Multifactorial analysis of the price formation in the terms of a risk-free rate

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Abstract. The prices being studied on the market must be tested in such a way as to identify the risks that exist, to identify the influencing factors and according to them to be able to assess whether the diversification of some prices on the market is real or is a momentary situation. The expected return of macro factors is not restricted by the null hypothesis and in this respect it is shown that this null hypothesis indicates to the investor the conditions to be taken into account in analysing the prices at which they place portfolios on the stock market, capital market or not. The factors can be grouped and they must be tested from a statistical point of view, to see if the parameters we have calculated can be a decision criterion in making the decision to place by purchase, to place by sale or to buy shares or other assets constituted in other types of portfolios. The models usually used are regression models that ensure the estimation and inference on the market, so to draw a definite conclusion on how it can be appreciated that prices are realistic, are those that have the level pursued by the investor in the sense of increase or decrease. Always, an analysis using the regression model is supplemented with a spectral analysis model to find the seasonal variation that could occur in the capital market. The purpose of this article is to test prices influenced by several factors and to identify a rate and a time when these prices are not fully risky.

Keywords: prices, capital market, returns, risks, models.

JEL Classification: C10, F40.
Introduction

We used the comparative study, the logical study of portfolios and assets on the capital market and we used the presentation of spectral and regression models that can be used in this regard by estimating the prospects to reach the optimal point of placement on the capital market, by selling or buying assets.

The price test is explained, one by one, in this article, showing that the model can be expressed in several forms.

We have resorted to some papers, such as Fama or French, which demonstrate that there are or are not high indices of return to the dimensions and values we have studied, a context in which it is suggested to adapt the analysis models so that they can or those that give meaning to the study conducted by the investor.

The article, in turn, covers all aspects of cross-cutting regressions useful for obtaining the factor risk premium if they are not traded and, based on it there is a logic of non-action on the capital market.

The factors that are chosen in construction are made explicit and concretized by simple examples, with graphic representations that give meaning to the analysis performed and ensure a more certain understanding of the meaning that the author pursued in the study of the capital market.

The testing of hypotheses, and therefore of prices, is a very important element and must take into account most micro and macroeconomic factors and study what the risk rate would be, being able to identify those moments when risks are triggered and can have large, special influences on the return that will be obtained from the placement on the capital market of the considered portfolios.

The value factor is always represented as the difference in profitability in a portfolio of stocks at high prices and a portfolio of stocks at low prices, in order to be able to inventory and estimate on the spot what is the return that the investor can obtain in entering the market trading in those portfolios.

It is sometimes considered that the averages of the risk factors are statistically significant over the entire period, for the period considered.

However, considering an unexpected inflation, we introduce in the regression parameter the effect that inflation can have. In the capital market, however, we can also talk about disinflation, in the context in which some assets, some portfolios lose their essence and can reach lower price levels.

Literature review

A number of researchers have turned their attention to price formation in the capital market. Thus, Andersen and Bollerslev (1998) pointed out that the square of daily returns is not an
exact measure of real volatility, as they are calculated at closing prices and therefore do not capture intra-day fluctuations of price.

Anghel, Iacob, and Dumbravă (2020) dealt with the creation of a model for the analysis of assets and decisions that can be taken in the study and forecasting of the capital market under the impact of inflation.

Bardsen et al. (2005) deal in their study with the construction and use of econometric models at the microeconomic level, and Engle (2001) publishes extensive material on the use of econometric models in the study of aspects of the capital market.

Fama and French (1993) are concerned with common risk factors in stock and bond yields.

Goyal and Santa-Clara (2003) approach in their paper a new perspective on the predictability of stock market returns with risk measures.

Granger and Poon (2003) analysed the literature on exchange rate volatility forecasting and concluded that models behave differently for different currencies.

Kasman, Kasman and Torun (2009) investigated the presence of dual long memory in eight emerging EU markets.

Martens and Zein (2004) showed that high frequency data can improve both measurement accuracy and forecast performance.

Hou Xue, and Zhang (2015) propose an empirical factor model consisting of the market factor, a size factor, an investment factor and a profitability factor, which largely summarize cross section of the average stock yield.

Methodology, data, discussions, results

We consider the case where there is a risk-free rate and some of the factors are the traded portfolios \((f_{1t} \in \mathbb{R}^{K_1})\), while others are macro factors \((f_{2t} \in \mathbb{R}^{K_2})\) which are not traded assets. The expected yield of macro factors is not restricted by the null hypothesis.

In this case, the unrestricted model is:

\[
R_t - R_{ft} = \alpha + B_1(f_{1t} - R_{ft}) + B_2f_{2t} + \varepsilon_t
\]

\(E(f_{2t}) = \mu_f\)  \hspace{1cm} \text{(1)}

The null hypothesis consistent with APT in this case is:

\[
\alpha = B_2r_2, \quad \text{for some strangers } r_2 \in \mathbb{R}^{K_2}. \hspace{1cm} \text{(2)}
\]

We can solve for \(r_2\) explicitly by \(r_2 = (B_2^T B_2)^{-1} B_2 \alpha\), assuming that \(B_2\) is of complete rank. It follows that the restrictions can be rewritten as \(M_2 \alpha = 0\), where
Matrix \( M_{B_2} \) of size \( N \times N \) is symmetrical and of rank \( N - K_2 \). Substituting \( \alpha \) in the return equation we obtain:

\[
R_t - R_{ft} = B_1(f_{1t} - R_{ft}) + B_2(f_{2t} - r_2) + \varepsilon_t
\]

which is linear in \( B \) given by \( r_2 \) (and linear in \( r_2 \) given by \( B_2 \)).

The restricted model can be estimated by the first conditioning \( r_2 \) and explicit resolution for \( \hat{B}(r_2) \) and then optimization according to parameters \( r_2 \). The simplest way to test this hypothesis is to use probability ratio statistics, which are unsympathetic - normal with \( N - K_2 \) degrees of freedom.

Next, consider the Wald test or rather a version of the Wald test that does not require the explicit use of the restriction. \( M_{B_2}\alpha = 0 \) (which is problematic due to the low rank).

Keeping \( \hat{\theta}_t = (\alpha_t, b_{21}^T) \), \( \hat{\theta} = (\hat{\theta}_1, ..., \hat{\theta}_N) \) \( \in \mathbb{R}^{(K_2+1)N} \) and \( \delta = (r_2^T, b_{21}^T, ..., b_{2N}^T) \) we will have the relationship:

\[
W = T \min_{\delta \in \mathbb{R}^{K_2(N+1)}} \left( \hat{\theta} - h(\delta) \right)^T \hat{H}^{-1}(\hat{\theta} - h(\delta))
\]

where:

\[
h(\delta) = (b_{21}^T r_2, b_{21}^T, ..., b_{2N}^T r_2, b_{2N}^T)^T \quad \text{and} \quad \hat{H} = \Omega_e(X^T X)^{-1}.
\]

If \( T \to \infty \), then \( W \to \chi^2_{N-K_2} \).

The two-section approach is essentially the same as in the Fama-Macbeth procedure for CAPM. In this case, including the square functions of all betas is quite cumbersome and usually not tried. However, the cross-section method may be useful for testing whether or not a factor determines a risk premium. For this we estimate \( b_{ij} \) by regressions by time series for each stock and we denote by \( \hat{B} \) matrix \( N \times K \) of estimates. Then, for each \( t \) we determine the risk premium of the ex post form factor:

\[
\hat{r}_t = (\bar{X}^T \bar{X})^{-1} \bar{X}^T Z_t \in \mathbb{R}^{K+1}
\]

where:

\( Z_t \) is the \( n \)-vector of excess yields and \( \bar{X} = (i, \hat{B}) \)

Under these conditions, we maintain the relationship:

\[
\hat{\rho} = (\hat{\rho}_0, ..., \hat{\rho}_K)^T = \frac{1}{T} \sum_{t=1}^{T} \hat{r}_t
\]

\[
\bar{\rho} = \frac{1}{T} \sum_{t=1}^{T} (\hat{r}_t - \hat{\rho}) (\hat{r}_t - \hat{\rho})^T
\]

In the end we will test if \( \tau_j \) is zero, using t-statistical FM statistics: \( \frac{\hat{\tau}_j}{\sqrt{\bar{\rho}_{jj}}} \), which is the normal standard asymptotic in the null hypothesis that \( \tau_j = 0 \).
Cross-sectional regressions are useful for obtaining the risk factor premium if no assets are traded.

In a series of papers, Fama and French (hereinafter referred to as FF) demonstrate that there were high rates of return associated with size and value. These size and value-for-profit indices are evident in the data for the period covered by the CRSP and Compost (FF (1992)) database, in previous US data (Davis (1994)) and in the non-US stock markets (FF (1998); Hodrick, Ng and Sangmueller (1999)).

FF (1993, 1996, 1998) argues that these profitability indices can be attributed to a rational asset pricing paradigm, in which size and value characteristics are proxies for asset sensitivities to sources of risk in the economy. This approach is now widely used in the analysis of empirical finance.

We will further present how they chose their factors. FF (1993) builds six double-sorted portfolios, formed on portfolios with two sizes and three to trade (B / M). They first sorted the stocks according to their size and divided them into two groups: large size and small size. Then we sort each of these groups according to the B / M value and divide them into three additional groups.

Sorting can also be done in reverse and in both ways six groups of stocks are obtained, according to Table 1.

<table>
<thead>
<tr>
<th>B / M</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>high/high</td>
<td>high/low</td>
</tr>
<tr>
<td>2</td>
<td>medium/large</td>
<td>medium/small</td>
</tr>
<tr>
<td>3</td>
<td>low/high</td>
<td>low/low</td>
</tr>
</tbody>
</table>

The stocks in each case are then weighted equally to produce six portfolios, one measuring the large and high B / W size, and on low-capitalized stocks and a portfolio of high-capitalized stocks, adjusted to have approximately equal price ratios (SMBs). Specifically, it is given by the relation:

$$SMB = \frac{1}{3} \left( \frac{\text{dimension high}}{B/M} - \frac{\text{dimension low}}{B/M} \right) + \frac{\text{dimension high}}{B/M} - \frac{\text{dimension low}}{B/M} + \frac{\text{dimension low}}{B/M} - \frac{\text{dimension high}}{B/M} + \frac{\text{dimension high}}{B/M} - \frac{\text{dimension low}}{B/M} + \frac{\text{dimension low}}{B/M} - \frac{\text{dimension high}}{B/M},$$

which we can write using the table as \([(1,1)-(1,2)+(2,1)-(2,2)+(3,1)-(3,2)]/3.

The value factor is the difference in profitability between a high-priced stock portfolio and a low-priced stock portfolio, adjusted to have an approximately equal capitalization (HML), i.e.:

$$HML = \frac{1}{2} \left( \frac{\text{dimension high}}{B/M} - \frac{\text{dimension low}}{B/M} \right) + \frac{\text{dimension high}}{B/M} - \frac{\text{dimension low}}{B/M} + \frac{\text{dimension low}}{B/M} - \frac{\text{dimension high}}{B/M}$$

which can be written as \([(1,1)-(3,2)+(1,2)-(3,2)]/2.
These factors are both portfolios of traded assets and can be written \( \sum w_{ts}R_{it} \) for certain weighting sequences \( w_{ts} \), to satisfy \( \sum_{t=1}^{N} w_{ts} = 0 \) is a zero cost portfolio.

Return to market factor is indicated by excessive return on a value-weighted market index (MKT).

SMB and HML for July of the year to June a \( t + 1 \) include all stocks (NYSE, AMEX and NASDAQ) for which there are market capitalization data for December \( t - 1 \) and June \( t \) and positive data (equivalent) for capital \( t - 1 \).

In Figure 1 we present the real percentage yields of the three factors, as well as the cumulated yields.

**Figure 1. Return to the market factor**

In Table 2 we present the annual average and the standard deviation of the monthly profitability series in the considered period.

To make the comparison between MKT and SMB HML, we added the risk-free rate in market profitability. We find that the average profitability of SMB and HML portfolios is quite low compared to that for market profitability, although they are consequently less risky.
This is not surprising, as both are arbitrage portfolios (positive weights for some securities cancelled out of negative weights over others). Indeed, during this period, the SME portfolio seemed to do even less than the risk-free asset.

Table 2. Annual average and standard deviation of the monthly return series

<table>
<thead>
<tr>
<th>Assets</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKT+Rf</td>
<td>11.1742</td>
<td>18.5558</td>
</tr>
<tr>
<td>SMB</td>
<td>2.5374</td>
<td>11.1241</td>
</tr>
<tr>
<td>HML</td>
<td>4.7858</td>
<td>12.1354</td>
</tr>
<tr>
<td>Rf</td>
<td>3.3313</td>
<td>0.8824</td>
</tr>
</tbody>
</table>

Table 3 shows the correlation matrix of the factors. This indicates that there is a fairly low correlation between the three assets.

Table 3. Factor correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>MKT+Rf</th>
<th>SMB</th>
<th>HML</th>
<th>Rf</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKT+Rf</td>
<td>1</td>
<td>0.3173</td>
<td>0.2421</td>
<td>-0.0176</td>
</tr>
<tr>
<td>SMB</td>
<td>0.1217</td>
<td>1</td>
<td>-0.0516</td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>0.0183</td>
<td>-0.0516</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Rf</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Figure 2 we present statistics of the variance ratio for the yield of the three factors. This shows the predictability of the HML portfolio over fairly short time horizons.

Figure 2. Variance ratio statistics

After identifying the factors, we form 25 test portfolios (by size and value characteristics) and it is done by multivariate regression:

$$R_{pt} = \alpha_p + \beta_{p,SMB}SMB_t + \beta_{p,HML}HML_t + \beta_{p,MKT}MKT_t + \epsilon_{pt}$$

(9)

This is a regression model of the panel with observed covariates, which are the same in each equation p, so that the estimation and inference are performed as described. APT
restrictions are tested ($\alpha_p = 0$) using statistics F. Thus, the size and value anomalies of CAPM are well explained.

However, subsequent work rejected APT restrictions in the three-factor model using different test portfolios, i.e. the model is not consistent with other anomalies. Recent work has extended the three FF factors to four, five, and even more to be included in the multiple regression.

For example, the impulse factor of Carhart (1997), the volatility factor of Goyal and Santa-Clara (2003), the liquidity factor of Amihud and Mendelson (1986) and Pastor and Stambaugh (2003). Hou, Xue and Zhang (2015) propose an investment factor. Fama and French (2015) themselves propose a five-factor model that has a similar motivation.

Another approach is to use macroeconomic variables as common sources of risk that determine asset prices. A problem that arises is that these series are often nonstationary and thus would result in an unbalanced equation for yields (which are usually much closer to stationary).

Chan, Chen and Hsieh (1985) and Chen, Roll and Ross (1986) used a number of surprise form factors, respectively:

$$f_t = m_t - E_{t-1}(m_t)$$

where:

$m_t$ are observable macroeconomic variables (potentially non-stationary).

Through this construction, we can anticipate that $f_t$ it is stationary. But, more importantly, the factors thus selected represent unanticipated information at time $t - 1$ and are not yet part of the prices. In this way, we seek to associate the new information in the factors with the yield achieved in the same period, as we do in the market model and the FF model.

In some cases, they take $E_{t-1}(m_t) = m_{t-1}$, such that $f_t$ contains only differentiated data. However, for inflation, the Fama and Gibbons (1982) model is used. This model is given by the relationship:

$$I_t = \alpha_{t-1} - \beta TB_{t-1} + \eta_t$$

where:

$I_t$ observes a monthly inflation during the period $t$, $TB_{t-1}$ is known at the time $t - 1$ and $\alpha_{t-1}$ is the negative of the unobserved real interest rate, which evolves according to the random circuit, i.e. at $\alpha_t = \alpha_{t-1} + u_t$, where the processes $u_t$ and $\eta_t$ are mutually independent with zero mean variation and finite dispersion.

The previous value of inflation is $\tilde{\alpha}_{t-1} - \tilde{\beta} TB_{t-1}$, where $\tilde{\beta}$ is estimated from differentiated data and $\tilde{\alpha}_{t-1}$ is obtained by the Kalman filter.

In Figure 3 we present the monthly inflation rate in the period 1950-2020.
The series is quite persistent compared to stock returns, and the first-order autocorrelation is around 0.5. The increase in industrial production (monthly series) is just as persistent in relation to stock returns.

The practice seems to be to ignore the preliminary estimation error in the calculation $f_t$, which can prejudice subsequent results (Pagan, 1984). Chan et al. (1985) used monthly data for the period 1958-1984.

Specific: percentage change in industrial output (remaining with a period), a measure of unexpected inflation expected change in inflation, difference in yield on low-yield corporate bonds and long-term government bonds and difference in yield on long-term government bonds long-term and short-term bonds.

20 firm-sized test portfolios were used at the beginning of the period. They estimate $b_t$ depending on the regressions of the time series and then perform regressions in section on $\hat{b}_t$ to estimate the risk premiums of the factor. Some of the macroeconomic factors are not traded assets.

They consider that the first averages of the risk factor are statistically significant over the whole period for industrial production and unexpected inflation. These include a market return, but notes that the associated risk index is not significant when risk indices associated with macroeconomic factors are included.

These models are commonly used in the field of asset management, i.e. for portfolio risk management, in combination with other techniques. Rosenberg (1974) considered the multifactorial regression model, where $f$ is treated as unknown parameters and B is related to the characteristics of observable stocks.

Other possible features include size and value, which are observed for all stocks.
Let's suppose that \( Z_{it} = \alpha_i + \sum_{j=1}^{K} b_{ij} f_{jt} + \epsilon_{it} \) is also valid for \( i = 1, \ldots, N \), getting the relationship:

\[
b_i = Dx_i,
\]

where:

\( x_i \) is the vector of the observed characteristics \( J \times 1 \) and \( D \) is an array \( K \times J \) of unknown parameters.

Substituting in the return equation, we obtain:

\[
Z_{it} = \alpha_i + b_i^T f_t + \epsilon_{it} = \alpha_i + x_i^T D^T f_t + \epsilon_{it} = x_i^T f^*_t + \epsilon_{it}
\]

where:

\( f^*_t = D^T f_t \) is a vector \( J \times 1 \) with specific factors for \( t = 1, \ldots, T \).

We can also write this as a transverse regression of form:

\[
Z_t = \alpha + X f^*_t + \epsilon_t
\]

for each time period \( t \), where \( X \) is the matrix \( N \times J \) of the specific characteristics of the observed firm.

It turns out that:

\[
E(Z_t/X) = \alpha + X \mu_{f^*} \; ; \; \text{var}(Z_t/X) = X \Omega_{f^*} X + \Omega_{\epsilon}
\]

where:

\( \mu_{f^*} \) and \( \Omega_{f^*} \) are its mean and variation \( f^*_t \), which is not supposed to vary over time.

This structure can be used in choosing the portfolio. That is, we can form portfolios to minimize conditional variation, given that a certain conditional average is obtained.

We'll assume that \( \alpha \) are orthogonal to the features so that \( X^T \alpha = 0 \). In this case we get:

\[
P_X Z_t = X f^*_t + P_X \epsilon_t \; ; \; M_X Z_t = \alpha + M_X \epsilon_t
\]

where:

\( P_X = X (X^T X)^{-1} X^T \) and \( M_X = I - P_X \) are the projection matrices associated with \( X \).

We can estimate the unknown yields of the characteristics \( f^*_t \) using the linear regression of the OLS section at the time moment, respectively:

\[
f^*_t = (X^T X)^{-1} X^T P_X Z_t = (X^T X)^{-1} X^T Z_t, \quad \tilde{\alpha} = M_X \tilde{Z}
\]

where:

\( \tilde{Z} = \sum_{t=1}^{T} Z_t / T \).
To reduce consistency $f_1^*$, we need that $N \to \infty$, which effectively means the use of individual assets rather than portfolios. It must also be at least $\lambda_{\text{min}}(B^TB) \to \infty$ with $B$ replaced with $X$. For its consistency $\alpha$, the $T$ should also be large.

Connor, Hagmann and Linton (2012) generalize this idea to allow beta functions to depend in an unknown way on the observed characteristics, according to the relation:

$$Z_{it} = f_{it} + \sum_{j=1}^{J} \beta_j(X_{ij})f_{jt} + e_{it}$$

A nonparametric method is proposed for estimating beta functions and then a transverse regression to obtain the factors. It is found that $\beta_j$ estimated to have nonlinear shapes. In this model, the expected yield varies continuously depending on the features, so:

$$\frac{\partial}{\partial x_j} E(Z_{it}|X_t = x) = \beta'_j(x_j)f_{jt}$$

If $X_j$ is the size characteristic then (19) measures how the expected return changes with size.

Conclusions

The first conclusion is that the analysis and study of price formation must be based on models with several factors of micro or macroeconomic influence.

Even if we analyse an influence of prices at the microeconomic level, we must also introduce a variable factor of macroeconomic problems that are essential in the transfer of the influence they have in the activity at the microeconomic level.

It is very clear that the study on price formation must be based on statistical-econometric models, regression or sequential, sectorial analyses that would allow a precise decision, at the time intervals for which the price evolution was estimated.

The capital market is one that also has the influence of the seasonal nature of some factors, for certain portfolios and not for the entire mass of portfolios in the capital market, which can act differently over a period of time.

For this, a spectral analysis must be used, showing price increases and decreases by virtue of the influence of micro and macroeconomic factors, the relationship between supply and demand on the capital market and, last but not least, the seasonal effect of a certain factor, or a group of factors on the change in prices.

It is clear that the change in prices is of the utmost importance as only in this way can a correct estimate be made of the evolution over time of the prices at which they will be traded or not on the capital market.
References


