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Multifractal analysis of equities. Evidence from the emerging and frontier banking sectors

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Abstract. *In this paper we analyse the degree of market efficiency present in bank equities for selected emerging and frontier economies. Employing the Multifractal Detrended Fluctuation Analysis (MFDFA), we inspect bank equity across thirty-seven banks in these economies. The MFDFA methodology enables to characterize the strength of multifractal complexities and to identify the sources of multifractal behavior. Furthermore, the time evolution of the Hurst exponents are calculated for banks having long-term temporal correlation as a significant source of multifractality. The results confirm that the time variant Hurst exponent can serve as a leading indicator for guiding investment decisions.*

Keywords: multifractality analysis, market efficiency, emerging and frontier markets, Hurst exponent, MF-DFA.

JEL Classification: G1, G2, G14.

Introduction

The fundamental way in which investors look at the world is by distinguishing the major market types i.e. Developed, Emerging and the Frontier Markets. In the last 2 decades, globalization and financial liberalization has ushered in an era marked by capital flows across borders; particularly towards the Emerging and Frontier Markets. What makes these markets so attractive apart from their high growth rates, is the fact that Emerging and Frontier Markets represent about four-fifths of the world population, and they account for nearly seventy percent of the global foreign exchange reserves. With India and China being strong consumption driven economies, Brazil, Nigeria, South Africa are the commodity rich nations, these countries today pull a large portion of foreign investment into their markets offering the prospect of exponential returns. These economies have managed to keep themselves independent to a large extent, from the effects of the global financial crisis because they were not so reliant on the Western banking system. Also, these countries have some of the world's leading commercial banks operating in their borders, offering a variety of financial products and services.

It is due to this the banks have found extensive success in their respective economies and make for attractive investment opportunities. Banks play a crucial role in many of the emerging and frontier economies because these economies unlike the west, are more of a savings driven economy. Therefore, this is where the financial intermediaries come into picture, mobilizing the savings from the public and channelling them to enable investment activities. To be able to deliver these services efficiently, a pre-requisite is sufficient capital. Capital raised via the equity channel has proven benefits such as lower funding costs, higher operative capacities and further lending possibilities which improves the performance of the banks in comparison to those institutions which operate with higher leverage ratios. Better performing banks make for attractive investment opportunities and the cycle reinforces back on itself giving better returns and prospects for the investors.

Efficient markets promote raising capital at a low cost and for this purpose, the financial markets and the banking system must be a tightly knit unit. Efficient markets are said to reflect all available information in their prices, implying that no single investor can beat the market. Secondly, the prices in an efficient market are also said to be random walk in nature, meaning they are impossible to predict using past values. The scenario in the real world is far from ideal though. Markets are prone to many forms of inefficiencies such as the momentum strategy, the "January Effect", market crashes and asset bubbles. Investors have also proven time and again that the markets can be outperformed. Thus, it is imperative to be able to exploit these inefficiencies to ones own advantage in order to get the best possible returns from the market and surrounding this, an entire area of study has emerged which is concerned with the understanding of stock price movements and market behavior. Models such as the CAPM, APT help price assets while models such as GARCH help modelling risk. But, these models being based on a set of unrealistic assumptions which may not always hold true, the models fail to accurately characterize the true nature of a financial time series.

To overcome this flaw, it was in the 1960s that Benoit Mandelbrot began developing the study of fractals as a tool to analyse financial markets. He was successful in coming up with a price model which made no prior assumptions about the data, had no constraints in its application and was accurately able to replicate the movement of prices when forecasted(1). This formed the bedrock upon which fractal models for financial data have been constructed and applied by several scholars over the years. It is with this idea serving as our foundation and the sheer importance of equity capital in enabling banks to deliver efficient services(2) serving as our motivation that we have undertaken a multifractal analysis of banking equities from selected Emerging and Frontier Markets to test for inefficiencies and identify investment opportunities.

The rest of the paper proceeds as follows: Section 2 provides a brief review of literature covering the application of multifractal methodologies to various asset classes such as derivatives, equities, interest rates, exchange rates and bonds. Section 3 highlights the data chosen for the purpose of the study. Section 4 expands upon in detail about the Multifractal Detrended Fluctuation Analysis and Section 5 presents the results and our observations from the study. Section 6 offers the concluding remarks.

Review of literature

Fama (1988) hypothesized the three forms of efficient markets stating that market prices of a security in-corporate all available information, therefore are random in nature making analytical and predictive tools pointless. The Efficient Market Hypothesis has been extensively studied using a variety of statistical and econometric tools in order to prove the existence of efficient markets or lack thereof. Lo (1999) has proven that prices indeed are not completely random and as Farmer (1999) has pointed out, a reason for this seemingly lack of efficiency in markets could be because the EMH itself is not a well posed and empirically refutable hypothesis. For the EMH to be operational, they highlight that it requires additional specifications such as investor preferences, information structure, etc.

One alternative to capturing the dynamics present in the financial markets are Agent Based Models which focuses on how human interactions and psychology influences economic decision-making process (see Lo 1991). Another path to understanding the behavior displayed and quantifying this behavior is the 'Fractal Market Hypothesis' proposed by Peters (1991) which focuses on different time horizons of different agents interacting in the market. Thus, in order to realistically model asset price movement, Mandelbrot (1997) proposed the Multifractal Model of Asset Returns which brought to light fresh aspects present in financial time series such as long memory in volatility(3), scale-consistency and multi-scaling. These properties are what make security prices in the markets fractal in nature. The presence of fractal properties has strong implications on our understanding of risk management and forecasting. Since then, various scholars have contributed to the literature on multifractals by detecting the presence of multifractals in financial time series using methods such as the Rescaled Range Analysis (Lo, 1991;Onali and Goddard, 2009), the Standard Partition function approach (Jiang, 2008; Wei, 2013), the Wavelet Transform Modulus Maxima approach (Los, 2004; Nikolaidis, 2010), the Detrended Fluctuation Analysis (Grech, 2008; Kim et al., 2011) and more recently the Multifractal Detrended Fluctuation Analysis (hereafter referred to as MFDFA).

Fractals are part of a growing extension of application of methods borrowed from physics in order to understand economic and financial systems[(4)]. Since then, several studies have been conducted to identify the fractal behavior displayed by different asset classes in financial markets. Although the literature, to our best knowledge, is sparse in terms of derivatives, an MFDFA of the KTB12 as a derivative security and the USB as a financial security (Thai Derivative Markets) revealed that both the derivatives and spot markets display different multifractal patterns and that the derivatives are non-linear in nature. The derivative also displayed a stronger multifractal degree as compared to the spot market instrument (Lim et al., 2007). The presence of multifractality was also confirmed in the Chinese agricultural futures market and the major source for such behavior was found to be long run temporal correlation. The futures also displayed different dynamics for large and small fluctuations respectively (Chen, 2010).

Analysis of commodities have shown that crude oil prices display a strong degree of persistence which is the result of long run memory effects with the price dynamics being made up of an extremely complex multifractal structure (Cisneros, Ibarra-Valdez and Soriano, 2002). A comparison between commodities and stocks also revealed that commodities tend to display a significantly broader multifractal spectrum when compared to stocks (Matia, Ashkenazy and Stanley, 2003). Anti-correlation of electricity spot prices was found to be significant in Spain’s electricity spot markets with fat-tailed probability density function being a major contributor to its multifractal nature (Dullaert and Rahmani, 2007).

Gold prices were also found to be multifractal for time periods of less than a month and more than a month. Multifractality in the short run i.e. less than a month was found to be present due to fat-tailed distribution while in the long run i.e. more than a month, long-term correlations also play a part in contributing to the multifractal nature of gold prices (Wang et al., 2011). The results were also confirmed by Bolgorian and Gharli (2011) that gold prices are long-run correlated and this is a source for its multifractal nature. Other studies found that agricultural commodity markets in an emerging market are less efficient that the US agricultural commodity market in terms of the efficient market hypothesis. The commodity markets also displayed strong temporal correlations, power-law distribution and nonlinearity, all signs of multifractality (Kim et al., 2011).

Applying a multifractal perspective to understanding the cross-correlation between the energy and emissions market revealed that Gas and CO2 displayed the largest multifractal degree and that Oil-CO2 and Gas-CO2 were positively correlated for the most part (Wang et al., 2014). Comparison of gold prices in India with the Global Gold Consumer Price Index showed that both series were highly fractal in nature and a random shuffle of the series did not weaken the degree of multifractality. However, the Global Gold CPI had a higher degree of long-term temporal correlation relative to the Indian Gold prices (Mali, 2014). Agricultural and energy commodities were shown to possess higher efficiency when evaluated on a daily scale. It was also proven that the agricultural and energy commodities exhibited very similar behavior when the properties of their multifractal spectra were investigated (Delbianco et al., 2016).

Furthermore, multifractal analysis has seen its application in understanding how exchange rates and interest rates behave. Initial studies proved that Euro-Yen deposit rates and the Euro-Yen term premiums possessed long-term dependence for Japan (Barkoulas and Baum, 1998). Long-range dependence was found to exist in the US interest rates also (McCarthy, 2004), thus providing the motivation for application of multifractals to monetary policy instruments. A multifractal study of the Iranian Rial-US Dollar exchange rate showed that the exchange rate displayed strong multifractality with small and large variations happening significantly due to long-run temporal correlation (Norouzzadeh and Rahmani, 2005). Testing of long-range dependence of LIBOR for five international currencies revealed a very high degree of multifractality for Indonesia suggesting that emerging markets do possess a stronger degree of multifractality. The testing also revealed that the degree of multifractality reduces with market maturity (Cajueiro and Tabak, 2007).

Measurement of the evolution of the British pound against the US Dollar showed that the exchange rate was multifractal in nature and had a time-varying degree of long-term dependence. The analysis also displayed a relationship between structural changes in the economy and the dynamics of the exchange rate (Souza et al., 2008). Multifractality due to temporal correlations have also been found to exist in the Asian foreign exchange markets with multifractality and market complexity positively related to the presence of high returns. The East Asian crisis also contributed significantly to an increase in multifractality (Oh et al., 2012).

Central and Eastern European Currencies have also been shown to possess a good degree of multifractality with non-linearity being a major determinant of the multifractal strength (Caraiani and Haven, 2014). Huge cross correlation was also found to exist between SENSEX fluctuations and the USD-INR exchange rate. The findings also revealed that as SENSEX decreases, the FX increases and vice versa (Dutta et al., 2016). Lastly, an investigation into the Chinese bond market interest rates has provided clear empirical evidence of multifractal features and long-run correlation being identified as the source for multifractality (Wang et al., 2016).

With respect to the equity markets, other than broad-tailed distribution and long-term temporal correlation, herding behavior has also been found to be one of the causes of multifractality. Analysis of the Japanese stock market provided evidence for the presence of multifractality due to the existence of herding behavior (Cajueiro and Tabak, 2007). Latin-American market indices when compared with the United States suggested that emerging markets do indeed have a higher multifractal degree with the market maturity determining the strength of multifractality (Zunino et al., 2008).

Studies on the Indian and Chinese markets showed that the BSE SENSEX, the NIFTY50 and the Shanghai Stock Price Index all display significant multifractal behavior with long-term temporal correlation and fat-tailed distribution contributing to the source of this multifractal nature (Kumar and Deo, 2008; Ying et al., 2009). Power-law relationship and significant multifractal range was observed in the Athens Stock Exchange General Index too (Stavroyiannis et al., 2010).

Evidence from emerging Eastern European Nations stock markets shows that the Global Financial Crisis has been influential in changing the shape of the multifractal spectra for these counties (Caraiani, 2012). The global financial crisis had a very negligible effect on the multifractal nature of the Chinese and Indian stock markets while the Japanese, Hong Kong, US, South Korean and Indonesian markets exhibited very strong multifractality during the crisis period (Hasan and Salim, 2014). In Brazil, the electricity and public utilities indices were found to be less efficient due to the persistent long term correlations while the other indices displayed a lack of such correlation thus were classified as more efficient (Stosic et al., 2019). Efficiency of the London stock markets was also found to deteriorate due to the Brexit vote uncertainty (Arshad et al., 2019).

From the vast array of literature, it is evident that self-similar patterns are not an anomaly, rather a regularity. The fractal patterns identified across various asset classes in multiple countries indicates that financial markets are not fully random in nature. The aim of our study is to analyze bank equity across 2 major market types i.e. Emerging Markets and Frontier Markets. Studies so far have ventured into either characterizing sectors and individual stocks within a particular country or a comparison of index performance across various countries. We contribute to the financial literature by comparing individual banks within and across nations which can help guide a more global investment strategy with respect to the banking sector.

Data and methodology

For the purpose of study, we have chosen the largest (in terms of asset size and market capitalization) publicly floated banks from a total of 10 economies, which are:

1. Emerging: Brazil, India, China, South Africa.
2. Frontier: Argentina, Kenya, Kuwait, Morocco, Nigeria, Sri Lanka.

The markets have been classified according to the MSCI classifications and the period of study is from January 2000 to December 2019 except for 3 countries i.e. China (January 2008 to December 2019), Nigeria (February 2012 to December 2019) and Kenya (June 2012 to December 2019). MFDFA will be applied to characterize the scale-invariant nature of the time series data sets and the scale chosen ranges from 10 to N/10.

Detrended Fluctuation Analysis has been a technique widely employed in determining the fractal properties. The technique has also been helpful in detecting the long range correlations present in non-stationary time series. Due to this versatility, it has seen its utility in understanding the dynamics of human gait, rainfall patterns, cloud structure, earthquake patterns, biotechnology and even in physics. The biggest advantage of this methodology is that it is able to avoid any spurious detection of correlations that have come to be the hallmarks of non-stationarity in a time series.

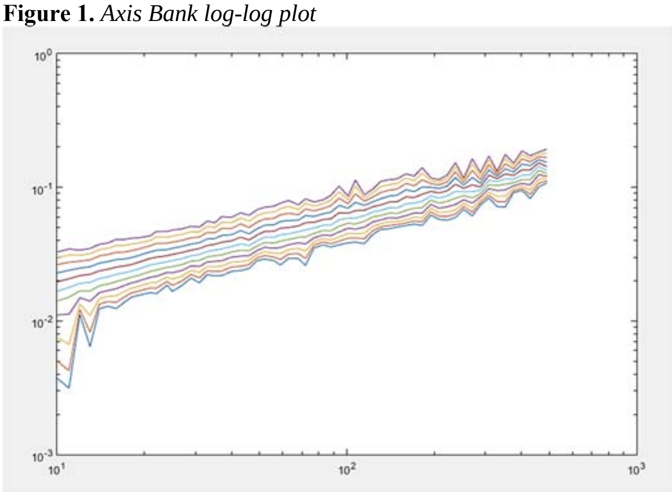
Data sets however may not always be mono-fractal in nature and thus multiple scaling exponents are required in order to extract a full description of this complex scaling behavior, calling into need Multifractal Analysis. Early models of multifractal analysis include the likes of the standard partition function approach and the wavelet transform modulus maxima (WTMM). A drawback of the standard partition function approach was its inability to give correct results for non-stationary time series affected by trends and could not be normalised. The WTMM method was not parsimonious and was di cult to program. Therefore, to tackle these drawbacks, Kantelhardt (2002) proposed a generalization of the Detrended Fluctuation Analysis (DFA) which is easier to program compared to the WTMM and can effectively analyse non-stationary series without the need for normalization.

Empirical results

Generalized Hurst Exponent Analysis

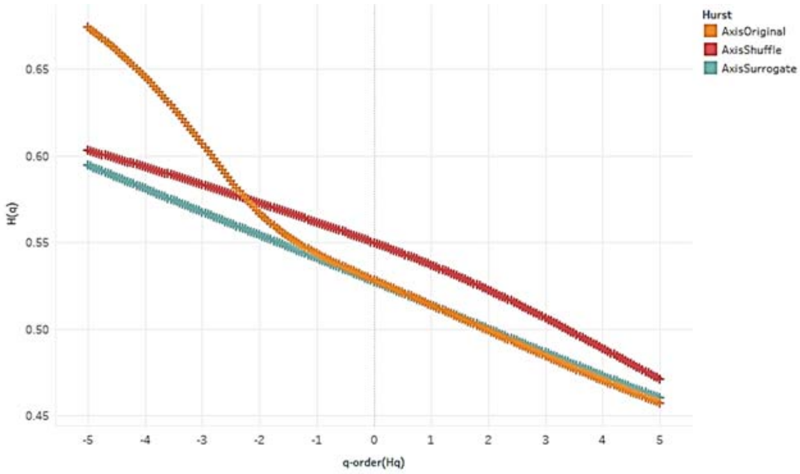
In the MF-DFA analysis, the values of the Generalized Hurst exponent *h(q)* is extracted from the slope of the *log-log* plots of $F_q(s)$ and the *scale* values. We obtain the value of *h(q)* for each of the *q-order* moments and these values serve as an indication of the fractal nature of a stochastic time series. For the purpose of our study, the values of *q* range from *-5* to *5*. Both positive and negative values of *q* are required to amplify the periods of large fluctuations and smaller fluctuations respectively. If the *log-log* plot is an increasing function and the value of *h(q)* is dependent on *q* then the time series is said to be multifractal in nature. Figure 1 illustrates a sample *log-log* plot obtained from the analysis of Axis Bank and it is evident that the series is multifractal based on the above description. Similar results were obtained for all the banks considered under the study.

Figure 1. *Axis Bank log-log plot*



From our analysis, we observed that the value of *h(q)* is not constant for any bank, rather they depend on *q* indicating that the return dynamics of all the banks analysed in the study have multifractal features. Furthermore, the values of *h(q)* for *q* less than zero are higher than the values of *h(q)* for *q* greater than zero indicative of the long-memory dynamics present in smaller fluctuations and anti-persistent or mean reverting dynamics present in large fluctuations of the returns. Figure 2 displays the dependence of *h(q)* on *q* graphically.

Figure 2. *h(q) dependence on q for the original, shuffled and surrogate series (Axis Bank)*



The non-linear dependence of $h(q)$ on q enables us to infer a measure of the multifractal degree i.e. the degree of complexity displayed by a series. A higher multifractal degree is directly related to a higher degree of market inefficiency. From the values of the generalized Hurst exponents, a measure of multifractality can be defined as:

$$Inefficiency = MF1 = \Delta h = h(q_{min}) - h(q_{max})$$

Where $h(q_{min})$ is equal to $h(-5)$ and $h(q_{max})$ is equal to $h(+5)$. The values of the generalized Hurst exponents for $h(-5)$ and $h(+5)$ along with the multifractal degree have been summarized in Tables 1 and 2.

Table 1. *Multifractal degree: emerging markets*

Country	Bank	h(-5)	h(+5)	MF1
Brazil	Banco Bradesco C4	0.5970	0.3215	0.2755
Brazil	Banco do Brasil	0.5790	0.4019	0.1771
Brazil	Itau Unibanco	0.5453	0.3098	0.2355
China	Bank of China	0.6345	0.3812	0.2533
China	Bank of Communications Ltd	0.6270	0.4361	0.1909
China	China Citic Bank Corporation	0.6645	0.3866	0.2780
China	China Construction Bank	0.6033	0.3778	0.2255
China	Industrial & Commercial Bank of China	0.6853	0.3825	0.3027
India	Axis Bank	0.6739	0.4572	0.2167
India	Bank of Baroda	0.6125	0.3855	0.2270
India	HDFC Bank	0.5589	0.3742	0.1847
India	ICICI Bank	0.5692	0.4102	0.1590
India	State Bank of India	0.6638	0.4238	0.2401
South Africa	Absa Group	0.5843	0.3483	0.2360
South Africa	FirstRand Bank	0.5262	0.3309	0.1953
South Africa	Investec Group Ltd	0.5705	0.3883	0.1822
South Africa	NedBank Group	0.5213	0.3656	0.1557
South Africa	Standard Bank Group	0.5136	0.3451	0.1685

Table 2. *Multifractal degree: frontier markets*

Country	Bank	h(-5)	h(+5)	MF1
Argentina	Banco Hipotecairo	0.7299	0.4695	0.2604
Argentina	Banco Macro	0.6020	0.4147	0.1873
Argentina	Banco Santander Rio	0.6717	0.4069	0.2648
Argentina	Grupo Financiero Galicia	0.6147	0.3828	0.2319
Kenya	Barclays Kenya	0.8855	0.3482	0.5373
Kenya	Standard Chartered Kenya	0.8090	0.3402	0.4689
Kuwait	Ahli United	0.5946	0.2834	0.3112
Kuwait	Kuwait Finance House	0.6874	0.3403	0.3471
Morocco	Attijariwafa Bank	0.6573	0.3697	0.2877
Morocco	Moroccan Bank of Foreign Commerce	0.8613	0.4918	0.3690
Nigeria	Access Bank	0.7344	0.3810	0.3534
Nigeria	Guaranty Bank	0.7398	0.3160	0.4230
Nigeria	United Bank for Africa	0.7602	0.4052	0.3549
Nigeria	Zenith Bank	0.7927	0.3189	0.4738
Sri Lanka	Commercial Bank of Cylon	0.8550	0.4297	0.4250
Sri Lanka	DFCC Bank	0.7216	0.4934	0.2282
Sri Lanka	Hatton National Bank	0.8179	0.5091	0.3088
Sri Lanka	Sampath Bank PLC	0.7760	0.4761	0.2999
Sri Lanka	Seylan Bank	0.6241	0.4204	0.2038

Among the banks considered in our study, Barclays Kenya has displayed the highest degree of multifractality with $MF1 = 0.5373$ and NedBank Group has the lowest multifractal degree with $MF1 = 0.1557$. As mentioned earlier in the methodology description, the standard Hurst exponent is $h(2)$. This value of $h(q)$ at $q = 2$ is the yardstick of a data sets' so called "mild or wild randomness". This value is also representative of the "index of long range dependence". It captures the tendency of a time series to either regress toward the mean or cluster in a particular direction. With the range of H being defined as $0 < H < 1$, the behavior displayed by a time series can be classified as persistent, anti-persistent or random walk. If the value of H lies between 0 to 0.5, we call this an anti-persistent series with an increase likely to be followed by a decrease and so on in the short run. For values of H between 0.5 to 1, increases are likely to be followed by more increases and vice versa for the short run. At H equal to 0.5, the series is known to display random walk behavior. Tables 3 and 4 summarize the values of the Hurst exponents for all the banks considered under our study.

Table 3. *Hurst exponent: emerging markets*

Country	Bank	Hurst Exponent
Brazil	Banco Bradesco C4	0.4596
Brazil	Banco do Brasil	0.4876
Brazil	Itau Unibanco	0.4200
China	Bank of China	0.4751
China	Bank of Communications Ltd	0.5203
China	China Citic Bank Corporation	0.4789
China	China Construction Bank	0.4876
China	Industrial & Commercial Bank of China	0.4727
India	Axis Bank	0.4990
India	Bank of Baroda	0.4718
India	HDFC Bank	0.4250
India	ICICI Bank	0.4727
India	State Bank of India	0.5086
South Africa	Absa Group	0.4274
South Africa	FirstRand Bank	0.4002
South Africa	Investec Group Ltd	0.4376
South Africa	NedBank Group	0.4280
South Africa	Standard Bank Group	0.4050

Table 4. *Hurst exponent: frontier markets*

Country	Bank	Hurst Exponent
Argentina	Banco Hipotecairo	0.5366
Argentina	Banco Macro	0.5016
Argentina	Banco Santander Rio	0.4735
Argentina	Grupo Financiero Galicia	0.4958
Kenya	Barclays Kenya	0.4604
Kenya	Standard Chartered Kenya	0.4590
Kuwait	Ahli United	0.4104
Kuwait	Kuwait Finance House	0.4423
Morocco	Attijariwafa Bank	0.4532
Morocco	Moroccan Bank of Foreign Commerce	0.5309
Nigeria	Access Bank	0.4773
Nigeria	Guaranty Bank	0.4439
Nigeria	United Bank for Africa	0.4972
Nigeria	Zenith Bank	0.4559
Sri Lanka	Commercial Bank of Cylon	0.5169
Sri Lanka	DFCC Bank	0.5434
Sri Lanka	Hatton National Bank	0.5768
Sri Lanka	Sampath Bank PLC	0.5552
Sri Lanka	Seylan Bank	0.4727

From Tables 1 and 2, we note that Hatton National Bank, Sri Lanka has the strongest long-range dependence or persistence with $H = 0.5768$ and FirstRand Bank Ltd has the strongest anti-persistent behavior with $H = 0.4002$. Out of the 37 banks listed, 7 banks have long-range dependence, 5 banks have dynamics close to a random walk structure and 25 banks have anti-persistent dynamics (with all South Africa banks displaying mean-reverting behavior).

Sources of multifractality and the multifractal spectrum

With the presence of multifractality being confirmed by the non-linear dependence of $h(q)$ on q, we then analysed the source of multifractal behavior. From the existing set of literature, it has been highlighted that majorly two sources contribute to the multifractal behavior of a time series which can be listed as: Multifractality due to a broad probability density function i.e. fat tails. Multifractality due to different long-range correlations of the small and large fluctuations. Both these cases can be tested to see which one contributes significantly to the presence of multifractal behavior. To test for the presence of long-term temporal correlation as a source, we randomly shuffle the series one hundred times in order to destroy any temporal correlations. To test for broad tailed probability distributions as a source, the series was run through the Iterative Amplitude Adjusted Fourier Transformation process to generate a surrogate series which retains the same autocorrelation structure of the original series. Re-running the MF-DFA on the shuffled and surrogate series and then plotting the singularity spectrum, we can assess the source of multifractality. If the spectrum of the shuffled series keeps close to the original, it implies that long-range correlations are not the cause of multifractality. Similarly, if the spectrum of the surrogate series keeps close to the original series spectrum, fat tails are not the cause for multifractality. However, if $\alpha_{original} > \alpha_{shuff}, \alpha_{surrogate}$ then neither of the two sources contribute to the multifractal nature of the series.

Tables 5 and 6 summarise the results from our study which specifies the multifractal degree for the original, surrogate and shuffled series with the multifractal degree defined as:

$$MF2 = \Delta\alpha = \alpha_{max} - \alpha_{min}$$

Where, the degree of multifractality is characterized by taking the difference between the largest and smallest values of the Holder exponents (α).

Table 5. *Multifractal degree emerging markets: original, shuffled and surrogate series*

Country	Bank	Original	Shuffled	Surrogate
Brazil	Banco Bradesco C4	0.5097	0.4738	0.4348
Brazil	Banco do Brasil	0.3358	0.1457	0.1822
Brazil	Itau Unibanco	0.5576	0.2666	0.2007
China	Bank of China	0.4342	0.4783	0.3723
China	Bank of Communications Ltd	0.3720	0.2551	0.2635
China	China Citic Bank Corporation	0.4893	0.2567	0.3176
China	China Construction Bank	0.4557	0.2997	0.2567
China	Industrial & Commercial Bank of China	0.5021	0.3453	0.2919
India	Axis Bank	0.3996	0.2638	0.2626
India	Bank of Baroda	0.4603	0.2649	0.2857
India	HDFC Bank	0.3835	0.1895	0.2754
India	ICICI Bank	0.3331	0.2939	0.3099
India	State Bank of India	0.4018	0.1534	0.2806
South Africa	Absa Group	0.4880	0.2604	0.2098
South Africa	FirstRand Bank	0.3802	0.1905	0.2430
South Africa	Investec Group Ltd	0.3603	0.2314	0.2694
South Africa	NedBank Group	0.3198	0.0948	0.2290
South Africa	Standard Bank Group	0.3077	0.2295	0.1982

Table 6. *Multifractal degree frontier markets: original, shuffled and surrogate series*

Country	Bank	Original	Shuffled	Surrogate
Argentina	Banco Hipotecairo	0.4970	0.2516	0.2693
Argentina	Banco Macro	0.3784	0.4290	0.4885
Argentina	Banco Santander Rio	0.4605	0.4604	0.6473
Argentina	Grupo Financiero Galicia	0.4512	0.4795	0.4723
Kenya	Barclays Kenya	0.8361	0.5768	0.6504
Kenya	Standard Chartered Kenya	0.8007	0.4883	0.6082
Kuwait	Ahli United	0.5049	0.1991	0.4100
Kuwait	Kuwait Finance House	0.4598	0.5073	0.6235
Morocco	Attijariwafa Bank	0.5337	0.1742	0.2406
Morocco	Moroccan Bank of Foreign Commerce	0.6141	0.3876	0.4018
Nigeria	Access Bank	0.5998	0.4127	0.2848
Nigeria	Guaranty Bank	0.7395	0.3049	0.4891
Nigeria	United Bank for Africa	0.6568	0.3559	0.4282
Nigeria	Zenith Bank	0.8278	0.2366	0.4143
Sri Lanka	Commercial Bank of Cylon	0.7556	0.3079	0.3156
Sri Lanka	DFCC Bank	0.4636	0.3833	0.3561
Sri Lanka	Hatton National Bank	0.5841	0.4142	0.2995
Sri Lanka	Sampath Bank PLC	0.5466	0.5651	0.3610
Sri Lanka	Seylan Bank	0.3285	0.3965	0.3684

The multifractal spectrum is one of the most used measures to understand the degree of multifractality because the spectrum is able to highlight the short and long-range dependence of the price dynamics on different scales. Furthermore, if the multifractal spectrum has a long left tail, it indicates that the multifractal structure is dominated by periods with large fluctuations and as a consequence, the multifractal structure is insensitive to the periods with small fluctuations. A long right tail is indicative of a multifractal structure dominated by periods with small fluctuations.

Local Hurst exponent

Financial markets are known to have characteristics such as non-linearity, dynamism and their movements being dictated by a variety of unknown parameters. This makes the prediction of the time evolution of a complex financial system very challenging. However, even though such prediction is challenging, it has been found that certain parameters lend themselves well to explain the behavior of the system. One indicator which we put to use in our study is the local Hurst exponent which plays the role of a "macroscopic indicator of complex, interior stock dynamics". The local Hurst exponent *H(t)* has an inherent advantage over the regular Hurst exponent because of its ability to identify time instant structural changes within the time series. Also, the method is more resistant to the presence of any long-term inaccuracies or distortions which emerge as a result of rapidly changing boundary conditions around the financial system. For the purpose of our study, only banks that have displayed long-term temporal correlation as a significant source of multifractal behavior have been chosen for calculation of the local Hurst exponent. A series correlated with its past values are ideal for forecasting and can be exploited for investment opportunities. The banks which fall under this classification are: Absa Group, Access Bank, Attijariwafa Bank, Barclays Kenya, Banco do Brasil, BMCE Bank, Bank of Baroda, China Citic Bank Corporation, Commercial Bank of Cylon, FirstRand Ltd, Guarantry Trust Bank, HDFC Bank, Industrial and Commercial Bank of China, Itau Unibanco, NedBank Group, State Bank of India, United Bank for Africa, Standard Chartered Kenya and Zenith Bank.
To identify clear trends and draw inferences about the presence of any cyclical behavior, we have chosen a 250-day rolling window to calculate the local Hurst exponent. The figures illustrating the plots of the local Hurst exponent and the prices against time are shown below:

Figure 3. *Absa Group, South Africa*

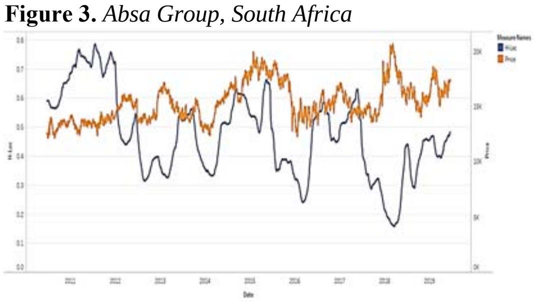


Figure 4. *Access Bank, Nigeria*

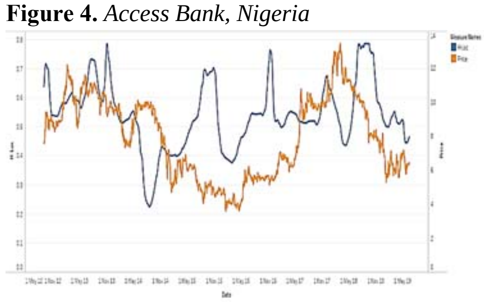


Figure 5. *Attijariwafa Bank, Morocco*

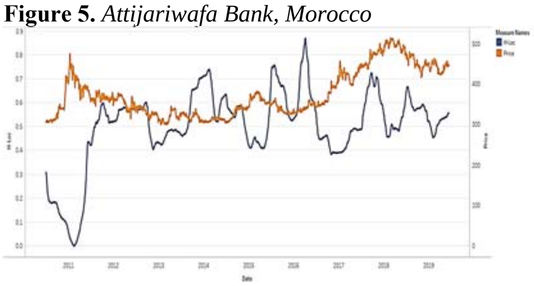


Figure 6. *Banco do Brasil, Brazil*

Figure 7. *Bank of Baroda, India*

Figure 8. *Barclays Kenya, Kenya*

Figure 9. *Banco Hipotecairo, Argentina*

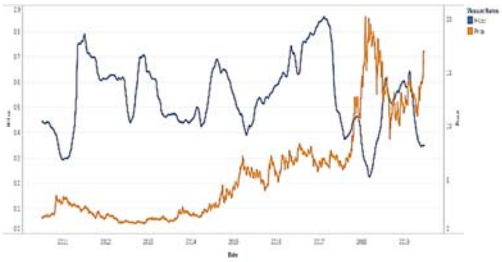


Figure 10. *China Citic Bank Corporation, China*

Figure 11. *Commercial Bank of Cylon, Sri Lanka*

Figure 12. *FirstRand Group Ltd, South Africa*

Figure 13. *Guaranty Trust Bank, Nigeria*

Figure 14. *HDFC Bank*

Figure 15. *Industrial and Commercial Bank of China, China*

Figure 16. *Itau Unibanco, Brazil*

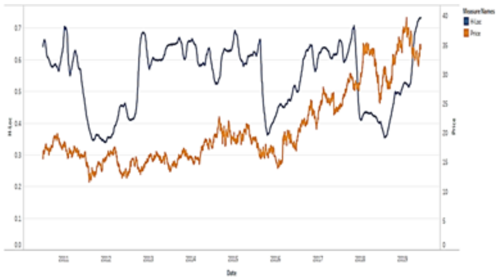


Figure 17. *NedBank Group, South Africa*

Figure 18. *SBI, India*

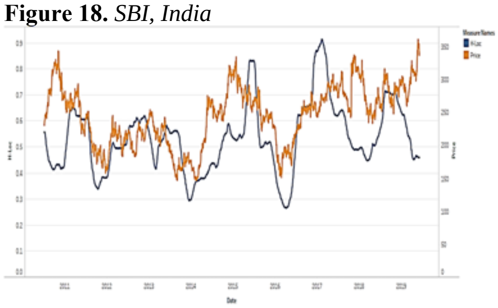


Figure 19. *Standard Chartered Kenya, Kenya*

Figure 20. *United Bank for Africa. Nigeria*

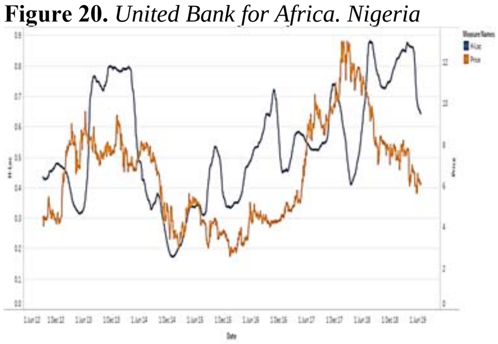


Figure 21. *Zenith Bank, Nigeria*

Observing the figures, we see that the time varying Hurst exponent is able to capture changes in the trends of the price movements before the actual shifts in price trends. The time dependent values of the Hurst exponent takes on a decreasing trend before the price arrives at major low points in its time evolution. Also, the price starts becoming anti-persistent in its behavior before arriving at periodic lows meaning the H_{loc} fluctuates below 0.5. Strong sell signals can be anticipated if the time evolution of the H_{loc} satisfies the following conditions: The local Hurst exponent takes on a decreasing trend, expect for small downward fluctuations. The local Hurst exponent is fluctuating below 0.5 while moving in a decreasing trend. Breach of threshold values for consecutive trading sessions can indicate impending crashes. Conclusions similar to these have also been verified in studies by Grech and Mazur (2004) and Grech and Pamula (2008). If behavior contrary to the conditions described above is being displayed by the security, it is a safe bet to purchase and hold the stock or to participate in the trading of the security. The time evolution of the local Hurst exponent can serve as a leading indicator for the price behavior since it allows us to place a certain degree of certainty in the stock markets. However, one has to be cautious in using these results since they cannot predict or foresee unpredictable external phenomena lying outside the control of investors and other market participants.

Conclusion

The importance of capital and the interlinkages between the banking sector and the financial markets have been sufficiently expanded upon in depth over the years by various scholars. Banks are required to perform critical intermediation to bring together savers and investors, serving as the medium to fund economic growth. Thus, how a bank decided to finance its operations has an impact on the efficiency with which it can deliver its services since higher leverage ratios are associated with higher operation costs and higher agency costs. Equities serve as a reliable source of capital for banks and the performance of these equities in the market have a bearing on the investment choices for the investors, which in turn has an effect on banking operations. Thus, to assess the performance of equities floated by major banks in the emerging and frontier markets, we apply the multifractal detrended fluctuation analysis to characterize the price behavior. Our study shows that the bank equities are indeed multifractal in nature and the more complex multifractal patterns are displayed by the frontier economies which is indicative of the still underdeveloped infrastructure necessary to cut down on the informational asymmetries that exist within the markets. Inefficient markets can deter investments and the multifractal degree serves as a reliable indicator of the market efficiency. We also confirm that market maturity does play a role in reducing the degree of multifractality vis-à-vis development of sophisticated infrastructure. An analysis of the Hurst exponents revealed that most banks have an anti-persistent nature. Furthermore, testing of the predictive capabilities of the local Hurst exponents has yielded promising results too. We have confirmed that the local Hurst exponent can serve as a leading indicator of the stock markets. Further studies can undertake forecasting of the local Hurst exponent to predict the time evolution of the "index of independence" in order to assess if there are any crashes which could possibly affect the markets in the future. The area is still academically young and has potential to be applied to a variety of financial models in order to better understand the markets.

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