Financial Securities Investments Analysis and Administration of Active Portfolio in Indeterminate Situations

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Abstract. *This article deals the problems of investments in securities. The purpose of this study is risk optimization and determination within a portfolio of risk value criteria when investments in financial titles are made in condition of undetermined situations. At the end, answers merge into questions mark. This provokes for reflection.*

Key words: indeterminate situations; return; risk; asymmetry; attitude towards risk.

1. The investments in securities. The determination of the individual titles and profitability

The investment represents the reason for many surprising actions for the one that owes the capital and is willing to gain. The investment in financial titles has a peculiar feature due to the volatility of the gains on relatively short terms and to the desire of the share holders to maximize their fortune.

The profitability of an action is given by the currency variation during its possession and by the distributed dividend so the share owners will be directly interested in its evolution and in the profit repartition for the dividends as it has to ensure a balance between their own resources used for development and the ones used for the remuneration of the capital possessions.

There was an old interest towards the concept of risk and the forms it can take. To present the risk as a common event would be a flagrant approach mistake. The existence of risk induces a series of internal manifestations, specific for the human subject psychology. If we wanted to characterize a situation, we would definitely make an appeal to the notion of risk.

B. Jaquillat and B. Solnik in „Les marchés financières et la gestion du portefeuille" define risk as follows:

- the sacrifice of an immediate advantage or absence of an immediate consumption in exchange for future advantages;
- the loss of a certain and immediate advantage related to a real good or a service consumption for a future and uncertain advantage, by investing in movable values;
- the incertitude regarding the value of a financial good that will be registered at a future date.

The behavior of the human subject toward risk occurrence is surprising. The reactions can be most of the time contradictory.

We were always trying to establish means of protection, of covering the risk. Why? Because many of us direct our actions towards the future, where the presence of risk is certain.

The subject's human profile influence his decisions of assuming the risk. The complex human nature is full of

contradictions and reactions many times impossible to understand. Maybe, that is why the science assigned the title of rational behavior for the participants to the economic and social life. Is there an irrational behavior as well? Of course there is, but the determining factors are divers: education, culture, religion, and environment are only a few of them.

Maybe the most beautiful description of risk can be found in the syntagm „The one who does not risk risks the most".

2. CAPM unifactorial model of investment in risky financial assets and the assets without risk

This model aims at optimizing the decision of investment in financial titles. The study of investment in financial assets shows that the analysis of the titles portfolio is realized using the Markowitz model that allows, after correlating two by two all the existing assets in portfolio, the determination of the portfolio with an absolute minimal variation. Markowitz's model allows us also to determine the efficiency frontier, which groups the portfolios which represents the best profitability for a certain risk.

Although slow and requiring a very large amount of information, this is the first method which allows the financial analysis of the titles in the portfolio taking into account the existing correlation between them. Thus, this method allows us to realize an optimum portfolio starting with a series of hypothesis like: we realize an investment of all the available funds; the operations short sales are not permitted; the adjusted profitability of the portfolio depending on risk is the objective of the investor. By forbidding the short sales we understand the fact that negative weight of the titles are not permitted in the portfolio, in other words you cannot sell titles you do not owe. To determine the investment opportunities we need to go through the following stages: the portfolio with an absolute minimal variance is determined; the titles weight of the portfolio is determined; the portfolios are classified in legal and illegal; by applying the principle of dominance the frontier of efficiency is determined, meaning that between two portfolios that have the same risk, the portfolio with the highest profitability is chosen, or that between two portfolios that have the same profitability it is chosen the one with the minimum risk. Thus, determining the optimum line of action involves a division of the possible solutions ensemble in two sets that comprise the efficient solutions and the dominant solutions followed by the determination of the efficient solution that maximizes the utility function of the investor and that has as parameters the portfolio profitability and risk.

According to Markwowitz's model for a portfolio formed of two titles, the associated variance can be calculated as follows:

If the profitability of two titles is independent then:

$$Var(R_P) = P_A^2 \text{ x } Var(R_A) + P_B^2 \text{ x } Var(R_B)$$

If the profitability of two titles is independent then:

$$Var(R_P) = P_A^2 \text{ x } Var(R_A) + P_B^2 \text{ x } Var(R_B) + 2P_A \text{ x } P_B \text{ x } Cov(R_A, R_B)$$

with

$$Cov(R_A, R_B) = \sum_{i=1}^{n} P_i[R_{Ai} - E(R_A)][R_{Bi} - E(R_B)].$$

However, the very large amount of necessary information to apply the model, respectively a number of dispersions equal to the number of considered title (n) and a number of covariance equal to n (n-1/2), determined the development of this model and led to the appearance of a simplified model for the portfolio analysis by Sharpe.

This proposes a new method of financial assets evaluation by objective criteria of the financial market; thus he proposes an unifactorial model which assumes that the profitability of any financial title is in a linear relationship with a macroeconomic factor. The amount of necessary information in this model is much lower being equal with 3n+2. This model eliminates the grouping of titles in the portfolio 2 by 2 and offers the possibility of an individual grouping depending on a chosen macroeconomic factor, usually this being identified with the medium profitability of the market. The expected profitability is influenced by two parameters: a position coefficient and a volatility index next to a macroeconomic factor. The title risk consists, according to Sharpe's theory, of two parts, respectively the systematic risk due to the capital market as a whole and explained by the dependence with the macroeconomic factor, and the risk specific for each title which can be removed by diversification.

The simplified equation of the market model without considering the residual influences is:

$$R_T = \beta \times R_M + \alpha$$

Where:

R_M - is the average profitability of the market or the variation of a macroeconomic factor.

β -is the volatility coefficient that quantifies the

relation between the evolution of the individual profitability of the title and the evolution of the average profitability on the market;

$$\beta = \frac{Cov(R_T \times R_M)}{Var(R_M)}$$

α - a coefficient equal to the individual profitability in the hypothetical situation when the profitability average rate on the market is zero.

Starting from the model extended with the residual influences we demonstrate the relation among the total risk, the systematic risk and the specific risk.

$$R_T = \beta \times R_M + \alpha + \varepsilon$$

$$\text{Var}(R_T) = \beta^2 Var(R_M) + Var(\varepsilon)$$

$$\sigma^2(R_T) = \beta^2 \times \sigma^2(R_M) + \sigma^2(\varepsilon)$$

$\sigma^2(R_T)$ = Total Risk

$\beta^2 \times \sigma^2(R_M)$ = Systematic Risk

$\sigma^2(\varepsilon)$ = Specific Risk.

This model, also known under the name of diagonal model, gave the possibility for a consequent development of the CAPM model, which sets the existence of the possibility for an investment on the capital market with assets having zero risk and a characteristic profitability, usually the bonds issued by the state. These models approach the issue of the portfolio, wishing to determine both the optimal proportion of the titles and the influence of a macroeconomic factor, considered on their level of profitability and risk. CAPM originated in the examination of the investors' behavior in a hypothetical model of economy, where they are operating only for a period. Actually, the investors operate for several periods, and that is why in the empiric examination of the CAPM using data from the capital markets it is necessary to make certain hypothesis with a presumption character. One of the basic hypothesis is that beta remains constant in time. This is not a reasonably enough measure, because the relative risk of the cash-flows is little possible to remain constant in time, without variations.

For the first time, the CAPM model was presented in its classical version by Sharpe (1964), followed by Lintner's comments (1965) and Mossin (1966, 1973).

The CAPM hypothesis:

The first fundamental hypothesis is now that the investors are concerned with the hoped profitability, closely connected with the risk associated to this.

Secondly, there is a set of traditional hypothesis related to the perfection of the capital market:

- there are no transition costs and assets which are not perfectly dividable;
- dividends are not taxed and plus values;
- lots of purchasers and vendors arise on the market, and non of them can influence the prices;
- All the investors can get loans at the rate of the interest without risk;
- All the investors can find any necessary information for a correct evaluation of the assets without charge;
- The period of the investment is the same for all the investors, the decisions for the investments are taken at the same time;
- All the investors have the same expectations about the future performances of the titles. This means that they agree with the hoped profitability, the dispersions and the associated co-variations. This hypothesis is known under the name of „idealistic certitude" hypothesis.

By introducing the asset without risk within the portfolio, several new elements are introduced:

- the rate of the interest without risk (Rf)
- the risk bonus, which is made up of two components:

a) the systematic risk $\left(\frac{E_M - R_f}{\sigma_M}\right)$

b) the specific risk (ε_i).

The CAPM model has the indisputable merit of identifying the two components of the normal profitability of all risky title.

For various portfolios: CML (capital market line)

$$E_p = R_f + \frac{E_M - R_f}{\sigma_M} \times \sigma_p$$

E_p – the profitability hope of the portfolio.

For individual titles: SML (security market line)

$$E_i = R_f + (E_M - R_f) \times \beta_i$$

E_i – the profitability hope of the a „i" title.

3. Optimizing the risk determination within a portfolio–risk value criterion

We can consider that the risk value is estimation for an interval of trust given to as much as we can sell of our portfolio for a given period.

To determine this value we need the following data: the present prices of all the assets in the portfolio, their volatility, as well as the correlation among them.

If within a portfolio we know the volatilities for all the assets in our portfolio and the correlation coefficient among them, we can calculate the risk value for the entire portfolio.

If the volatility of the „i" asset is σ_i and the correlation coefficient of the i asset with article j is ρ_{ij}, then the risk value of a portfolio made up of n assets owing Δ_i of the i asset is:

$$-\alpha(1-c)\delta_t^{\frac{1}{2}}\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{n}\Delta_i\Delta_j\sigma_i\sigma_j\rho_{ij}}\ .$$

The risk value can also be used to measure the performances of a portfolio starting with the relation:

$$\frac{\text{The daily profit or loss}}{\text{The daily risk value}}\ .$$

4. Choosing investments in financial titles for undetermined situations

In such cases it is impossible to appreciate the probability of realizing an event, and that is why we shall classify the events in two categories, favorable and unfavorable events. The investor has subjective criteria, and in this case his attitude towards risk is relevant.

The attitude of the human subjects towards risk divides them in two categories:

- pro-risk;
- against risk.

The pro risk type is the one who assumes a risk in exchange for remuneration according to the assumed risk.

The against risk type is the one who tries to diminish his risk all the time.

Minimizing the risk represents one of the fundamental purposes of the assets portfolio management and allows an optimization of the capital holdings according to the attitude of the investor towards risk. Therefore the utility function was defined as a function expressing the relative interest of an investor for the different levels of enrichment. The absolute level of the utility is a totally abstract measure.

Von Neuman-Morgenstern's comment

Von Neuman-Morgenstern calculates the first derivate of the expected utility in uncertainty conditions. They assumed that, generally, the investors choose lotteries, where lottery means a variable with specific possible incomes associated with probabilities.

This approach was criticized because it starts from the assumption that the agents know (used in the sense that they guess) the probabilities to realize the incomes. Therefore we are in the presence of a game of chance where there are probable objectives that can be assumed. The probabilities are known and have an objective character.

The critic aims at the fact that we can not specify the probability of the distributions because the investors cannot characterize the choices like a lottery.

Savage's comment

Savage states on the expected utility that it takes the form of a choice, having mostly the characteristic of a probable state than that of a lottery. Unlike von Neuman-Morgenstern, for Savage the probabilities are obtained, rather than data with a strong subjective character.

What is specific for Savage's approach is the idea according to which if the agents' preferences regarding a future probable state are dominated by some axioms, then they have a representation of the expected utility according to their convictions.

Savage's approach, starting with this situation, is actually immune to the objections which stipulated the fact that the investors do not know the probabilities.
Savage states that if the agents are able to choose then they behave as if they had already known the probabilities, which are actually subjective and differentiated.

Friedman and Savage's comment

Friedman and Savage emphasized the relation between the investors' behavior towards risk and the mathematic symbol for the second derivate of the utility function.

The utility function for an investor who dislikes risk has a down oriented concavity. The second derivate for this utility function is negative.

$$\frac{d^2u(w)}{dw} < 0$$

In other words, the marginal utility of an investor's richness who dislikes risk decreases according to the increase of the respective investor's income.

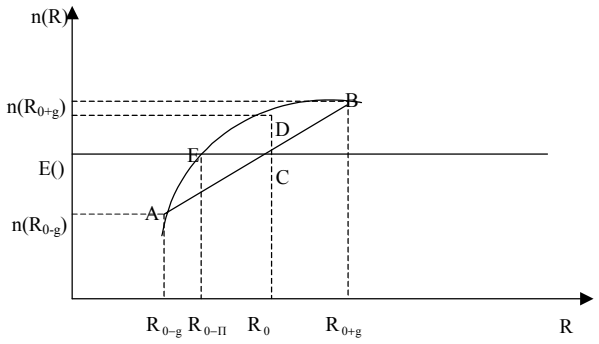

Figure 1. *The utility function for an investor who dislikes risk*

In the diagram we have:

R_0 – the investor's initial richness

g – the investor's probable gain

$R_{0-\Pi}$ – the moment when the investor's hoped utility equals the hoped utility of the lottery.

For an investor who likes the risk, the utility function has an up oriented concavity. The second derivate for this utility function is positive.

The marginal utility of the investor's richness who likes the risk is increasing, this investor giving an extra utility to each unit gained when he is richer.

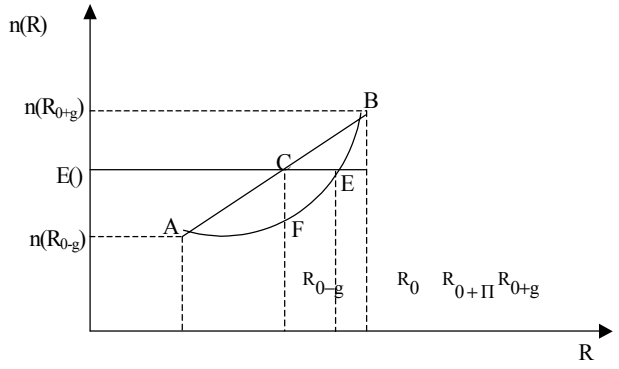

Figure 2. *The utility function for an investor who likes risk*

Therefore it is obvious that the investment analysis made by the capital owner will consider both the economic-financial situation of the economic organization where he wants to invest and his attitude towards risk, in other words the features of his own utility function.

To evaluate investments we use a series of criteria arisen especially from the subjects' attitude towards risk. These criteria are:

- *Laplace's criterion*, which considers that possible states have the same occurrence probability and who uses the arithmetic average as a calculus instrument of the possible results. We shall choose the decision that maximizes the arithmetic average.
- *Wald's criterion (minimax).* This criterion encourages a cautious attitude towards risk, choosing a minimization of the maximal losses previewed as the right decision.
- *Maximax criterion.* This criterion encourages an expansive attitude towards risk, choosing the decision that maximizes the gain by comparing the biggest values.
- *Hurwicz's criterion* starts from the necessity of associating some probabilities of realizing the optimist and the pessimist scenario in such a way as to result a sure event.
- *Savage's criterion,* which recommends the determination of the "regret" that corresponds to the difference between the most favorable case and the particular case.
- *Choosing the criteria* consists of comparing all the other criteria and choosing the decision that appears selected most of the times.

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