Credit Risk Evaluation

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Abstract. In the environment in which a bank functions there are many risk sources that determine the reduction of the profitability. These risk sources must be attentively identified, measured and taken into consideration for the elaboration of a bank’s general strategy of monitoring and disproof of the risks. The risk is generally defined as: the adverse effect that certain distinct incertitude sources exert over the profitability. The measurement of the risk requires that both the incertitude and the potential adverse effect over the profitability be surprised and evaluated.

Key words: credit risk; the structure of the portfolio; modeling of risk factor; transformation function; distribution of the credit loss.

In the 90’s the credit risk became the major preoccupation of the risk managers in the financial and regulation institutions. This was due to several factors:

- Although the market risk is much better evaluated, the largest part of the capital of the commercial banks is used for the credit risk. The complexity of the standards measuring models, the analysis and the management of the credit risk might, therefore, not be in accordance with their importance.

- Propelled by the liberalization and integration of the European market, the new distribution channels through e-banking, the financial un-intermediation and the entrance on the market of the insurance companies and investment funds, the pressure of the competition between the financial institutions grew and determined the diminution of the credit limits.

- Meanwhile, the number of companies’ bankruptcies stood still or grew in the majority of European countries. A great number of insolencies and reorganizations of the banks’ activities have been influenced by a previous bankruptcy of the creditors.

- The regulation methods imposed by Basel Committee to prevent the risk that the financial institutions to go bankrupt because of the credit losses. These regulations include, for example, certain limits of the display according to the capital reserve. The Basel Committee of Banking Supervision promotes a system of minimal capital calculus necessary for the credit risk (the Internal Ratings-Based), based on new models of credit risk determination.

The essence of credit risk management is the measurement and attenuation of unexpected losses for the credit display in a portfolio. A simple example of a situation that involves the credit display is the granting of a loan; the creditor of the loan cannot be entirely sure that the debtor will honor the agreement and will return the loan – that is why it suffers a display until the loan reaches maturity. This is where the risk concept appears; the display always introduces a risk.

The typical examples of the products that represent the credit display are: the loans, the bonds and the currencies,
but there is a variety of other financial instruments that involve the display. In the quantitative terms the display is defined as the maximum loss by the non-payment of a counter-party; the granting of 1 leu loan means an initial display of 1 leu – this is how much it is expected to be lost in the beginning by the one that grants the loan, if the debtor does not have the possibility to pay any money (the non-payment status).

The management of the credit risk nowadays is a result of the effort of the banking industry for the avoidance of non-payment experience from the end of 80’s and the beginning of the 90’s. A result of this effort is also the relatively new and powerful market of credit derivates – instruments that can be use by institutions to protect themselves against non-payment situations.

1. Requirements regarding the modeling of the risk of credit portfolios

Until the present day the development of the models of the risk of the crediting portfolios has been possible because of the theoretical approaches. The modeling begins by understanding the nature of the credit risk and its components which determine the model’s algorithms. Such an approach tends to create models that are rather interesting than useful, as their results determine incompletely the objectives of credit risk measurement of the risk managers, but also the regulators.

Thus, a basic model, for the regulation and management of the portfolio, must be determined by pragmatic objectives. First of all, it must include all the important types of risk from a credit portfolio. Secondly, it must be applicable to any kind of significant credit to which the bank exposes itself. Thirdly, the model must be fast, stable and correct. Finally, it must determine the necessary results to support the vital applications of the risk management.

An important ingredient in evaluating the credit risk is the determination of the distribution of the non-payment rate. The non-payment risk of the credit is the risk that the debtor cannot pay the financial obligations. The risk of the credit limit appears in the portfolios in which for the credit limit the market value of a product is treated and used. The quality of a debtor’s credit is determined by the number of investors towards which he believes he may have financial obligations – on short, medium or long term. This trust may be modified in time (the migration of the credit’s rating), thus the distribution of the non-payment rate is described naturally by a continuous stochastic variable.

The purpose of modeling

The fundamental object in evaluating the credit risk is the purchase of the distribution of the credit loss. Once this distribution is known, we can both answer the main question regarding the losses expected in a given portfolio and determine how great the capital reserve must be.

One of the most important aspects for the building an adequate model for the credit risk and the questions that appear after the purchase of the distribution of the credit loss are:

1) Which are the non-payment rates and the recuperation rates for the possible non-payments?

In many cases the non-payment takes place because of the debtor’s bankruptcy. Usually, the debtor presents a number of goods that can be sold so that a part of the debts to be paid.

The degree from which the debtor (statistically) can recover the entire debt or a part of it as a result of the closing, liquidation, reorganization or sale is called the recovery rate.

2) For what period the credit is evaluated?

The choice of a horizon large enough is of course very important if we want the possibility of fixing the attenuation actions, in case of any drastic situation. Usually, a year is considered to be the adequate horizon, but the portfolio’s maturity is also an adequate horizon.

3) Which are the variable factors of the modeling process?

This includes the risk process, the variable parameters and the error’s model. The risk process appears because of the random nature of the current observations and it is evaluated by the model’s results for a high enough level of trust. The variable parameters and the error’s model, on the other side, appear because of the difficulties in using the entry data kit, previously observed, to propose a unique model and to estimate its parameters.

4) What fundamental factors affect the non-payment rates?

Various economic factors, such as the rate of economic growth and the level of the interest rates, affect the non-payment rates. An increasing economy may determine a small number of non-payments. These fundamental factors are usually treated in the model by incorporating the volatility of the non-payment rate of the debtor. This approach gives birth to the distribution functions of the loss with fat tails.

5) Which is the size of the loss for a given level of trust?

The answer to this question may be given if the loss’s distribution function is obtained, at least for all the events with probabilities of reasonable appearance.

6) Which are the possible extreme incomes?

The answer is hard to be given as the data of the possible events in this case are rare. The usual model to solve the extreme events is the settlement of the limits regarding the size of the parties of the same economic sector from a portfolio. All these to prevent the correlations that are too big between the portfolio’s components. Such restrictions are called the limits of concentration. Another possibility is the simulation of an adequate model a large number of times for various situations to observe the frequency of the extreme events.

Although it is obvious, any truly useful model for the credit risk must be capable to represent the entire credit
portfolio. Even if the model covers all the types of credit risk, its results will mistake the correct representation of the risk if it is not wide-range regarding the covering of the types of lessees and the credit’s instruments.

The majority of the models are projected from the perspective of commercial loans and bonds’ portfolios. When these models are applied to the displays for retail or for the small business they will have unsatisfactory results. The modeling of the concentration or diversification of effects over different segments is critical, and the spread practice to use separate models for different sub-portfolios fails in this case. For example, a portfolio that includes loans in the auto sector, accounts for credit cards in Michigan, auto indirect loans and residual auto rentals create concentrations that are ignored if they are not treated in the same model.

On the other side, the request for the covering as wide as possible must be tempered by practice considerations and especially by the volume of entries for the parameterization of a wide-range model. The bias of the commercial loan and of the bonds portfolio determines a transactional level of the structure of entries for the majority of the models. Unfortunately, the extension on the entire typical portfolio of the bank is not feasible. Given the size of the banking portfolios and the diversity of the offered products, the specific detail on the level of the transaction extensive systems will be necessary achieved for the entry processing, and the performances of the model will a real problem.

The challenge, therefore, is the finding of a structure of the input which has enough flexibility to recognize the individual transactions where the word risk is significant, but keeps only the important information for the remained displays.

An additional complication comes from the variety of the rating systems used by different banks and from the probability that multiple rating systems are used in different parts of the portfolio of the same bank. To be usable for the entire portfolio, the structure of the entries must be sufficiently flexible to accept any combination of the internal ratings, their action and the results of the scoring model, with different types of different portfolios of a bank or similar portfolios of different banks.

Further, a model of the credit portfolio’s risk must, also, be capable to adapt to any instruments included in a portfolio which a manager might want to evaluate.

The components of the credit risk

A wide-range model of the credits portfolio risk includes the approach of four types of distinct risk:

1. Specific non-payment. A loans portfolio with the probability for fixed non-payment presents the risk that more loans to enter in non-payment than it is expected. The non-paid loans are relatively rare, especially in the commercial loans portfolios, but the impact of each non-paid loan may be significant. This risk may be diminished by diversification – insuring the existence of a large number of diverse lessees in the portfolio – but a bank usually attracts a too great display to be removed from the rest of the portfolio.

2. Systematic non-payment. In a portfolio, the non-paid loans tend to group in certain moments. Intuitively, many lessees enter in non-payment when the economy is poor and fewer in case the economy is increasing. The models must take into consideration the effects of these fluctuations by the systematic non-payment. Effectively, the non-payment probability is not fixed, but it is a risky variable of itself.

3. The economic loss without non-payment. The value of a loan may decrease without the lessee to enter into non-payment. A deterioriation of the value of the credit perceived, either because of the formal diminution of the rating, either to the systematic changes in the expectations of future non-payments of the loan. For some types of portfolios this fact presents as “market value” adjusted to reflect changes of the expectations as well as of the price on the credit risk market.

4. The loss given by the non-payment. The importance of the losses suffered for non-payment may vary because of the factors specific to the efficiency of each individual loan, but especially because of the systematic variations of the non-paid loans. The loss given by the non-payment may be correlated to the systematic non-payment rates and may result from each or from their combination the volatility of the recovery rate or the volatility of the display.

The regulation tends to include all these four types of risk, since it develops the approach of a basical model. The IRB provided in the new Agreement from Basel already makes appeal to three of the four types, with a certain imprecision. The so-called formula “Benchmark Risk Weight” is concentrated on the fluctuations of the systematic non-payment rate. The IRB also makes appeal to the economic losses without non-payment by “adjusting the maturity” (with the maturity settled at three years for the “IRB Founding”) and the individual risk of a non-paid loan by “adjusting the granularity”.

The majority of the models of the credit portfolio risk integrates the risks: systematic non-payment and specific non-payment. There is a theoretic general consensus, not algorithmic, regarding the approach of the non-payment risk – the non-payments take place randomly and specifically around the fluctuations of the systematic non-payment rates – thus the various models produce, as the equivalent entry parameters, disagreements almost identical and shapes for the distribution of the losses almost similar.

The risk of the economic losses without non-payment is extremely non-consistent. Some of the models fully omit this risk, with the suggestion that the general model is supplemented with a separate model regarding the value of the risk for the limits of the credits. Those models that directly include this risk tend to consider the perspective of “the
rating’s migration”, in which there is a number of states of the credit’s efficiency (opposite to the models with a single state, in which there are only payment or non-payment).

The entries for the models with several states include a matrix of the transition of the ratings (gives back the probability that a credit passes from a rating to another) and a reevaluation program of each credit on each rating, including the non-payment. The degree from which the reevaluation of the systematic risk is appealed to depends on the mechanisms that form the basis of pursuit of the systematic non-payment rates and the transition probabilities. If the ratings are “doubtful” or somehow “clinically-neutral” (that is the changes of the rating do not keep up with the changes of the expected losses) this method will minimize the risk of systematic changes in future expected losses.

Finally, the losses given by non-payment have been until now less accentuated in the development of the methods of the credit portfolios risks, which concentrated more on the incidence of the non-payment. This seems reasonable – in the end, that the non-payment represents the primary source of the risk. More methods assume the loss given by the ascertained non-payment, some of them include the variability of the recovery rates based on the “loan by loan”, but none includes the systematic variation of the displays or of the recovery rates, less the possibility of the correlation with the variation of the systematic non-payment rate.

If the inclusion of the specific and systematic recovery rate’s volatility is not successful in the model of the risk of the credit portfolio this is equivalent to the attribution of the zero value to the volatility of the recovery and/or its correlation with the systematic risk of non-payment. Some results point out the fact that it is usually a bad enough attribution. It is even more unsuitable for certain sub-portfolios where the correlation represents a main characteristic regarding the crediting decision – the real commercial value, the rental of price goods and their return, the registries of the returned loans – as the value of the guarantees would suffer from the same factors that cause the increase of the systematic non-payment rate. This is a risk that cannot be neglected in the pragmatic modeling of a real portfolio for credits.

The performance of the model

The objectives of the performance assume speed, stability and accuracy. The calculus time is of major importance for the dynamic and applications in real time such as the price of the loan, the evaluation of guarantees, the testing of sensitivity, “what if scenario” and others. The lack of stability or accuracy gives back untrustworthy results of the model.

Unfortunately, many models of the risk of the credit portfolio use the Monte Carlo simulation, which is slow and instable for the wanted results. Some of the models abuse of simplifying presumptions for the increase of the speed and stability, but with the cost of the diminution of the accuracy if the presumptions do not reflect adequately the conditions of the real world.

The model’s functionality

Finally, a model of the risk of the credit portfolio must be of course projected to produce useable results. The first objective for the analysis of the risk and the capital is to create a distribution of the probabilities for the credit’s losses. This is useful itself for the control of the risk as well as for the applications of capital adequacy based on the risk.

Some of the most important decisions-support of the applications of a model of the risk of the credits portfolio – the price of the loan based on the risk, the settlement of the dynamic limits and the analysis of the concentration for the identification of the constraints or the optimization of the opportunities – require rather the contributions of the risk than the distribution of the aggregated losses. For these applications it is not important only to be capable of associating the contributions to the risk of the loans existent in the portfolio, but to foresee the solutions to determine the contributions to the risk of the hypothetic loans that are not part yet of the portfolio.

Several models of the risk of the credits portfolio do not offer support for the modeling of the impact of the economic scenarios over the expected losses of the credit and surely neither for the distribution of the credit losses in a particular scenario. This type of analysis of scenario is critical for the support of the intuition of the management and the planning of the unanticipated events, as well as the communication of the risks to the regulators, the rating agencies and the equity annalists.

2. The non-payment model

The model implies a systematic factor, which can be considered as representing the general economic conditions. This factor leads to variations through the non-payment rates in the portfolio.

The distribution of the probabilities of the non-payment rate of the portfolio may be built integrating the distributions of the conditioned non-payment rate with the distribution of the systematic non-payment rate for all the systematic non-payment rate values.

The model of non-payment details the behavior of the conditional factor of the non-payment rates only, leaving other variables of the loss (the recovery rate, the display, etc.) fixed. In the next paragraph the structure of the model’s entries is described.

The portfolio representation

To keep the performances in a portfolio of thousands of loans, the model takes into consideration the advantage offered by the following sentence: The loans that are part of the same portfolio are not significantly different, if they have similar non-payment probabilities and are of similar size.
Such loan groups can be represented rather through an aggregated display than through an individual registration. The entries of the model consisted of, therefore, portfolios of the “homogenous cells”.

A sub-portfolio contains similar loans from industry, geography etc., but which may have different non-payment probabilities or different measures of the display. A homogenous cell groups further the loans of a sub-portfolio which have approximately the same non-payment probability and size of the display. The matrix of a sub-portfolio of loans groups therefore the loans by rating and displays.

This structure is capable of summing up more loans in a compact manner, but it can be as detailed as it is wanted, with numerous loans individually admitted for the elimination of the proposed rounding. The entire portfolio can be segmented in this manner, and the structure permits the easy adaption to different types of sub-portfolios (for example corporations vs. retail). The fact that this approach treats the displays smaller than the average in the interval of the small sizes of the displays, as a small number of large displays must be noted. This will not have an important effect over the results of the model as long as the size of the displays of this interval is small enough contributing very little to the specific non-payment risk of the portfolio.

The factor’s structure

The model requires a function that makes the connection between the probability of non-payment for an individual loan and the systematic factor. In the “Merton model” a loan enters into non-payment when the value of the latent factor (most of the times analog with the value of the goods of the corporations) declines under a critical value (also, analog to the value of the corporation’s liabilities). Assuming that the standard modifications of the latent factor are normally distributed, the modification of the latent factor for the loan \( i \) can be expressed thus:

\[
\Delta A_i = \rho \times m + \sqrt{1-\rho^2} \times \epsilon_i
\]

(1)

where:

\( m \) represents the systematic factor of the sub-portfolio (\( m \sim N(0,1) \)), \( \rho \) is the correlation between the systematic factor of the sub-portfolio and the latent factor of the lessee, and \( \epsilon \) is the specific component (\( \epsilon_i \sim N(0,1) \)).

For a given cell of the matrix of the loans of the sub-portfolio with the rating \( r \) and the corresponding non-payment probability \( p_r \) (the unconditioned average non-payment rate), the conditioned non-payment rate may be expressed as:

\[
P_{r|m} = \Phi \left( \frac{\Phi^{-1}(p_r) - \rho \times m}{\sqrt{1-\rho^2}} \right)
\]

(2)

where:

\( \Phi \) is the cumulated standard density function. This expression of the conditioned non-payment rate respects the known restrictions and the empirical information being delimited between zero and one and convex.

A conditioned non-payment rate \( P_{r|m} \) being given, the exact distance of the non-payments of a homogenous cell of the sub-portfolio is given by the binominal distribution. As the number of loans becomes bigger and bigger, the nominal distribution becomes refractory. Anyway, the package of described entries allows the user to specify the size of the intervals which will maintain reasonable the effective number of loans.

As the non-payment rate of each cell of the sub-portfolio is independent, having the dependency conditioned by the systematic factor, the aggregation of the binominal distributions of these to the conditioned distribution of the sub-portfolios is common. The distribution of the unconditioned loss is evaluated by the integration of the conditioned distribution of the sub-portfolio with the distribution of the systematic factor over the entire interval of values of the systematic factor.

The algorithm for the evaluation of the distributions of the loss

The non-payment rate \( d \) for one homogenous cell having the fixed average non-payment rate \( p \) (the same for all the lessees of the cell, by definition) with the total display \( e \) and the average of the size of the display \( s \) the binominal distribution will follow:

\[
Pr(d) = \frac{k}{n} = B[k,n,p] = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}
\]

(3)

where:

\( k \) is the number of non-payments and \( n \) is the number of the debtors in the cell (determined by the rounding at integral of \( \frac{e}{s} \)).

The distribution of the losses of this cell, with the average fixed recovery rate \( r \) for the non-payment loans, then the simple distribution of the non-payment rate re-associated with the amounts of the calculated losses as a product of the number of non-payments, the size of the display and one minus the recovery rate:

\[
Pr[l_s = ks(1-r)] = B[k,n,p]
\]

(4)

The distribution of the losses for the entire portfolio, with each cell having the size of its display and its non-payments rate (its recovery rate, if this detail is required), is obtained by combining the distributions for each cell, assumed independent (necessary consequence for the fixed conditioned non-payment rates). This can be obtained iteratively combining the right-pairs of cells, with the probability of each total loss \( c \) resulted from a pair of cells A and B equal to the amount of the probabilities for all the combinations of the common losses that produce a total loss \( c \).

\[
Pr[l_{A+B} = C] = \sum_{i=0}^{c} Pr[l_A = c-i] Pr[l_B = i]
\]

(5)
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Summarizing up to this point, a set of fixed non-payment probabilities being given, this algorithm will present the distribution of the losses for the $f[l\ p]$ portfolio, where $p$ is the non-payment probability settled for each cell (corresponding to different values of $p$, for different risk ratings). Using Merton’s transformation function, which connects the systematic factor of the sub-portfolio to the non-payment rate of each cell ($P_f[m]$) as the function of the non-payment probability and of the factor of correlation of the lessee – to – the sub-portfolio, the expression of the rate of the sub-portfolio losses may be expressed as $f[l\ m]$.

This distribution may be called “the distributions’ – factor of the conditioned losses” of the sub-portfolio. The distribution of the unconditioned losses of the sub-portfolio is then evaluated for any amount of the losses given by the average of the distributions’ – factor of the conditioned losses calculated for all the possible values of the factor, determined according to the distribution of the probability of the factor’s values:

$$f(l) = \int_{-\infty}^{\infty} f(l|m) \varphi(m) dm \quad (6)$$

where:

$\varphi(m)$ is the density function of the normal standard probability.

More practical, the distribution of the unconditioned loss of the sub-portfolio may be evaluated considering the distribution of the factor’s values as discrete and forming a “matrix of the distribution – factor of the conditioned losses”, which will contain the distributions’ – factor of the conditioned losses for each of a number of samples of the factor’s values (with the probability associated to each). The algorithm may be accelerated by introducing the loss unit $u$ for the amounts of the determined losses in equation 5. The conceptual flow of the algorithm is illustrated in Figure 1.

![Figure 1. The conceptual algorithm for the evaluation of the loss distribution of the sub-portfolio](image)

1. For the value of the systematic factor of the sub-portfolio, $m$.
2. The conditioned systematic non-payment rates are calculated for different ratings.
3. The distribution of the conditioned losses is determined for all the cells of the matrix of the loans sub-portfolio.
4. These distributions of the losses are iteratively combined (the distributions of two cells are combined, a third distribution is added to the resulted distribution, then a fourth distribution and so on) to produce the distribution of the conditioned loss of the sub-portfolio for the given value of the systematic factor $m$.
5. The distribution of the unconditioned losses is evaluated to each value of the loss by integrating in the entire row of values of the systematic factor.

References


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