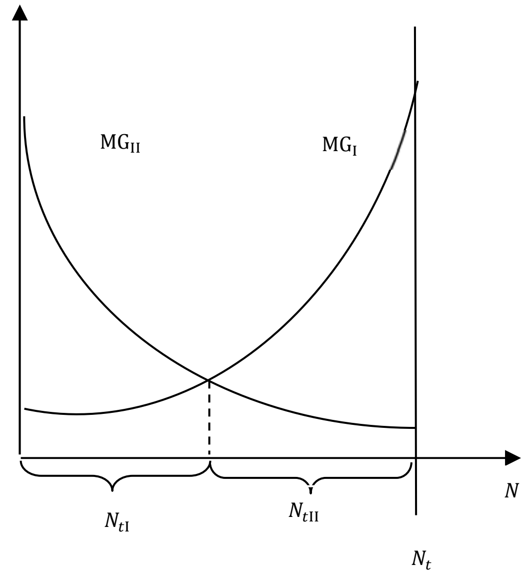
The aggregate final output for the two sectors is given by:

$$y_{tj} = \frac{1}{\alpha_j} N_{tj} A_t \chi_j^{-\frac{\alpha_j}{1-\alpha_j}} \tag{10}$$

Since labor is intersectorally mobile, there will be the same wage level for both sectors in a two-sector economy. Labor can be used in sector one as well as in sector two. Furthermore, there are no local restrictions in place. Thus, labor is flexible intersectorally and intrerregionally and it results an equilibrium wage, shown in figure 1. In closer examination of every single sector the respective market clearing wage is equal to the marginal product of labor.

$$w_j = \frac{1-\alpha_j}{\alpha_j} A_t^{1-\alpha_j} x_{tj}^{\alpha_j} N_j^{-\alpha_j} \tag{11}$$

Figure 1. *The equilibrium wage of the labor market*



2.1. World technology frontier and properties of the entrepreneurs

Just as Acemoglu, Aghion, Zilibotti (2006) starting from the premise, that the most productive country presents the world technology frontier $\bar{A}_t$. The productivity at the frontier grows with rate g, so that

$$\bar{A}_t = \bar{A}_0(1+g)^t \quad (12a)$$

In the following there are two different cases considered, concerning the world technology frontier. The first one is the above-mentioned world technology frontier of equation (12a) which is exogenous. The observed country is not able to change this frontier by innovating technologies. The considered country has a consistent technological growth rate g, which results in the world technology frontier. Possible innovations and scale effects do not affect the frontier because the impact is too small. These innovations are called microinventions as introduced in Mokyr (1990). For instance, a new watering system might be helpful for relatively less agricultural characterized countries, but will not be significant for relatively more developed countries.

The second case emphasizes an endogenous world technology frontier, which will raise with every new technology.

$$\bar{A}_{tj} = \bar{A}_0 (1+g_j)^t \quad (12b)$$

According to Mokyr (1990) macroinventions increases the state of technological development with every new innovation. The invention of the internet illustrates the importance of this innovation for the whole world. To begin with the analyze of the exogenous world technology frontier of equation (12a) up to a closer look follows in chapter 3.

It is assumed that the small country's state of technology A_t is lower than the frontier technology, $A_t \leq \bar{A}_t$. The productivity of intermediate goods produced by firm υ at time t is denoted as

$$A_t(\upsilon)= s_t(\upsilon)\, [\eta\, \bar{A}_{t-1}+\gamma_t\, A_{t-1}(\upsilon)] \quad (13)$$

Here, $s_t(\upsilon)$ is the size of the project or investment size, either a small project $s_t(\upsilon) = \sigma <1$, or a large project $s_t(\upsilon) =1$ is implemented.

The term $\eta\ \bar{A}_{t-1}$ denotes the productivity increase caused by adopting existing technologies. The knowledge for imitation is based on the technological level of the world technology frontier. The other term, $\gamma_t\, A_{t-1}(\upsilon)$, denotes the skill level of the entrepreneur and the local knowledge of the discussed economy. The properties of the entrepreneur determine the firm productivity as well as the investment size. Additionally, it is assumed, that entrepreneurs can be low-skill with $\gamma = 0$, or high-skill with $\gamma >1$.

Beside the skill differentiation there is another distinction about the age of the entrepreneurs. They can be young or old. If a firm decides to hire a young entrepreneur the skills will be unknown at the beginning. Old entrepreneurs already worked for a time period and because of these experiences it is known if they are high or low-skill.

Young entrepreneurs are only able to operate small projects, but old entrepreneurs operate big projects. The retained earnings of previous projects will be invested in the current project, whereby big projects are financeable. Old entrepreneurs, independent of their skill endowment, can offer financial support to the firms. But as stated by Acemoglu, Aghion, Zilibotti (2006) it is assumed that there are two different options to finance the projects: Either there are the owners of the firms, the capitalists, or the old entrepreneurs by contributing their retained earnings.

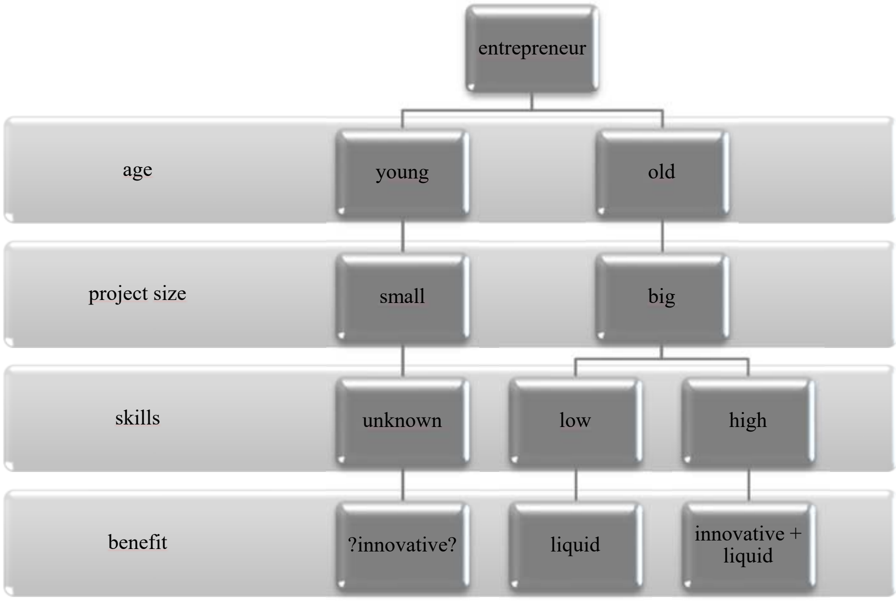
The skills and abilities determine the success of innovative activities. High skilled entrepreneurs are able to innovate and to imitate technologies. Entrepreneurs with low skills use existing technology of the frontier to adopt them. Figure 2 gives an overview about the entrepreneur´s characteristics.

There are three decisions for firms about the age, the skills and the project size which depends on the age.

But not each firm can choose freely between these three alternatives. Young firms have to hire young entrepreneurs and only a possibility of innovative activities. An old low-skill entrepreneur is very unpopular regarding innovations but very helpful to finance the project. Bigger projects need to be financed by the support of old entrepreneurs and small projects are accompanied with young entrepreneurs. This will be shown in chapter 2.3. Since the age of the entrepreneurs of young firms are determined, the skills are unknown.

The productivity growth which is caused by innovations depends on the economy’s state of technological development and on the selection of the entrepreneur.

Figure 2. *Possible characteristics of entrepreneurs*



2.2. Trade

To analyze the consequences of trade, a second final good sector and a second region to trade with are needed, guided by the ideas of Krugman (1991).

In the following part a small open economy will be looked at, called 'the Home'. This is exactly the same as the closed economy which was already mentioned. The second region, the rest of the world, is defined as 'the World Market'.

There are no transport costs. For the sake of simplicity, it is assumed that every produced good will be consumed. There is no variety regarding the preferences between regions, but there are different market sizes. On the one hand there is our exemplary economically small home country and on the other hand there is the World Market.

In this analysis it is also presumed that the factors of production are labor and intermediates. As Krugman (1991) stated, the same endowment with production factors is assumed in both regions. The workers are mobile among the sectors whereas the firms cannot trade between them. As considered above the wages are equal between both sectors and due to the mobility of labor it is also equal between the regions. Barriers to trade and complete specialization are excluded.

As Acemoglu, Aghion, Zilibotti (2006) the same technology for the two final good sectors are denoted and unlike Krugman, not for the different regions. The final goods are tradeable. The World Market differs from the Home economy especially in productivity.

$$A^{WM} > A^{H} \tag{14}$$

The world is better endowed with technological knowledge than the small Home country. So, it is more expensive for the Home country to produce good one. However, the production in sector two is relatively labor-intensive. Labor is relatively low-priced in the Home country and thus the domestic price of good two is smaller than the one of the World Market. It is the ambition of this paper to facilitate the technical know-how and thereby the productivity A^H by an export promoting policy. The gap of the technologies determines several price relationships. The price of the final good is P_j. So, with increasing productivity the price ratio decreases. It is an inverse link and therefore it appears reasonable that the Home price ratio is higher than the World Market price ratio.

$$\left(\frac{P_I}{P_{II}}\right)^H > \left(\frac{P_I}{P_{II}}\right)^{WM}, \text{ if } A^{WM} > A^H \qquad (15)$$

Because of this variation of price ratios and the small country's openness encourages trade between Home and World Market. The effect of this is one common price ratio for both regions.

As a consequence, it is more profitable for the domestic producers to offer the relatively more expensive good two. The suppliers substitute good one for good two. As a result, the supply of good one is reduced. The Home country supplies more of product two compared to the autarky situation.

Now the factor markets will be focused. The lower supply of good one releases a lot of intermediate goods and a small amount of labor. But these production factors are necessary to manufacture the higher amount of good two. In the second sector the production factor labor is relatively more important and the demand increases. At the labor market there is a high demand of sector two and a lower demand of sector one. Thus, the wage increases because of excess demand, according to (11). The price χ of the intermediate good $x(\upsilon)$ decreases because of an excess supply. This factor is more important for the production of good one. To conclude the decreasing final good price relationship results in an increasing factor price relationship in the Home region.(1) The consumers act in a contrary way. The relative increase in the price of good two is less attractive to the consumers and causes decreasing demand. The consumers substitute good two for the relatively cheaper good one. The demand of product one increases.

To sum up, trade leads to a reduction of the production in sector one and a increasing demand. Therefore, good two will have a higher supply and a lower demand. A closer view to sector two shows that the firms of the Home region produces more than the domestic households need for consumption. The surplus of good two is the export for the World Market. So good one is the import good, because the demand of product one is bigger than the offer of the same product in the Home region. The home region specializes in sector two and reduces its focus on sector one.

This is the new setup and the next step is to introduce the export supporting policy. The export sector is able to manage bigger projects than the import sector, because of the export promotion. Resulting scale effects lead to the fact that a grown sector can require an increased demand, but also learning-by-doing effects raise the productivity.(2) But is this also connected with a higher growth rate of the technological knowledge? Does a country with export support converge with the world technology frontier on an earlier development stage than without support?

The model by Acemoglu, Aghion, Zilibotti (2006) starts from the premise that there are two different project sizes:

small project: $s_t(\upsilon) = \sigma$, with $\sigma < 1$

large project: $s_t(\upsilon) = 1$;

In the following we differentiate the project size in three categories:

small import project: $s_t(\upsilon) = \sigma_I$ with $\sigma_I < 1$

small export project: $s_t(\upsilon) = \sigma_{II}$ with $\sigma_{II} < 1$

$$\sigma_{II} > \sigma_I \quad (16)$$

large project: $s_t(\upsilon) = 1$

It is to be determined, whether the technology development stage A_{t-1} of a country grows faster with trade by promoting exports.

2.3. Individual optimization

The main decisions in this model depends on the project size and the different kinds of entrepreneurs.

Young entrepreneurs´ skills are unpredictable unlike those of old entrepreneurs. With the knowledge about the skill characteristics a firm can decide to replace the old entrepreneur by a young one. It is obvious, that high-skill entrepreneurs will never be replaced, because of their high potential to finance projects and furthermore their ability to innovate new technologies. The only question is whether a low-skill entrepreneur will be retained or not.

Acemoglu, Aghion, Zilibotti (2006) show that old low-skill entrepreneurs will be retained if the firm prefers large projects, which is along with old entrepreneurs because of self-finance. Small investments accompanied by young entrepreneurs mean replacing an old low-skill entrepreneur. To prove the decision, it is important to compare the firm values resulting from both alternatives.

$$V_{tj}(\upsilon | \, s = 1, e = o, z = L) > E_t V_{tj}\big(\upsilon | \, s = \sigma_j, e = y\big) \quad (17)$$

with $V_{tj}(\upsilon | \, s = 1, e = o, z = L) = [(1-\mu)\delta_j N_j \eta \bar{A}_{t-1} - \max(\kappa \bar{A}_{t-1} - RE_t, 0)$

$$(18)$$

and $E_t V_{tj}\big(\upsilon | \, s = \sigma_j, e = y\big) = (1-\mu)\delta_j N_j \sigma(\eta + \lambda\gamma a_{t-1j})\bar{A}_{t-1} - \phi\kappa\bar{A}_{t-1}$ (19)

The characteristics of the entrepreneurs are the age and the skill level. Entrepreneurs are young or old with $e \in \{y, o\}$ and can be high skill or low skill, $z \in \{H, L\}$. Depending on the age and the skills there are the different investments sizes, $s \in \{\sigma_j, 1\}$. RE_t are the total retained earnings of an entrepreneur. The profits are shared between the entrepreneurs and the capitalists. The entrepreneurs get an amount of μ and the capitalist get $(1-\mu)$ of the total gain δ. The costs of investments are the following for the two different project sizes:

$$k_t(\upsilon|s) = \begin{cases} \phi\kappa\bar{A}_{t-1} & \text{if } s = \sigma_j \\ \kappa\bar{A}_{t-1} & \text{if } s = 1 \end{cases} \quad (20)$$

The main decision of firms is about the replacement of old low-skilled entrepreneurs by young one and described in equation (17). If the value of a firm V_{tj} operating a large project with an old low-skill entrepreneur is higher than the expected value of a firm E_tV_{tj} with a small project and a young entrepreneur then there will be no replacement of the old entrepreneur. It is to verify whether the value of an old low-skill entrepreneur is higher than the expected value of a young skill-unknown entrepreneur with smaller investments. If this is not the case, the decision of the firm won't be profitable. On the one hand, there are bigger feasible projects with the financial support of old entrepreneurs; on the other hand, they do not have the know-how to innovate products and processes. Big projects cause higher costs which reduce the profit. But old entrepreneurs bring their retained earnings in to finance part of the investment costs.

Regarding the expected value of young firms with young entrepreneurs, they may have the technological knowledge because of the unknown skills, but caused by missing financial resources they cannot sponsor higher investments. In this case the skills are unknown as well as the profit. High-skill entrepreneurs are more profitable than low-skill entrepreneurs, because they can advance products and increase the sales volume. As a result of smaller investments, the costs of the projects are also smaller. But the returns are also according to the project size and smaller projects gain less than big ones. To retain old low-skill entrepreneurs it must be weight up if the retained earnings compensate the higher costs more than the possible profit due to innovations by young entrepreneurs.

3. The macroeconomic equilibrium

To determine the distance to frontier first it is necessary to define the average productivity for young firm, old firms which retain their low-skill entrepreneurs for operating bigger projects and for old firms which replace the low-skill entrepreneurs.

Acemoglu, Aghion, Zilibotti (2006) assume that a young entrepreneur is high-skill by λ and low-skill by (1-λ). Young firms solely employ young skill-unknown entrepreneurs. They use the technologies of the frontier $\bar{A}_{t-1}$ to adopt existing processes or products. An entrepreneur is high skilled, γ, with the probability λ and applies the domestic technological knowledge, A_{t-1}, for innovative activities. Young entrepreneurs have no retained earnings and without this financial support young firms only operate small projects, σ_j.

$$A^y_{tj} = \sigma_j(\eta\bar{A}_{t-1} + \lambda\gamma A_{t-1}) \quad (21)$$

Old firms can select their entrepreneurs. They benefit by old high-skilled entrepreneurs twice, they are innovative and provide financial support. That is why they will always stay in the firm. The decision-making of the old firms about the old low-skilled entrepreneurs is more interesting. The first case describes the productivity of old firms, which retain their low-skill entrepreneurs, (R_{tj}=1). Because of the financial support they operate big projects, s=1. The productivity of all retained entrepreneurs composes of high-skill entrepreneurs and of old low-skill entrepreneurs.

$$A^{o}_{tj}\left(R_{tj}=1\right)=\eta\bar{A}_{t-1}+\lambda\gamma A_{t-1} \qquad (22)$$

If an old firm replace the old low-skilled entrepreneurs by young ones, (R_{tj}=0), the following productivity results. The high-skill entrepreneurs, with a probability of λ, are kept and the low-skilled entrepreneurs with (1- λ) are replaced by young entrepreneurs, operating small projects.

$$A^{o}_{tj}\left(R_{tj}=0\right)=\lambda(\eta\bar{A}_{t-1}+\gamma A_{t-1})+(1-\lambda)\sigma_j(\eta\bar{A}_{t-1}+\lambda\gamma A_{t-1}) \qquad (23)$$

Assuming that half of the firms are old, the whole productivity will be defined as:

$$A_{tj}\equiv\int_0^1 A_{tj}(\upsilon)\,d\upsilon=\frac{(A^{y}_{tj}+A^{o}_{tj})}{2} \qquad (24)$$

The distance to frontier at time t is defined as: a_{tj}. After reflection of all elements (21) - (24) the following will be obtained for each sector:

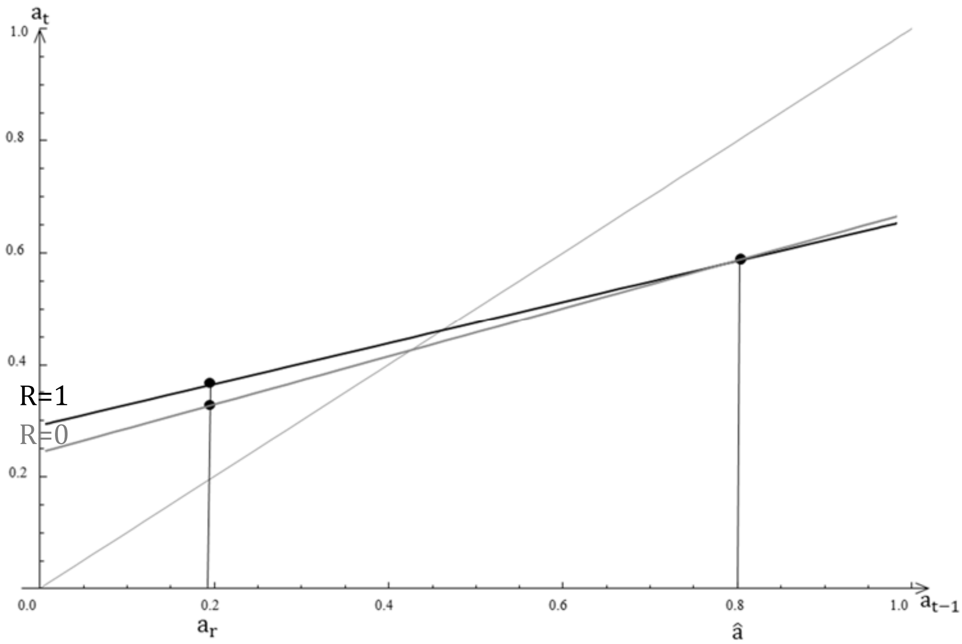
$$a_{tj}=\begin{cases}\frac{1+\sigma_j}{2(1+g)}\left[\eta+\lambda\gamma a_{t-1j}\right] & \text{if } R_{tj}=1\\ \frac{1}{2(1+g)}\left[\left(\lambda+\sigma_j+(1-\lambda)\sigma_j\right)\eta+\left(1+\sigma_j+(1-\lambda)\sigma_j\right)\lambda\gamma a_{t-1j}\right] & \text{if } R_{tj}=0\end{cases} \qquad (25)$$

A comparative view at both technological growth rates shows, that for the retaining case, R_{tj}=1, the gain of new technological knowledge is smaller than in the replacing case, R_{tj}=0.

A firm can decide between the retention of an entrepreneur, $R_{tj}=1$, or to terminate him, R_{tj}= 0.[(3)] If a firm retains an old low-skill entrepreneur then the focus is on the financial support of the project and to imitate or adopt products. Particularly, it is easier to manage big projects because of the retained earnings and entrepreneurs´ experiences. Also scale effects facilitate the production and a bigger production volume is feasible. But if a firm terminates entrepreneurs to get young ones, then innovations are the main aim. For the continuous development of a firm and their products, high-skill entrepreneurs are needed. New ideas lead to new variations of goods. Optimized processes improve the efficiency of production. This creates new needs and an increasing sales volume. With young entrepreneurs there is a chance to bring fresh knowledge in to the firm. Most profitable are the old high-skill entrepreneurs. They have the necessary technological knowledge, the essential experience and provide retained earnings. Therefore, they are not terminated and will stay in the firm, independent of the decision.

Both strategies are illustrated in Figure 3 for one sector with an exogenous world technology frontier. The determined project size of the frontier constituting firm or country is 1. Thus, it is not possible for smaller projects to reach the frontier. A firm pursue the innovation-based strategy ($R_{tj} = 0$) or the imitation-based strategy ($R_{tj} = 1$).

Figure 3. *One sector – exogenous world technology frontier*



with:

$\sigma_{I/II} = 0.5,\ \lambda = 0.5, \gamma = 10, \eta = 4, \kappa = 1, n = 1, \phi = 0.5, \delta = 0.5, i = 0.02, \mu = 0.5$

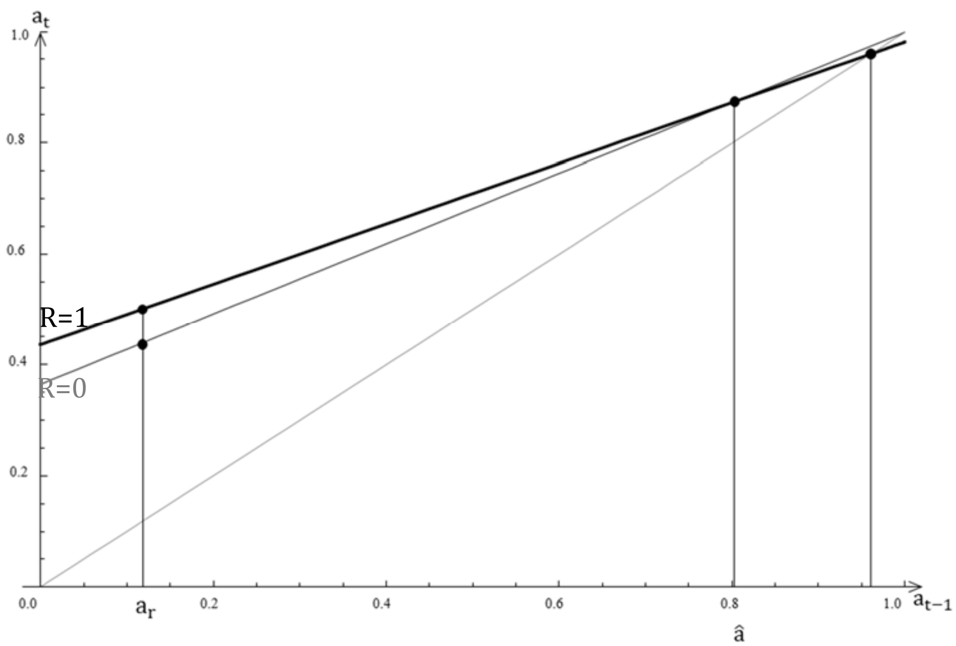
Every development stage at the time t-1, a_{t-1}, comes to a higher development stage a_t. The future a_t depends on the chosen strategy. From here on it will be distinguished between the investment-based / imitation-based strategy and the innovation-based strategy. Different sectors have different world technology growth rates and are determined by the innovations all over the world. The technologically most developed country represents the world technology frontier. The firm with the most developed technology in a country stands for the stage of the technological development of this country. Figure 3 illustrates the distance to frontier or the stage of development of a country at both points of time t and $t-1$. The value $\hat{a}_j$ is the threshold where the innovation strategy is more productive than the imitation strategy. From there on the innovation-based strategy leads to a higher development stage a_t than the imitation strategy. In reference to the productivity $\hat{a}_j$ is the key switching point. The gap to the world technology frontier decreases than with the investment-based strategy. If the development stage of a country is smaller than $\hat{a}$, a higher technological stage is reached, by adapting existing goods. If the development stage of a country is higher than $\hat{a}_j$ then innovations lead to a higher technological growth than imitation. Considering the costs and the profitability, a_{rj} is the corresponding threshold. From there on it is too expensive to retain old low-skill entrepreneurs and it is more productive to risk the unknown capabilities of young entrepreneurs. This threshold shows the point, where the old low-skill entrepreneurs are too costly for the firm, despite their retained earnings. The possibility of profits made by new products or a more efficient production process is higher than the financial support of the old entrepreneurs. As you can see in equation (26) this decision depends on the cost structure:

$$a_{r_j}(\mu, \delta) = \frac{\left[(1-\mu)(1-\sigma_j)+\frac{1+r}{1+g}\mu\sigma_j\right]\eta-\frac{\kappa(1-\phi)}{\delta N_j}}{(1-\mu)\sigma_j\lambda\gamma} \quad (26)$$

From this stage of development, a replacement of an old low-skill entrepreneur starts to be profitable.[(4)]

For the endogenous case the world technology frontier extends with every innovation. This situation is illustrated in figure 4 for one sector. An innovative firm can catch up the distance to frontier or constitute the frontier by itself. The path of development with the innovation-based strategy will necessarily end in the world technology frontier. But the frontier will extend with every innovation. For relatively less developed countries the imitation strategy is the more interesting one and $\hat{a}_j$ is relatively high. Consulting the rentability of a strategy the switching point is far apart from $\hat{a}_j$ the productivity threshold. From this point of view the innovation-based strategy fits for most countries.

Figure 4. *One sector – endogenous world technology frontier*



with:

$\sigma_{I/II} = 0.5,\ \lambda = 0.5, \gamma = 10, \eta = 4, \kappa = 1, n = 1, \phi = 0.5, \delta = 0.5, i = 0.02, \mu = 0.5$

4. Export supporting policy

In an attempt to highlight the role of trade different investment sizes are introduced. Initially the export sector one operates smaller projects than sector two. Since the project size of the export sector starts to increase and therefore the size of the import sector projects decreases. Bigger projects lead on to economies of scale and the capacity of the export sector two offsets these projects. The following section is about a strategy with export support by subsiding projects for the export sector.

In big countries the project size correlate with the growth of the world technology frontier, so the world technology frontier is endogenous. If the distance to the world technology frontier a_{tj} depends on the skills of the entrepreneurs and the project size, then the growth rate also depends on the different project sizes and there are different growth rates for the two sectors.

$$g_j = \tfrac{1}{2}\left(\left[\lambda + \sigma_j + (1-\lambda)\sigma_j\right]\eta + \left[1 + \sigma_j + (1-\lambda)\sigma_j\right]\lambda\gamma\right) - 1 \qquad (27)$$

An innovation extends the world technological frontier, like a macroinvention (Mokyr, 1990). Not only the technologically leading firms or countries influence the knowledge which is available for the World Market, the growth rate is independent of the size of a country or the development stage (27). By neglecting the skills of the entrepreneur there are two very interesting effects caused by the project size on a_{tj}.

$$a_{tj} = \begin{cases} \frac{1+\sigma_j}{2(1+g_j)}\left[\eta + \lambda\gamma a_{t-1j}\right] & \text{if } R_{tj} = 1 \\ \frac{1}{2(1+g_j)}\left[\left(\lambda + \sigma_j + (1-\lambda)\sigma_j\right)\eta + \left(1 + \sigma_j + (1-\lambda)\sigma_j\right)\lambda\gamma a_{t-1j}\right] & \text{if } R_{tj} = 0 \end{cases} \qquad (28)$$

First a higher investment is along with the project size $s_t(\upsilon) = \sigma$, which increases the productivity $A_{tj}(\upsilon)$ as concluded by equation (13). A higher number of goods let mistakes appear and the manufacturing can constantly be improved. Also, workers with a routine work faster and produce a higher amount of output. Overall, the production gets more efficient and that is why this effect is called the efficiency effect and it appears positive on the stage of the technological development of a country.

A second effect should also be considered, the growth effect. The leverage of the world technology growth rate depends also on the project size σ. A higher σ induces a bigger growth rate. But this creates a negative effect on the distance, a_{tj}, which increases. With the same growth rate, a firm with bigger projects can reach the world technology frontier very quickly, but with the rise of the project size the frontier also raises. The question is what will happen with the distance to frontier if the export sector two gets bigger projects than before. Which of the above-mentioned effects dominates?

This question will be answered by having a closer look to equation (28). Induced by promoting the exports, the export sector two has allocated small export projects and the import sector small import projects. The allocation of big projects $\sigma = 1$ is independent of the skills of the entrepreneur. Due to the variation of the project size σ both effects occur again in a change of the distance to the world technology frontier a_t. In the case, that the entrepreneurs will be retained (R_{tj}=1) the negative growth effect dominates the effect of increased efficiency. For instance, this relationship is explained at the import sector where the project size decreases. Big projects get more important and the small import projects are less profitable for innovative projects. Additionally, the distance to frontier decreases by less technological growth at the frontier by small import projects. This case is different from the replacement case of the innovation-based strategy (R_{tj}=0). The growth effect is dominated by the efficiency effect. Looking at the export sector the project size increases. Especially young firms benefit by this support and short-term relationships are preferred.

These effects lead to a specific company structure in the different sectors. The import sector one focuses on long term relationships with more imitations. The export sector specializes mainly on innovations

If a small country is analyzed, innovations may not affect the world technology frontier, because microinventions like Mokyr (1990) mentioned are assumed. For these countries the world technology frontier is exogenous. Hence the technological knowledge of the world and the frontier cannot be changed by an economically small country and are independent of the project size. In the following subsections we will analyze the impacts on and not the effects of the different country sizes.

4.1. Technologically small countries with an exogenous world technology frontier

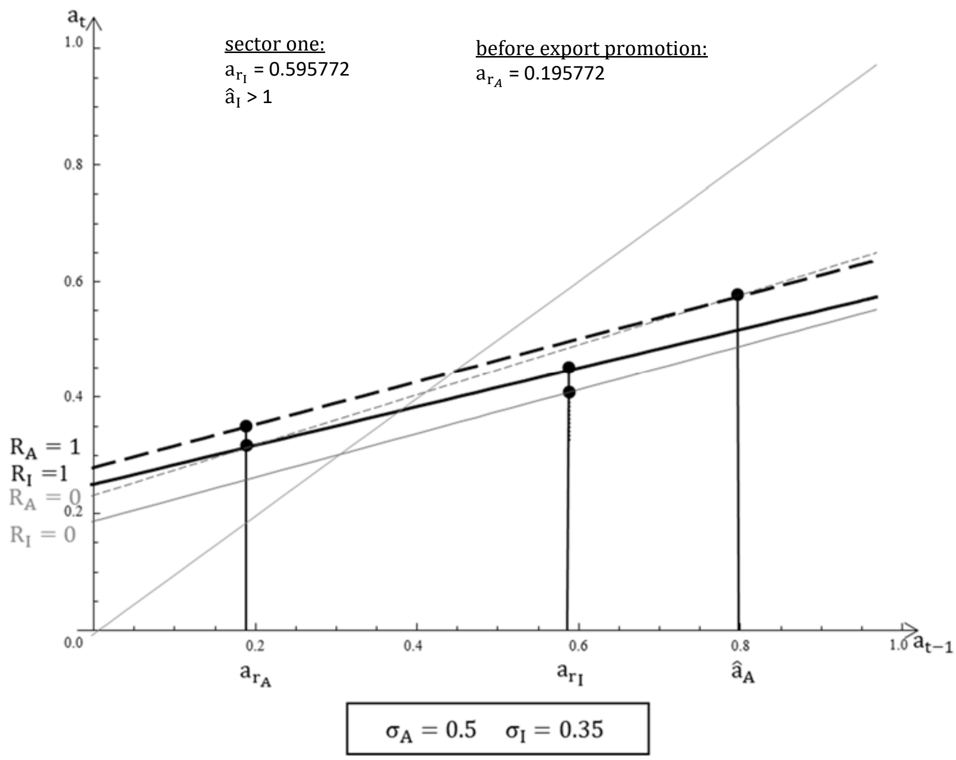
The reaction of the export support for a technologically small country with an exogenous world technology frontier is obvious plain. According to equation (25) there is a positive correlation between the project size and the stage of technological development a_{tj} for each strategy. A rising project size leads to higher productivity and there is no other effect which influences the growth rate. But the distance to the frontier depends also on the skills.

The two illustrations figure 5 and figure 6 show each just one sector; before and after export promotion. In the situation before the export support it is assumed, that both sectors have the same investments, illustrated by an average project size, $\sigma = 0.5$. With trade policy the project size of the export sector decreases,σ_I=0.35 and the project size of the import sector increases, σ_{II}= 0.75. The dashed lines show the situation of a sector before the export supporting policy. The solid lines represent the strategy for the export and the import sector after export promotion. These images illustrate the different strategies by every possible level of technological development. There are bigger projects in the export sector two than in the import sector one.

Beginning with the import sector one the projects sizes decrease after export support. This leads to lower technological development stages a_t, independent of the pursued strategies. There is less growth than before export promotion. It is too much effort to develop new individual methods for every new small project. Furthermore, there are no magnitude effects which could justify the more expensive innovations. For each technological development stage, which is smaller than $\hat{a}$, it is recommended to start with the imitation strategy and switch to the innovation-based strategy if the threshold $\hat{a}_j$ is overstepped. The increase of technological knowledge is higher by the imitation strategy (R_{tA}=1), till $\hat{a}_j$.

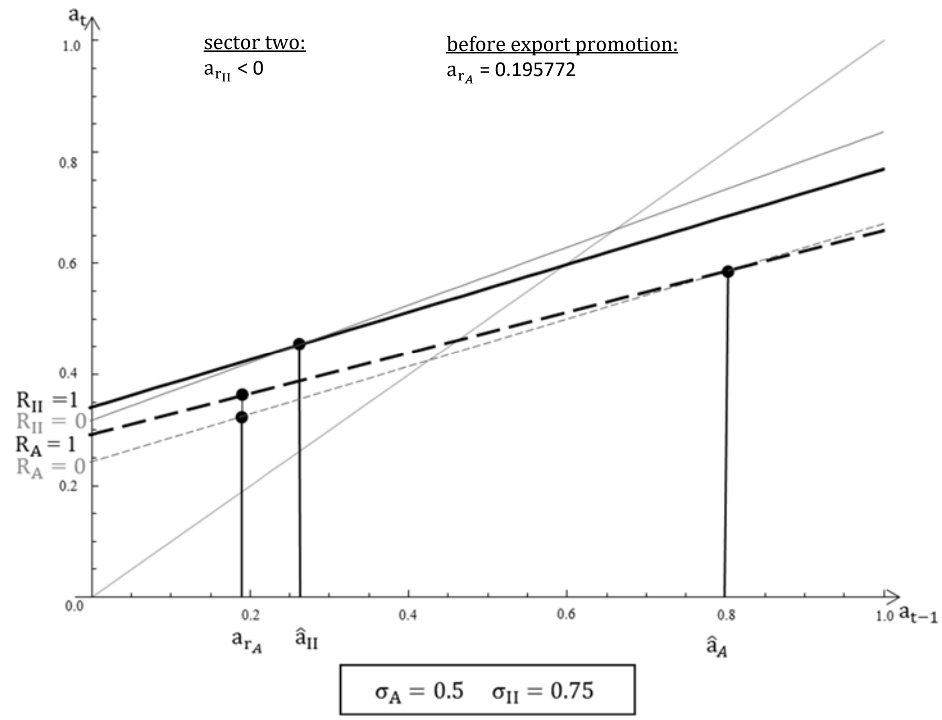
Only after this threshold the technological growth is higher by the innovation strategy (R_{tA}=0). By comparing the prior situation with the supported import sector there is a higher $\hat{a}_I$ than without trade. Because of the smaller projects, the intersection of both strategies $\hat{a}_j$ shifts to the right, with $\hat{a}_I$>1. The switching point is later referring to the technological development stage. Countries with a development stage smaller than $\hat{a}_I$, should follow the imitation strategy (R_{tj}=1). The imitation strategy leads to a higher productivity and should be followed anyway. This strategy focuses on the old low-skill entrepreneurs, which are retained. The experiences and financial support benefit more than potential innovative activities of young skill-unknown entrepreneurs. For this reason, the strategy is also called the investment-based strategy. Most of the products of sector one will be imported. Thus, the production of this good is not important as it is for good two.

Figure 5. *Trade with exogenous technology frontier – import sector*



with:
$\lambda = 0.5 \quad \phi = 0.5 \quad N = 1 \quad \eta = 4 \quad \gamma = 10 \quad r = 0.02 \quad \kappa = 1 \quad \delta = 0.5 \quad \mu = 0.5$

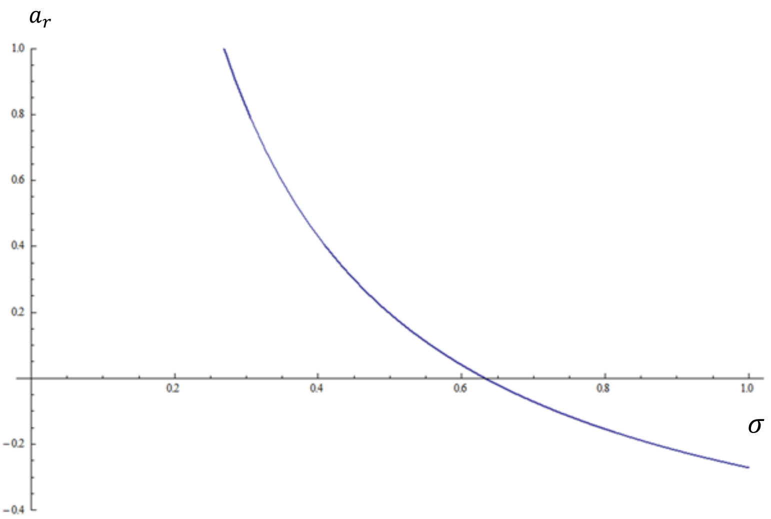
Figure 6. *Trade with exogenous technology frontier - export sector*



with:
$\lambda = 0.5 \quad \phi = 0.5 \quad N = 1 \quad \eta = 4 \quad \gamma = 10 \quad r = 0.02 \quad \kappa = 1 \quad \delta = 0.5 \quad \mu = 0.5$

Figure 6 illustrates the export sector two which increases by the export promotion. Here the continuous lines are above the dashed lines. For every development stage there is a higher increase of technological knowledge than before with smaller projects without the supported exports. The intersection shifts to the left and the threshold $\hat{a}_{II}$ is smaller than without trade. Only the less developed countries with a technological development stage $a_{t-1j} < \hat{a}_{II}$ should follow the imitation-based strategy (R_{tII}=1) up to $< \hat{a}_{II}$. Compared to the situation in sector one in sector two exist a very small $\hat{a}_{II}$. Thus, bigger projects induce an earlier change to the innovation-based strategy. Bigger projects lead to more profits; therefore, it is important to optimize the production process of the export sector. The old low-skill entrepreneurs will be replaced for young ones with expected new ideas. In this case it is worth to change old low-skill entrepreneurs with experience and financial means for young skill-unknown ones. The value of the retained earnings is smaller than the value of the potential innovations caused by young entrepreneurs. The advantages of technological growth outweigh because of the innovation-based strategy (R_{tII}=0). But the most important threshold is a_{rj} and considers the profit as in equation (26). By analyzing this threshold in particular there is an inverse relationship between a_{rj} and the project size σ, as you can see in figure 7.

Figure 7. *Impact of the project size*



Increasing project size σ induces a smaller a_{rj}. Because bigger projects are more expensive and higher investments are needed, it comes to an earlier switch of the strategies due to the cost. At this point it is relatively more important to have efficient methods of production and new technologies. Reflecting figure 5 the import sector one has a higher threshold a_{rj} than the situation without trade. So smaller projects need less investments and cause less costs. The imitation strategy is the suggested strategy for countries with a development stage $a_{t-1j} < a_{rI}$, which is higher than a_{rA}. We can observe countries, which follow the investment-based strategy under political control and have now a higher development stage than before. Hence smaller costs lead to higher gains and the change of the entrepreneurs is not yet needed. With a look at the export sector two, bigger project sizes lead to a smaller threshold a_{rII} compared to the situation before export support. Because the threshold a_{rII} is small enough (< 0) only the innovation strategy is plausible for bigger projects and the imitation strategy can be ignored. In the export sector two are no old low-skill entrepreneurs employed. Bigger projects, means a larger production volume, more sales and thus higher gains, despite higher costs. It is more profitable to produce with newer and more efficient technologies than using the old ones. The higher costs are compensated.

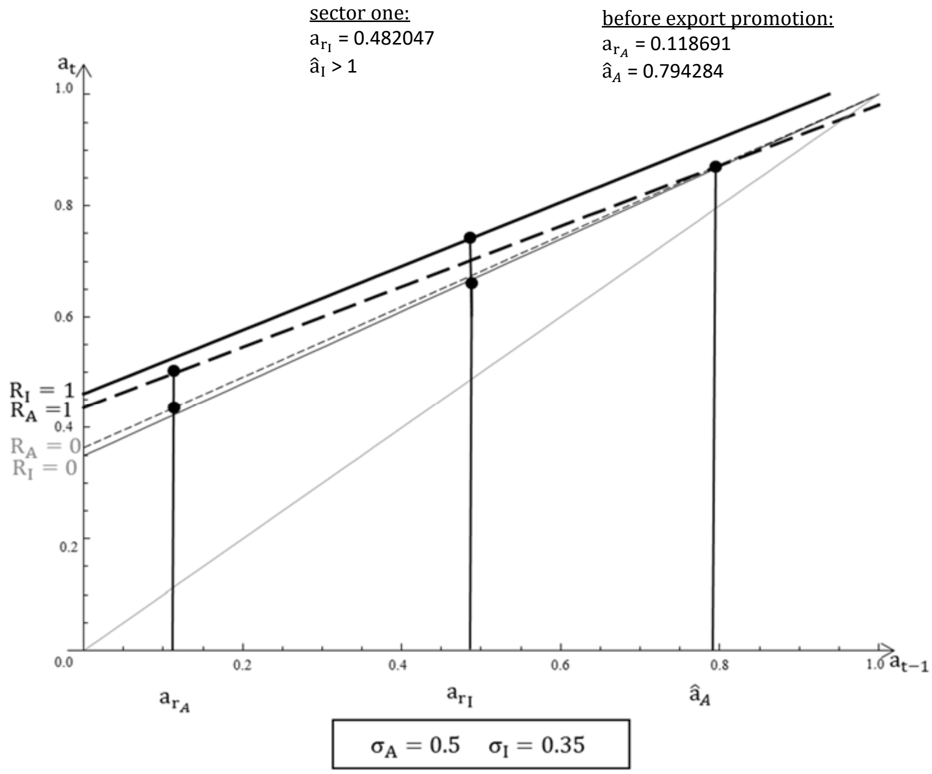
This example demonstrates that the export support encourages innovations. Even for small economic countries with less or no effects on the World Market there is a difference because of openness to trade. Especially for the export sector young firms and young entrepreneurs are much more important caused by the supported project size.

For this reason, the imitation strategy should not be neglected for the export sector. There is still a higher growth rate although the innovation-based strategy is not the preferred one. Thus, it is quite possible that technologically less developed countries also derive benefit from a supported export sector.

4.2. Technologically big countries with an endogenous world technology frontier

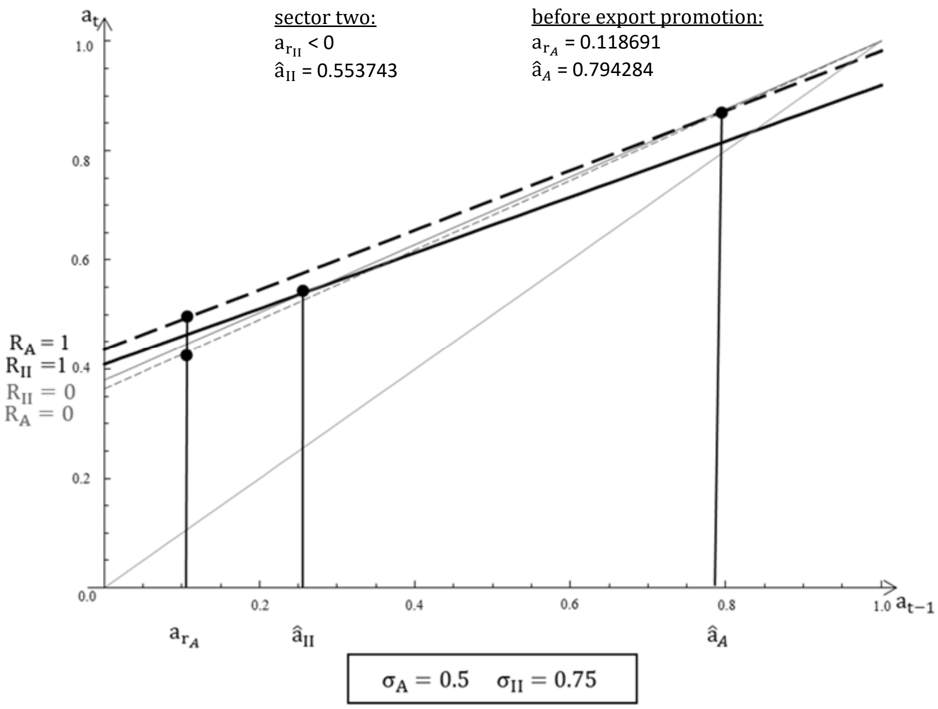
Quite different is the reaction of a technological big country on the endogenous world technology frontier. Assuming macroinventions, every innovation increases the world technology frontier. An export supporting strategy effects the endogenous frontier unambiguously as illustrated in figure 8. The smaller import projects in sector one lead to a higher prospective development stage by the imitation strategy over all possible development stages. By following the innovation-based strategy a degradation of all possible development stages results compared to the autarky situation. Thus, the old low skilled entrepreneurs are more important for the import sector in a country with export support. Even the thresholds a_{r_I} and $\hat{a}_j$ increase with the trade policy and indicate a later shift to the innovation-based strategy. To conclude, the import sector is characterized by the imitation based strategy and consequently focuses on long relationships with old low-skilled entrepreneurs.

Figure 8. *Trade with endogenous technological frontier - import sector*



with:
$\lambda = 0.5 \quad \phi = 0.5 \quad N = 1 \quad \eta = 4 \quad \gamma = 10 \quad r = 0.02 \quad \kappa = 1 \quad \delta = 0.5 \quad \mu = 0.5$

Figure 9. *Trade with endogenous technological frontier – export sector*



with:
$\lambda = 0.5 \quad \phi = 0.5 \quad N = 1 \quad \eta = 4 \quad \gamma = 10 \quad r = 0.02 \quad \kappa = 1 \quad \delta = 0.5 \quad \mu = 0.5$

This kind of general statement does not hold for the export sector, which is illustrated in figure 9. Even if the export promotion improves the productivity for each development stage compared to the autarky situation, the imitation based strategy for relatively less developed countries will be the preferable option. But by following the imitation based strategy the export sector of this economy has to deal with a smaller increase of the development stage compared to the autarky situation. Hence there is a negative effect of the export promotion on the imitation based strategy. By considering the thresholds $a_{rII} < 0$, the innovation-based strategy is the only reasoned strategy.

5. Conclusion

Trade affects development and economic growth. The underlying idea was to question if and how a country could benefit by trade. To clarify the trade effects export support is implemented in the modified growth model. As a result, we can deduce development strategies for different development stages of countries. To analyze the trade impact, we differ between technological small and big countries with exogenous respectively endogenous world technology frontier. The political intervention affected the sectors differently.

Independent of the development stage and the country size the export sectors focus on the innovation-based strategy and the import sectors on the imitation-based strategy. This paper shows that an export promotion strategy encourages innovations explicit in the export sector. The innovation-based strategy operates with young entrepreneurs and their hidden talents. Indeed, bigger projects need more investment than small ones, but a newly invented and more efficient method of production leads to reduced costs. So bigger projects are relatively less investment intensive. Especially economies of scale result from a rising project size. Consequently, a higher growth rate of technological knowledge will result. The investment-based strategy, which is the most recommended strategy for the import sector, is based on well-known techniques and methods which are implemented by the old low skill entrepreneurs.

The technological development of technological small countries with an exogenous world technology frontier are driven by the export sector. Whereas the import sector focus on the imitation-based strategy. The results for technological big countries with an endogenous world technology frontier are not that clear. The export sector of relatively less developed countries does not benefit by trade and the political support has only an effect after a specific development stage. To this point the technological development is driven by the imitations of the import sector. But overall big countries benefit by the export support, even if the technological development is mainly caused by the imitating import sector.

This paper proves that the project size is a driver for the growth of technological knowledge. The export support policy does not only stimulate a growing export sector that a country could benefit from. It stimulated also the incentives to innovate. Additionally, the paper shows, that imitations are at least as important as innovations for the technological development of a country.

To sum up all countries improve the development potential and benefit by trade.

Notes

(1) As well-known from the Stolper-Samuelson theorem (Samuelson and Stolper 1941).

(2) Romer 1986 says that technological knowledge supports growth because of learning-by-doing effects, among other things. Operating experiences raise the labor productivity and therefore a higher developed stand of technological knowledge is achieved for a country due to investing in physical capital. But it has also to be mentioned, that trade may have negative effects on growth. Young (1991), Matsuyama (1992) and, Galor and Mountford (2008) consider that the import sector fails to benefit from learning-by-doing opportunities.

(3) $R_t = \begin{cases} 0 & if E_t V_t^*(e=y) \geq V_t^*(e=o, z=L) \\ 1 & if E_t V_t^*(e=y) < V_t^*(e=o, z=L) \end{cases}$

(4) Another key factor by Acemoglu, Aghion, Zilibotti (2006) is the non-convergence trap a_{trap}. If the technological development stage is smaller than a_{trap} and following the investment-based strategy it will fall into a non-convergence trap. The economy converges to a specific level of technology and cannot reach the world technology frontier. According to that with $a_r < a_{trap}$ the non-convergence trap can be neglected. This will be the case in the following and this threshold will not be considered anymore.

$$a_{trap_j} = \frac{(1+\sigma_j)\eta}{2(1+g_j) - (1+\sigma_j)\lambda\gamma}$$

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Mathematical Appendix

The distance to frontier of sector j is defined as

$$a_{tj} = \frac{A_{tj}}{\overline{A}_t} \quad (29)$$

It is assumed that half of the firms are young

$$A_{tj} = \frac{A^y_{tj} + A^o_{tj}}{2} \quad (30)$$

and are using the following world technology frontier in sector j at time t

$$\overline{A}_t = \overline{A}_{t-1}(1-g). \quad (31)$$

Equation (29)-(31) taken together leads to:

$$a_{tj} = \frac{A^y_{tj} + A^o_{tj}}{2\overline{A}_{t-1}(1-g)} \quad (32)$$

It is to distinguish between retained or terminated entrepreneurs.

$$a_{tj}(R_{tj} = 1) = \frac{\sigma_j(\eta\overline{A}_{t-1} + \lambda\gamma A_{t-1j}) + (\eta\overline{A}_{t-1} + \lambda\gamma A_{t-1j})}{2\overline{A}_{t-1}(1+g)} \quad (33)$$

Considering $\overline{A}_t = 1$ and $A_{t-!j} = a_{t-1j}$ the distance to frontier for a retained old entrepreneur ($R_{tj} = 1$) results.

$$a_{tj}(R_{tj} = 1) = \frac{\sigma_j(\eta + \lambda\gamma a_{t-1j}) + \eta + \lambda\gamma a_{t-1j}}{2(1+g)} \quad (34)$$

and simplified, it yields:

$$a_{tj}(R_{tj} = 1) = \frac{1+\sigma_j}{2(1+g)}\left[\eta + \lambda\gamma a_{t-1j}\right] \quad (35)$$

Unlike equation (36) describes the distance to frontier for an old entrepreneur who is terminated ($R_{tj} = 0$):

$$a_{tj}(R_{tj} = 0) = \frac{\sigma_j(\eta + \lambda\gamma a_{t-1j}) + \lambda(\eta + \gamma a_{t-1j}) + (1-\lambda)\sigma_j(\eta + \lambda\gamma a_{t-1j})}{2(1+g)} \quad (36)$$

After simplification it appears:

$$a_{tj}(R_{tj} = 0) = \frac{1}{2(1+g)}\left[\lambda + \sigma_j + (1-\lambda)\sigma_j)\eta + (1 + \sigma_j + (1-\lambda)\sigma_j)\lambda\gamma a_{t-1j}\right] \quad (37)$$

and equation (25) is deduced.

$$a_{tj} = \begin{cases} \frac{1+\sigma_j}{2(1+g)}\left[\eta + \lambda\gamma a_{t-1j}\right] & \text{if } R_t = 1 \\ \frac{1}{2(1+g)}\left[\left(\lambda + \sigma_j + (1-\lambda)\sigma_j\right)\eta + \left(1 + \sigma_j + (1-\lambda)\sigma_j\right)\lambda\gamma a_{t-1j}\right] & \text{if } R_t = 0 \end{cases} \quad (38)$$

The variation of the different project sizes can be seen in the derivation of a_{tj}. The following paragraph there are differed two possibilities, the world technology frontier could be assumed as exogenous or endogenous. The mainly considered situation is an exogenous frontier. If the entrepreneurs are getting retained for ($R_{tj} = 1$)the a_{tj}-intercept increase by a rising project size:

$$\frac{\partial \chi_1}{\partial \sigma_j} = \frac{(1+\sigma)\eta}{2(1+g)} > 0 \qquad (39)$$

with the a_{tj}-intercept $\chi_1 = \frac{(1+\sigma_j)}{2(1+g)} * \eta$.

and the grade ascends also if the project size starts to rise.

$$\frac{\partial \xi_1}{\partial \sigma_j} = \frac{(1+\sigma)}{2(1+g)} * \left(\lambda * \gamma * a_{t-1j}\right) > 0 \qquad (40)$$

with the grade $\xi_1 = \frac{(1+\sigma_j)}{2(1+g)} * \lambda\gamma a_{t-1j}$.

The a_{tj}- intercept rises as well for the case of termination (R_{tj}=0) caused by bigger projects.

$$\frac{\partial \chi_0}{\partial \sigma_j} = \frac{\eta(2-\lambda)}{2(1+g)} > 0 \qquad (41)$$

with the a_{tj}-intercept $\chi_0 = \frac{1}{2(1+g)} * \left(\lambda + \sigma_j + (1-\lambda)\sigma_j\right)\eta$ and the grade rises if the project size also rises for (R_{tj}=0).

$$\frac{\partial \xi_0}{\partial \sigma_j} = \frac{a_{t-1j}\gamma(2-\lambda)\lambda}{2(1+g)} > 0 \qquad (42)$$

with the grade $\xi_0 = \frac{1}{2(1+g)} + \left(1 + \sigma_j + (1-\lambda)\sigma_j\right)\lambda\gamma a_{t-1j}$.

Now we consider the more complex case of a world technology frontier, which rises with every innovation.

$$a_{tj} = \begin{cases} \frac{1+\sigma_j}{2(1+g_j)}\left[\eta + \lambda\gamma a_{t-1j}\right] & \text{if } R_t = 1 \\ \frac{1}{2(1+g_j)}\left[\left(\lambda + \sigma_j + (1-\lambda)\sigma_j\right)\eta + \left(1 + \sigma_j + (1-\lambda)\sigma_j\right)\lambda\gamma a_{t-1j}\right] & \text{if } R_t = 0 \end{cases} \qquad (43)$$

Beginning with the exogenous case that the entrepreneurs are getting retained for (R_{tj}=1) the a_{tj}-intercept decrease by a rising project size:

$$\frac{\partial \chi_1}{\partial \sigma_j} = \frac{\partial \chi_1}{\partial \sigma_j} + \frac{\partial \chi_1}{\partial g_j}\frac{\partial g_j}{\partial \sigma_j} \qquad (44)$$

with the a_{tj}-intercept $\chi_1 = \frac{(1+\sigma_j)}{2(1+g_j)} * \eta$

$$\frac{\partial\chi_1}{\partial\sigma_j} = -\frac{\eta(\eta(2-\lambda)+\gamma(2-\lambda)\lambda)(1+\sigma_j)}{(\gamma\lambda(1+\sigma_j+(1-\lambda)\sigma_j)+\eta(\lambda+\sigma_j+(1-\lambda)\sigma_j))^2} + \frac{\eta}{\gamma\lambda(1+\sigma_j+(1-\lambda)\sigma_j)+\eta(\lambda+\sigma_j+(1-\lambda)\sigma_j)} \quad (45)$$

simplified: $\frac{\eta(\gamma\lambda(-1+\lambda)+\eta(-2+2\lambda))}{(\eta(\lambda(-1+\sigma_j)-2\sigma_j)+\gamma\lambda(-1+(-2+\lambda)\sigma_j))^2} < 0$ (46)

and the grade falls if the project size starts to rise.

$$\frac{\partial\xi_1}{\partial\sigma_j} = \frac{\partial\xi_1}{\partial\sigma_j} + \frac{\xi_1}{\partial g_j}\frac{\partial g_j}{\partial\sigma_j} \quad (47)$$

with the grade $\xi_1 = \frac{(1+\sigma_j)}{2(1+g_j)} * \lambda\gamma a_{t-1j}$

$$\frac{\partial\xi_1}{\partial\sigma_j} = -\frac{a_{t-1j}\gamma\lambda(\eta(2-\lambda)+\gamma(2-\lambda)\lambda)(1+\sigma_j)}{(\gamma\lambda(1+\sigma_j+(1-\lambda)\sigma_j)+\eta(\lambda+\sigma_j+(1-\lambda)\sigma_j))^2} + \frac{a_{t-1j}\gamma\lambda}{\gamma\lambda(1+\sigma_j+(1-\lambda)\sigma_j)+\eta(\lambda+\sigma_j+(1-\lambda)\sigma_j)} \quad (48)$$

Simplified: $\frac{a_{t-1j}\gamma\lambda(\gamma\lambda(-1+1.\lambda)+\eta(-2+2.\lambda))}{(\eta(\lambda(-1+\sigma)-2\sigma)+\gamma\lambda(-1+(-2+\lambda)\sigma))^2} < 0$ (49)

But for the case of termination (R_{tj}=0) the a_{tj}- intercept rises because of bigger projects.

$$\frac{\partial\chi_0}{\partial\sigma_j} = \frac{\partial\chi_0}{\partial\sigma_j} + \frac{\partial\chi_0}{\partial g_j}\frac{\partial g_j}{\partial\sigma_j} \quad (50)$$

with the a_{tj}-intercept $\chi_0 = \frac{1}{2(1+g_j)} * \left(\lambda + \sigma_j + (1-\lambda)\sigma_j\right)\eta$

$$\frac{\partial\chi_0}{\partial\sigma_j} = -\frac{\eta(\eta(2-\lambda)+\gamma(2-\lambda)\lambda)(\lambda+\sigma+(1-\lambda)\sigma)}{(\gamma\lambda(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma))^2} + \frac{\eta(2-\lambda)}{\gamma\lambda(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma)} \quad (51)$$

Simplified: $\frac{\gamma\eta\lambda(2-3\lambda+\lambda^2)}{(\eta(\lambda(-1+\sigma)-2\sigma)+\gamma\lambda(-1+(-2+\lambda)\sigma))^2} > 0$ (52)

and the grade also falls if the project size starts to rise for (R_{tj}=0).

$$\frac{\partial\xi_0}{\partial\sigma_j} = \frac{\partial\xi_0}{\partial\sigma_j} + \frac{\xi_0}{\partial g_j}\frac{\partial g_j}{\partial\sigma_j} \quad (53)$$

with the grade $\xi_0 = \frac{1}{2(1+g_j)} * \left(1 + \sigma_j + (1-\lambda)\sigma_j\right)\lambda\gamma a_{t-1j}$

$$\frac{\partial\xi_0}{\partial\sigma_j} = -\frac{a_{t-1j}\gamma\lambda(\eta(2-\lambda)+\gamma(2-\lambda)\lambda)(1+\sigma+(1-\lambda)\sigma)}{(\gamma\lambda(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma))^2} + \frac{a_{t-1j}\gamma(2-\lambda)\lambda}{\gamma\lambda(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma)} \quad (54)$$

Simplified: $\frac{a_{t-1j}\gamma\eta\lambda(-2+3\lambda-1.\lambda^2)}{(\eta(\lambda(-1+\sigma)-2\sigma)+\gamma\lambda(-1+(-2.+\lambda)\sigma))^2} < 0$ (55)

The threshold a_{rj} (26) works out the level of proximity to frontier, where termination of the relationship to an old low skill entrepreneur starts to be profitable. The decision if an entrepreneur is retained depends of the different values of a firm. The expected value of a young entrepreneur realizing small projects is:

$$E_tV_{tj}(\upsilon|s=\sigma, e=y) = (1-\mu)\delta_jN_j\sigma_j\left(\eta + \lambda\gamma a_{t-1j}\right)\bar{A}_{t-1} - \phi\kappa_j\bar{A}_{t-1} \quad (56)$$

And the value or an old low skill entrepreneur doing big projects is:

$$V_{tj}(\upsilon|s=1, e=o, z=L) = \left[(1-\mu)\delta_j N_j \eta \bar{A}_{t-1} - \max\left(\kappa_j \bar{A}_{t-1} - RE_t, 0\right)\right] \tag{57}$$

with the retained earnings of an old entrepreneur given by

$$RE_t = \frac{1+r}{1+g_j}\sigma_j \mu \delta_j N_j \eta \bar{A}_{t-1}] \tag{58}$$

$$V_{tj}(\upsilon|s=1, e=o, z=L) = \left[(1-\mu)\delta_j N_j \eta \bar{A}_{t-1} - \max\left(\kappa_j \bar{A}_{t-1} - \frac{1+r}{1+g}\sigma_j \mu \delta_j N_j \eta \bar{A}_{t-1}, 0\right)\right] \tag{59}$$

Simplified:

$$V_{tj}(\upsilon|s=1, e=o, z=L) = \left[(1-\mu)\delta_j N_j \eta - \kappa_j + \frac{1+r}{1+g}\sigma_j \mu \delta_j N_j \eta\right] \tag{60}$$

$$E_t V_{tj}(\upsilon|s=\sigma, e=y) = (1-\mu)\delta_j N_j \sigma_j\left(\eta + \lambda\gamma a_{rj}\right) - \phi\kappa_j \tag{61}$$

To compare both values, they will be equal:

$$V_{tj}(\upsilon|s=1, e=o, z=L) = E_t V_{tj}(\upsilon|s=\sigma, e=y) \tag{62}$$

$$\left[(1-\mu)\delta_j N_j \eta - \kappa_j + \frac{1+r}{1+g_j}\sigma_j \mu \delta_j N_j \eta\right] = (1-\mu)\delta_j N_j \sigma_j\left(\eta + \lambda\gamma a_{rj}\right) - \phi\kappa_j \tag{63}$$

$$\left[(1-\mu) + \frac{1+r}{1+g}\sigma_j \mu\right]\delta_j N_j \eta - \kappa_j + \phi\kappa_j = (1-\mu)\delta_j N_j \sigma_j \eta + (1-\mu)\delta_j N_j \sigma_j \lambda\gamma a_{rj} \tag{64}$$

$$\left[(1-\mu)\left(1-\sigma_j\right) + \frac{1+r}{1+g}\sigma_j \mu\right]\delta_j N_j \eta - \kappa_j(1-\phi) = (1-\mu)\delta_j N_j \sigma_j \lambda\gamma a_{rj} \tag{65}$$

and getting the threshold: $a_{rj} = \frac{\left[(1-\mu)(1-\sigma_j)+\frac{1+r}{1+g}\sigma_j\mu\right]\eta - \frac{\kappa_j(1-\phi)}{\delta_j N_j}}{(1-\mu)\sigma_j\lambda\gamma}$ (66)

The connection between the project size σ and the threshold a_{rj} is shown by the derivation of a_{rj}.

$$\frac{da_{rj}}{d\sigma} = \frac{\partial a_{rj}}{\partial\sigma} + \frac{\partial a_{rj}}{\partial g} * \frac{\partial g}{\partial\sigma} + \frac{\partial a_{rj}}{\partial\delta}\frac{\partial\delta}{\partial\sigma} \tag{67}$$

$$\frac{(\eta+\lambda\gamma)(1-\mu)}{\gamma\lambda(1-\mu)\sigma} + \frac{\eta\left(-1+\mu-\frac{2(1+r)(\eta(2-\lambda)+(2-\lambda)\lambda\gamma)\mu\sigma}{\left(\lambda\gamma(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma)\right)^2}+\frac{2(1+r)\mu}{\lambda\gamma(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma)}\right)}{\gamma\lambda(1-\mu)\sigma} - \frac{-(\eta+\lambda\gamma)(1-\mu)(1-\sigma)+\eta((1-\mu)(1-\sigma)+\frac{2(1+r)\mu\sigma}{\lambda\gamma(1+\sigma+(1-\lambda)\sigma)+\eta(\lambda+\sigma+(1-\lambda)\sigma)})}{\gamma\lambda(1-\mu)\sigma^2} \tag{68}$$

and simplified as:

$$-\left[\frac{0+\frac{\eta^2(-4+2\lambda+i(-4+2\lambda))\mu\sigma^2}{(\eta(\lambda(-1+\sigma)-2\sigma)+\lambda\gamma(-1+(-2+\lambda)\sigma))^2}}{\gamma\lambda(-1+\mu)\sigma^2} + \frac{\lambda\gamma(1+\mu(-1+\frac{\eta(-4+2\lambda+i(-4+2\lambda))\sigma^2}{(\eta(\lambda(-1+\sigma)-2\sigma)+\lambda\gamma(-1+(-2+\lambda)\sigma))^2}))}{\gamma\lambda(-1+\mu)\sigma^2}\right] \tag{69}$$

Since the derivation is negative there is an inverse connection between the project size and the a_{rj}, seen in Figure 7.