R.M. Solow Adjusted Model of Economic Growth

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Abstract. *Besides the models of M. Keynes, R.F. Harrod, E. Domar, D. Romer, Ramsey-Cass-Koopmans etc., the R.M. Solow model is part of the category which characterizes the economic growth.*

The paper proposes the study of the R.M. Solow adjusted model of economic growth, while the adjustment consisting in the model adaptation to the Romanian economic characteristics.

The article is the first one from a three paper series dedicated to the macroeconomic modelling theme, using the R.M. Solow model, such as: "Measurement of the economic growth and extensions of the R.M. Solow adjusted model" and "Evolution scenarios at the Romanian economy level using the R.M. Solow adjusted model".

The analysis part of the model is based on the study of the equilibrium to the continuous case with some interpretations of the discreet one, by using the state diagram. The optimization problem at the economic level is also used; it is built up of a specified number of representative consumers and firms in order to reveal the interaction between these elements.

Key words: reprezentive consumer; reprezentive firm; macroeconomic model; macroeconomic equilibrium; state diagram; evolution scenarios.

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1. The model hypothesis[(1)]

An economy with householders and firms - each carrying out specific hypothesis - is considered.

Hypothesis at the householders' level:

- at the economy level there is a number of identical householders *G(t)*, at the *t* moment;
- for each householder, the consumers are assumed identical;
- the householders number is increasing with a r_1 rate, so $G(t) = G(0) \times e^{r_1 t}$ for the continuous case and $G(t) = G(0) \times (1 + r_1)^t$ for the discreet one.

Observation: It can be noticed that the growth rate of the householders is the same with the one of employers at the economy level.

- the householders' goal is to maximize the utility for an indefinite period of time;
- the householder1s utility function is:

$$U(c(t), t) = \int_0^\infty e^{-\alpha t} \times U(c(t)) \times \frac{L(t)}{G(t)} dt \tag{1}$$

where:

$c(t)$ - the consumption of each householder member;

$U(c(t))$ - the utility function for each family member, at *t* moment;

$L(t)$ - the employees at the economy level, at *t* moment;

$\frac{L(t)}{G(t)}$ - the average number of members at the householder level;

$U(c(t))\frac{L(t)}{G(t)}$ - the utility at the householder level, at t moment;

α - adjustment rate: if is increasing, the future consumption value is lower, considering the current one;

- each member of the household offers a labour unit at each time moment;
- householders rent to the firms their capital;

Hypothesis at the firms' level:

- at the economy level there is number of identical firms, *F(t)*, at the *t* moment;
- each firm production function has replaced factors and Harrod technical progress(2) such as(3):

$$\tilde{q}(t) = \tilde{q}(a(t)\times L(t), K(t)) = a(t)^{\gamma_1} \times L(t)^{\gamma_1} \times K(t)^{1-\gamma_1},$$

with $0 < \gamma_1 < 1$ (2)

where:

$\tilde{q}(t) = \frac{Q(t)}{F(t)}$ - production at the company level;

Q(t) - synthesis indicator at macroeconomic, at *t* moment;

K(t) - total capital at the economy level, at *t* moment;

$a(\cdot)$ - represents a function of technical progress and shows the integration on labour factor, with the increase rate;

- the goal of the firms is to maximize profit for an indefinite period of time(4):

$$\pi(\cdot) = \int_0^{\infty} e^{-\alpha t}[p\times\tilde{q}(a(t)\times L(t), K(t)) - w(t)\times L(t) - r(t)\times K(t)]dt \quad (3)$$

where the capital price (marginal cost of the capital) at t moment is supposed equal with the real interest rate:

$$r(t) = f'(k(t))$$

and the labour price or the real wage at t moment is:

$$W(t) = a(t)\left[f(k(t)) - k(t)\times f'(k(t))\right]$$

where the real wage on labour unit will be:

$$w(t) = f(k(t)) - k(t)\times f'(k(t))$$

- the firms are owned by householders to whom the profit reverts;

Notations:

K(0) - the initial capital (wealth) at the economy level;

$k(0) = \frac{K(0)}{G(0)}$ - the initial endowment (wealth) with capital of each householder;

Macroeconomic hypothesis:

- there is no depreciation at the capital level(5);
- income is split, at each moment, between consumption and savings;
- the production function at macroeconomic level is neoclasic (linear homogenous), with two replaced production factors, the labour and the capital;

$$Q(t) = Q(a(t)L(t), K(t)) \quad (4)$$

Supplementary notations:

C(t) - consumption at the economy level, at *t* moment;

I(t) - investments at the economy level, at *t* moment;

S(t) - savings at the macroeconomic level, at *t* moment;

$i(t) = \frac{I(t)}{L(t)}$ - investments volume per each employee;

$s(t) = \frac{S(t)}{L(t)}$ - savings volume per each employee;

$q(t) = \frac{Q(t)}{L(t)}$ - production at the representative employee level from the economy;

$\tilde{i}(t)$ - the investment requisite per capita, corresponding to the natural increase of the population, in conditions of maintaining the actual level of the techniques.

2. The economic and mathematic model

Considering the laws of J.M. Keynes, from a threshold the persons touch a welfare level and if the income increases, the gap between this and the consumption is higher.

In conclusion, the savings growth rate, $\frac{\dot{S}(t)}{S(t)}$, is higher than the income one, $\frac{\dot{Q}(t)}{Q(t)}$, as $\frac{\dot{S}(t)}{S(t)} > \frac{\dot{Q}(t)}{Q(t)}$

Considering the two possible cases for the income, the psychological law determines:

- if the income increases to $\frac{\dot{S}(t)}{S(t)} > \frac{\dot{Q}(t)}{Q(t)} > \frac{\dot{C}(t)}{C(t)}$;

- in the case of decreasing to $\frac{\dot{S}(t)}{S(t)} < \frac{\dot{Q}(t)}{Q(t)} < \frac{\dot{C}(t)}{C(t)}$.

Conclusion: For an important economic growth it is necessary to stimulate the investments into the country economy.

R.M. Solow starts, for building his model, from the general condition of the macroeconomic equilibrium: aggregate demand is equal to the aggregate offer: $D(t) = Q(t)$, and

$$D(t) = C(t) + I(t) \tag{5.1}$$

$$Q(t) = C(t) + S(t) \tag{5.2}$$

we have the relation:

$$I(t) = S(t), \text{ cu } \dot{K}(t) = I(t) = S(t) = sQ(t) \tag{6}$$

and supposing that: $\frac{\dot{L}(t)}{L(t)} = \frac{\dot{K}(t)}{K(t)} = r_1$ (7)

Dividing the relation (6) with L obtain:

$$\frac{I(t)}{L(t)} = s \times \frac{Q(t)}{L(t)} \tag{8}$$

or in average terms, the condition of the macroeconomic equilibrium is:

$$i(t) = s \times q(t) \tag{9}$$

or equivalent: $i(t) = s \times f(k(t))$

From equation (7) we have: $\dot{K}(t) = r_1 \times K(t) = I(t)$. Although it is the relation $\tilde{i}(t) = r_1 \times k(t)$, and the equilibrium condition requires the solving of differential equation:

$$\dot{k}(t) = i(t) - \tilde{i}(t) = s \times f(k(t)) - r_1 \times k(t) \tag{10}$$

which has the solution noted with .

This is the fundamental equation of the R.M. Solow model, with it can be analysed the stability of the dynamic equilibrium, using the state diagram(6).

If we introduce the depreciation rate of capital, ρ, the previous equation becomes:

$$\dot{k}(t) = i(t) - \tilde{i}(t) = s \times f(k(t)) - r_1 \times k(t) \tag{11}$$

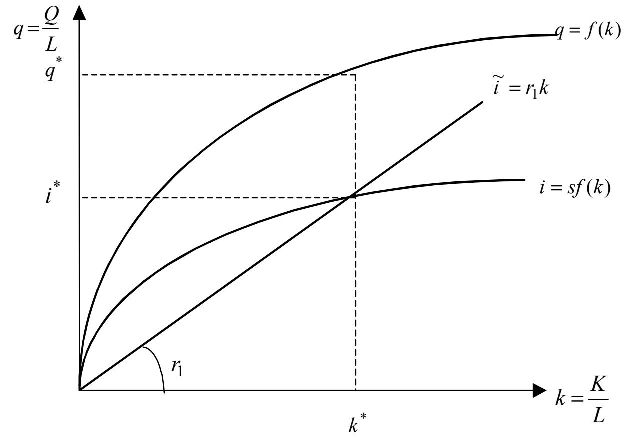

Figure 1

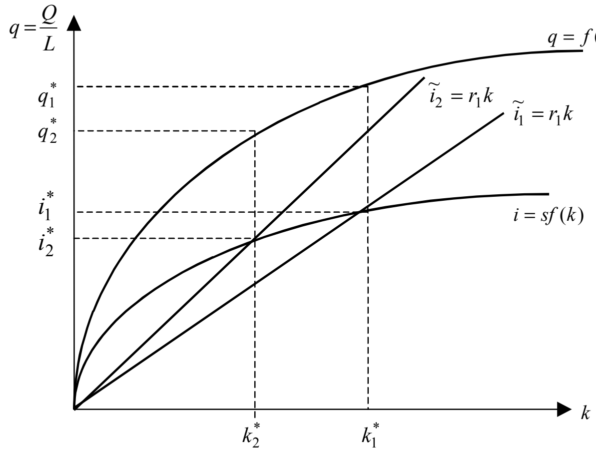

Figure 2

In figures 1 and 2 the equations (10) and (11) are represented.

Observations: 1. The saving rate change has a *level effect* and not one of increasing, doesn't altering the rate of growth of $\frac{Q}{L}$.

2. The altering of growth rate $\frac{Q}{L}$ will change the equilibrate increase trajectory of the output per capita;

3. The R.M. Solow adjusted model shows that the important differences between countries, considering the national income per capita, don't have as principal cause the accumulations.

Notes

(1) R.M. Solow s-a ocupat de relaxarea ipotezei coeficienților constanți ai producției din modelul Harrod. În acest model, rapoartele capital-output și capital-muncă nu sunt fixate, ca în modelul Harrod-Domar.

(2) a se vedea Stancu, S., Microeconomie-comportamentul agenților economici. Teorie și aplicații, Editura Economică, București, 2006, pp. 236-245.

(3) $\tilde{q}(\cdot)$ este presupusă a fi o funcție de producție liniar omogenă, cu $\tilde{q}'(\cdot) > 0$ și $\tilde{q}''(\cdot) < 0$.

(4) s-a presupus aceeași rată de actualizare ca și la gospodării.

(5) această ipoteză poate fi relaxată, conducând la un model mai laborios.

(6) a se vedea pentru aceasta Stancu S., Modelarea matematică a fenomenelor și proceselor de conducere. Teorie și aplicații, Editura Academiei de Studii Economice, București, 2007, pp. 151-156 și pp. 165-173.

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