Theoretical and Applied Economics
Volume XXIX (2022), No. 4(633), Winter, pp. 197-204

Two conditions which induce Giffen behavior in any numerical analysis if applied to the Wold-Juréen (1953) utility function

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Abstract. *The present paper extends the existing literature on the relationship between the Wold-Juréen (1953) utility function and Giffen behavior. This we do by applying two conditions, one of which is due to Sproule (2020). In particular, this paper demonstrates that these two conditions, when taken together, induce the inferior good (Good 1) to exhibit Giffenity. This finding serves to underscore the general perception that the Wold-Juréen (1953) utility function offers both analytical simplicity and much pedagogical value to the definition and to the study of Giffen behavior.*

Keywords: Wold-Juréen (1953) utility function, inferior good, Slutsky decomposition, Giffen paradox, pedagogy.

JEL Classification: A22, A23, D11.

Introduction

In 2011, Heijman and van Mouche (2011a) published an edited volume of conference proceedings under the title, "New Insights into the Theory of Giffen Behaviour". The sole motivation for this tome was the further exploration of the theory of Giffen behavior. In the introductory chapter, Heijman and van Mouche (2011b) stated that the genesis for their book sprang from a solitary question – a question which was motivated by one of the volume's contributors, Moffatt (2011). That one question is this:

"(T)o what extent are Giffen goods, for a single consumer, theoretically possible in the neoclassical framework of utility maximization under a budget restriction?" (Heijman and van Mouche, 2011b, pp. 1-2).

One of several objects-of-analysis within this monograph is the particular utility function due to Wold and Juréen – a utility function which we refer to hereafter as the Wold-Juréen (1953) utility function. The reason for the interest in this one utility function is the pivotal role that it has played in the study of Giffen behavior. Its role has been so pivotal, that Sproule (2020, page 2) dubbed it the progenitor of all theoretical research on Giffenity, which use those two-good utility functions, which have the potential to exhibit Giffen behavior. Sproule's proclamation is no more than a crystallization of the aforementioned statement by Moffatt (2011), who wrote:

"Ever since Wold and Jureen's attempt to illustrate the Giffen paradox by specifying a particular direct utility function, there has been a stream of contributions from authors pursuing similar objectives, for example Spiegel (1994), Weber (1997 and 2001), Moffatt (2002), and Sørensen (2007).(1) One of the lessons learned from this strand of literature is that it is not easy to specify a direct utility function that predicts 'Giffen behaviour' and simultaneously satisfies the basic axioms of consumer theory." (Moffatt, 2011, p. 127)

While the Wold-Juréen (1953) utility function has proven central to the study of Giffen behavior, so too has a related paper by Weber (1997), which provides the first thorough enumeration of the essential properties of the Wold-Juréen (1953) utility function.(2)

Since the appearance of the tome by Heijman and van Mouche (2011a), and the prior appearance of the paper by Weber (1997), no other significant contribution to this literature has been made; that is, until the appearance of the paper by Sproule (2020). In this paper, Sproule (2020) presented two main arguments:

Argument 1: Using the Wold-Juréen (1953) utility function, Weber (1997) demonstrated that the Giffenity of Good 1 depends upon the relative magnitudes of the decision maker's (hereafter DM's) income vis-à-vis the price of the normal good (Good 2). In particular, Weber stated that: "Giffen behavior is more likely for higher … incomes" and that the Giffenity of Good 1 "is more likely at lower values of the price for Good 2". (Weber, 1997, p. 40) In response to these two claims, Sproule (2020, p. 2) stated: "Weber's precondition is so vague that it lacks broad appeal" and that Weber's precondition does not accord "with a core tenet of microeconomics, which is that economic decision-making is predicated on (changes in) relative prices."

Argument 2: Sproule (2020) then went on to offer a new precondition for Giffen behavior under the Wold-Juréen (1953) utility function – one which does accord with the core tenet of microeconomics regarding relative prices. In particular, Sproule's (2020) precondition (or prediction) is this: if the DM has the Wold-Juréen (1953) utility function, and if the price of the inferior good (Good 1) is greater than or equal to the price of the normal good (Good 2), then the inferior good (Good 1) is a Giffen good.

The purpose of this paper is to extend Sproule's (2020) analysis of the Wold-Juréen (1953) utility function and Giffen behavior to the domain of numerical analysis. In particular, the purpose of this paper is to provide an analytical framework for a particular two-good utility function, in which one of the goods is an inferior good (Good 1), so that this same good (when placed in a numerical domain) can be shown to exhibit Giffen behavior, when two restrictions are imposed on the selection of the numerical values. That utility function is the Wold-Juréen (1953) utility function, and the origins of these two restrictions are: (a) one of the core properties of the Wold-Juréen (1953) utility function itself, and (b) the analysis by Sproule (2020).

This paper is organized as follows. The next section (the second section) provides a terse overview of the literature. In particular, the second section highlights the key points reported in Weber (1997) and in Sproule (2020). The third section presents our analytical framework. In particular, in this section, we present the two conditions which (when combined) induce Giffen behavior in the inferior good (Good 1) for any numerical analysis when they are applied to the Wold-Juréen (1953) utility function. In the fourth section, we undertake a numerical demonstration of the validity of the analytical framework. In particular, we show that if the set of numerical values for the exogenous variables (viz., the prices of Good 1 and Good 2 and the DM's income) satisfy the two conditions defined in our analytical framework, then that requirement alone is sufficient to induce the inferior good (Good 1) in the Wold-Juréen (1953) utility function to exhibit Giffen behavior. Summary remarks are offered in the fifth and final section.

Previous research

Suppose that the DM resides in a two-good world, (x_1, x_2), where x_1 and x_2 denote the quantities of Goods 1 and 2. In this world, suppose that p_1 and p_2 denote the market prices of these two goods, and m denotes the DM's income.

Next suppose that the DM's utility function is the Wold-Juréen (1953) utility function. That is, suppose that the DM's utility function is defined as $U = \frac{(x_1-1)}{(x_2-2)^2}$ where $x_1 > 1$ and $0 < x_2 < 2$ (see Wold and Juréen, 1953; Weber, 1997, and Sproule, 2020). If so, then the DM's ordinary or Marshallian demand functions are defined as follows:

$$x_1^* = 2 + \frac{2p_2 - m}{p_1} \tag{1}$$

$$x_2^* = 2\left(\frac{m - p_1}{p_2} - 1\right) \tag{2}$$

where $x_1^* > 1$ and $0 < x_2^* < 2$ (see Weber, 1997 and Sproule, 2020). We note that Equation (1) represents the Marshallian demand function for the inferior good (Good 1) and Equation (2) represents the Marshallian demand function for the normal good (Good 2).

As mentioned previously, two papers are needed to summarize the relationship between the Wold-Juréen (1953) utility function and Giffen behavior or (in common parlance) "Giffenity". These are Weber (1997) and Sproule (2020). Key elements of these two papers are defined next in Propositions 1 and 2.

Proposition 1 (Weber, 1997, p. 40): If the DM has the Wold-Juréen (1953) utility function, then:

a) $\text{Sign}(\text{TE}) = \text{sign}\left(\frac{\partial x_1^*}{\partial p_1}\right) = \text{sign}(m-2p_2)$ where TE denotes the total effect of a change in the price of the inferior good (Good 1).

b) If m is "relatively large" and if p_2 is "relatively small", then $\text{TE} = \frac{\partial x_1^*}{\partial p_1} > 0$ and the inferior good (Good 1) is a Giffen good.

Sproule (2020) found that Weber's precondition for Giffenity (viz., $m > 2p_2$ in Proposition 1) to be unacceptable a priori. As a consequence, Sproule proposed an alternative precondition, which is this:

Proposition 2 (Sproule, 2020): If the DM has the Wold-Juréen (1953) utility function, and if $p_1 \geq p_2$, then x_1^* is a Giffen good (that is, $\frac{\partial x_1^*}{\partial p_1} = \text{TE} > 0$).

As mentioned previously in Arguments 1 and 2 above, Sproule (2020) argued that his precondition (viz., $p_1 \geq p_2$) is more appealing than Weber's because Sproule's precondition accords with a core tenet of micro-economics, which is that economic decision-making is predicated on relative prices or on a change in relative prices.

The analytical framework

Our strategy for inducing Giffen behavior in any numerical analysis when applied to the Wold-Juréen (1953) utility function flows from two propositions. These are reported next in Propositions 3 and 4:(3)

Proposition 3 (Sproule, 2020): If the DM has the Wold-Juréen (1953) utility function, then $p_1 + p_2 < m < p_1 + 2p_2$

Proof: Since $x_1^* = 2 + \frac{2p_2 - m}{p_1}$ for $x_1^* > 1$ [see Equation (1)], it then follows that $p_1 + 2p_2 > m$ (3)

And since $x_2^* = 2\left(\frac{m - p_1}{p_2} - 1\right)$ for $0 < x_2^* < 2$ [see Equation (2)], it also follows that

$$p_1 + p_2 < m < p_1 + 2p_2 \quad (4)$$

Finally, we note that, given Equation (4), Equation (3) is redundant. •

Proposition 4: When using numerical analysis to determine the presence of Giffen behavior in the context of the Wold-Juréen (1953) utility function, success requires that the numerical values be chosen so that they satisfy these two conditions, $p_1 + p_2 < m < p_1 + 2p_2$ and $p_1 \geq p_2$.

Proof: See Proposition 2 and Proposition 3 above. •

Inducing Giffen behavior in a numerical analysis

In this section, we confirm that the model outlined in the last section is sufficiently simple that it can be used in a microeconomics course at any level. In this section, we show that if a given set of numerical values fully accords with Proposition 4 above, then the numerical values for the component parts of the Slutsky Equation (viz., the total effect, substitution effect, and income effect) for the Wold-Juréen (1953) utility function can be deduced and that the sign of numerical value of the TE can be shown to be positive.

Remark 1: One set of numerical values which satisfy the two conditions defined in Proposition 4 is this set:

$p_1 = 10$

$p_1' = 9$

$p_2 = 5$

$p_1 + p_2 = 15$

$p_1 + 2p_2 = 20$

$m = 18$

Remark 2: To verify that the set of numerical values contained in Remark 1 accords with Proposition 4, we present these two numerical tests:

$$\text{Test 1: } (p_1 + 2p_2 = 20) > (m = 18) > (p_1 + p_2 = 15)$$

$$\text{Test 2: } (p_1 = 10) > (p_2 = 5)$$

Remark 3: The numerical values reported in Remark 1, and then vetted in Remark 2, yield the three points of equilibrium which are integral to the determination of the magnitudes of the associated total effect, substitution effect, and income effect for Good 1 when the DM's utility function is the Wold-Juréen (1953) utility function. These three points we term here as Point A, Point B, and Point C.[(4)]

Point A: Point A is the point of initial equilibrium; that is, Point A is the equilibrium point where the exogenous variables are the DM's income and the initial values of the price of Good 1 and Good 2. Hence, we denote the general value of Good 1 at Point A as:

$$x_1^A = x_1^A(p_1,p_2,m) = 2 + \frac{2p_2 - m}{p_1}.$$

Point C: Point C is the point of final equilibrium; that is, Point C is the equilibrium point where the exogenous variables are the DM's income, the initial price for Good 2, and the final price for Good 1. In summary, we denote the general value of Good 1 at Point C as:

$$x_1^C = x_1^C(p_1^{'},p_2,m) = 2 + \frac{2p_2 - m}{p_1^{'}}.$$

Point B: Point B is the third point of equilibrium; that is, Point B is the equilibrium point where the exogenous variables are the initial price for Good 2, the final price for Good 1, and the DM's income so adjusted that the set of equilibrium values at Point B, $\left(x_1^B(p_1^{'},p_2,m^{'}),x_2^B(p_1^{'},p_2,m^{'})\right)$, is equally affordable as the set of equilibrium values at Point A, $\left(x_1^A(p_1,p_2,m),x_2^A(p_1,p_2,m)\right)$, where $m' = \Delta p_1 . x_1^A + m$. For more on the algorithm for adjustment of the DM's income to m′, see Varian (2014, Chapter 8). With this material in hand, we can now denote the general value of Good 1 at Point B as:

$$x_1^B = x_1^B(p_1^{'},p_2,m^{'}) = 2 + \frac{2p_2 - m^{'}}{p_1^{'}}$$

Remark 4: Given Remark 1 and Remark 3, we can now state the numerical values for Good 1 at Points A, B, and C as follows:

Point A: $x_1^A = x_1^A(p_1,p_2,m) = 2 + \frac{2p_2 - m}{p_1} = 2 + \frac{2(5) - 18}{10} = 1.2000$

Point B: $m' = \Delta p_1 . x_1^A + m = (-1)(1.2) + 18 = 16.8$.

Since $m' = \Delta p_1 . x_1^A + m = (-1)(1.2) + 18 = 16.8$, therefore

$$x_1^B = x_1^B(p_1^{'},p_2,m^{'}) = 2 + \frac{2p_2 - m^{'}}{p_1^{'}} = 2 + \frac{2(5) - 16.8}{9} = 1.2444$$

Point C: $x_1^C = x_1^C(p_1^{'},p_2,m) = 2 + \frac{2p_2 - m}{p_1^{'}} = 2 + \frac{2(5) - 18}{9} = 1.1111$

Remark 5: With Remark 4 in tow, we are now in a position to define the numerical values of the three component parts of the Slutsky Equation, viz., the total effect (TE), the substitution effect (SE), and the income effect (IE). In view of Remarks 1 and 4, these numerical values are as follows.

Total Effect: The numerical value of the TE is:

$$TE = \frac{x_1^C - x_1^A}{\Delta p_1} = \frac{1.1111 - 1.2000}{-1} = \frac{-0.0889}{-1} = 0.0889 > 0.$$

We note here that the numerical value of the TE is positive, which indicates that Good 1 is a Giffen good, just as predicted by Proposition 4 and Remark 2.

Substitution Effect: The numerical value of the SE is:

$$SE = \frac{x_1^B - x_1^A}{\Delta p_1} = \frac{1.2444 - 1.2000}{-1} = \frac{0.0444}{-1} = -0.0444 < 0.$$

Income Effect: The numerical value of the IE is:

$$IE = \frac{x_1^C - x_1^B}{\Delta p_1} = \frac{1.1111 - 1.2444}{-1} = \frac{-0.1333}{-1} = 0.1333 > 0.$$

We note here that the numerical value of the IE is positive, which indicates that Good 1 is an inferior good and which accords with a general feature of the Wold-Juréen (1953) utility function.

Remark 6: The results reported in Remark 5 accord with all expectations for the Slutsky decomposition using the analytical framework defined in this section and in a prior section, viz.,

$$SE = -0.0444 < 0$$

$$0 < IE = 0.1333.$$

$$TE = SE + IE \Leftrightarrow 0.0889 = -0.0444 + 0.1333$$

$$TE = 0.0889 > 0 \text{ because } |SE| < |IE|.$$

In summary, we note (once again) that the TE is positive (which indicates that Good 1 is a Giffen good), and that this property arises: (a) because the IE is positive and (b) because the absolute value of the IE dominates the absolute value of the SE.

Conclusion

The present paper extended the existing literature on the relationship between the Wold-Juréen (1953) utility function and Giffen behavior. This was achieved by the application of two conditions to a numerical analysis of the Wold-Juréen (1953) utility function. Stated more precisely, these two conditions, when taken together in a numerical analysis of the Wold-Juréen (1953) utility function, induce the inferior good (Good 1) to exhibit Giffen behavior.

The present finding serves to underscore the general perception that the Wold-Juréen (1953) utility function offers both analytical simplicity and much pedagogical value to the study and to the definition of Giffen behavior.

Notes

(1) Other examples might include: Haagsma (2012), Landi (2015), and Nachbar (1998).
(2) It would appear that the primary motivation of Weber's (1997) paper is the definition of Giffen behavior for a utility function due to Spiegel (1994), and that the secondary motivation of Weber's (1997) paper is the definition of Giffen behavior for the Wold-Juréen (1953) utility function.
(3) The reader ought to keep in mind that, henceforth, we shall employ one of two possible variants of the substitution effect in the Slutsky equation, viz., we shall use the "Slutsky substitution effect" rather than the "Hicksian substitution effect". For further details, see Chapter 8 of Varian (2014).
(4) Should the reader wish to see the graphical counterpart of this sort of analysis, he or she is directed to Figure 8.1 in Varian (2014).

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