Measurement of the Economic Growth and Add-on of the R.M. Solow Adjusted Model

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Abstract. *Besides the models of M. Keynes, R.F. Harrod, E. Domar, D. Romer, Ramsey-Cass-Koopmans model etc., the R.M. Solow model is part of the category which characterizes the economic growth.*

The paper aim is the economic growth measurement and add-on of the R.M. Solow adjusted model.

Key words: reprezentive consumer; reprezentive firm; economic growth; technical progress; macroeconomic model; macroeconomic equilibrium; human capital; physical capital.

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We have to notice that a complete analysis of the economic growth must explain the social changes and the progress steps registered by the production technologies, also.

The measurement element in R.M. Solow adjusted model is showing that, on long term, the economic growth depends only by the technological progress, but on short term is depending also on the capital accumulation.

The add-on of the R.M. Solow model with human capital come to conclude that the output differences for each employee are having as source the physical and human capital also.

It is known that the process of economic growth is changing the society, because is having a strong impact over the living and the working, on one hand, and is realising transformations over groups which have to adept to these new production exigency, on the other hand.

In the sequel, a complete analysis of the economic growth has to explain the social changes and the progress steps registered by the production technologies, also.

Although it is noticed that the majority of the economic growth models are based on a close economy – the theoretical one – without taking in account the extern sector, on one hand because of the simplicity and on the other hand the main element of the economic growth is a consequence of the internal economy.

1. Brief presentation of the economical and mathematical model

Adequate to the J.M. Keynes laws, from a threshold when the people are having a specific level of the welfare, if the income is increasing, the lag between this and consumption is becoming bigger.

As a result, the rhythm of savings increasing, $\frac{\dot{S}(t)}{S(t)}$, is higher than the income one, $\frac{\dot{Q}(t)}{Q(t)}$, namely $\frac{\dot{S}(t)}{S(t)} > \frac{\dot{Q}(t)}{Q(t)}$.

R.M. Solow is starting, for building his model, from the condition of general macroeconomic equilibrium: aggregate demand is equal to the aggregate offer - $D(t) = Q(t)$, and because:

$$D(t) = C(t) + I(t)$$

$$Q(t) = C(t) + S(t)$$

and we deduce the relation:

$$I(t) = S(t), \text{ cu } \dot{K}(t) = I(t) = S(t) = sQ(t)$$

or, in average terms, after calculation, the condition of macroeconomic equilibrium is: $i(t) = s \times q(t)$ or:

$$i(t) = sf \times (k(t))$$

Therewith, from $\tilde{i}(t) = r_1 \times k(t)$, the equilibrium condition leads to the next differential ecuation:

$$\dot{k}(t) = i(t) - \tilde{i}(t) = s \times f(k(t)) - r_1 \times k(t)$$

which has the solution noted with k^*.

The last relation it is the fundamental equation of the R.M. Solow model, wherewith can be analysed the dynamic stability of the equilibrium, using the state diagram (Stancu, 2007, pp. 151-156, 165-173).

If we consider the case with the depreciation capital rate, ρ, the former equation becomes:

$$\dot{k}(t) = s \times f(k(t)) - (r_1 + \rho) \times k(t)$$

2. Measurement of the economic growth with the R.M. Solow model

In the R.M. Solow model, the economic growth on long term depends only on the technological progress, while on short term depends on the capital accumulation, too.

Using the relation from the R.M. Solow model (Rosca, Stancu, 2007, pp. 3-6), which shows that the production function, at the macroeconomic level, is a neo-classical one (linear homogeneous), with two interchangeably production factors, labour and capital:

$$Q(t) = Q(a(t)L(t), K(t))$$

and doing the differential, we obtain:

$$\dot{Q}(t) = \frac{\partial Q(t)}{\partial L(t)}\dot{L}(t) + \frac{\partial Q(t)}{\partial a(t)}\dot{a}(t) + \frac{\partial Q(t)}{\partial K(t)}\dot{K}(t) \quad (1)$$

with $\frac{\partial Q}{\partial L} = \frac{\partial Q}{\partial (aL)}a$, $\frac{\partial Q}{\partial a} = \frac{\partial Q}{\partial (aL)}L$

Dividing the relation (1) to $Q(t)$, we have:

$$\frac{\dot{Q}(t)}{Q(t)} = \frac{K(t)}{Q(t)} \times \frac{\partial Q(t)}{\partial K(t)} \times \frac{\dot{K}(t)}{K(t)} + \frac{L(t)}{Q(t)} \times$$

$$\div \frac{\partial Q(t)}{\partial L(t)} \times \frac{\dot{L}(t)}{L(t)} + \frac{a(t)}{Q(t)} \times \frac{\partial Q(t)}{\partial a(t)} \times \frac{\dot{a}(t)}{a(t)} =$$

$$= \beta_L(t)\frac{\dot{L}(t)}{L(t)} + \beta_K(t)\frac{\dot{K}(t)}{K(t)} + R_S(t)$$

relation which emphasizes the growth rate of the income, where $\beta_K(t)$ is the output elasticity, at the macroeconomic level, against the capital, $\beta_L(t)$ is the output elasticity, at the macroeconomic level, in respect with the labour, with $\beta_L(t) + \beta_K(t) = 1$, from the linear homogeneity hypothesis of the production function at macroeconomic level, and $R_S(t)$ it is *Solow residuum* which reflects other sources of growth, except of the capital accumulation.

From $\beta_L(t) = 1 - \beta_K(t)$, and deducting $\frac{\dot{L}(t)}{L(t)}$ from both members, the last relations can be written:

$$\frac{\dot{Q}(t)}{Q(t)} - \frac{\dot{L}(t)}{L(t)} = \beta_L(t))\frac{\dot{L}(t)}{L(t)} + \beta_K(t)\frac{\dot{K}(t)}{K(t)} + R_S(t) - \frac{\dot{L}(t)}{L(t)}$$

$$= (1 - \beta_K(t))\frac{\dot{L}(t)}{L(t)} + \beta_K(t)\frac{\dot{K}(t)}{K(t)} + R_S(t) - \frac{\dot{L}(t)}{L(t)}$$

$$= \beta_K(t)\left(\frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)}\right) + R_S(t) \quad (2)$$

Observations:

1. Growth rate of *K* and *L* are measured through empirical data;
2. Growth rate of the ratio output on employee depends on the output elasticity per capita, considering the labour technical endowment, the margin between growth capital rate and labour one and the Solow dross.

3. The extension of R.M. Solow model with human capital

In this model, the resources assignment between human capital and the physical one is exogenous.

The total quantity of goods and services made by the economy's employees is given by the function:

$$Q(t) = \left[a(t)\widetilde{L}(t)\right]^{\gamma_2}\left[K(t)\right]^{1-\gamma_2}, \text{ cu } 0<\gamma_2<1 \quad (3)$$

The dynamic equations of the capital, labour and technical progresses are:

$$\dot{K}(t) = s \times Q(t) - \rho \times K(t) \quad (4)$$

extending:

$$\dot{K}(t) = s \times Q(t) - \rho \times K(t) - r_2 \times K(t) \quad (4')$$

$$\frac{\dot{L}(t)}{L(t)} = r_1 \quad (5)$$

$$\frac{\dot{a}(t)}{a(t)} = r_2 \quad (6)$$

The human capital of each employee is depending on education the number of studying years), E.

Supposing the same education for all the employees, constant in time, E, then the human capital is described by the next relation:

$$\widetilde{L}(t) = L(t) \times k_{\min}(E)$$

where $k_{\min}(E)$ is the minimum of human capital on each employee, considering the studying years, with $k'_{\min}(\cdot) > 0$, $k''_{\min}(\cdot) > 0$.

Supposing that $k_{\min}(E) = e^{\tau \times E}$, $k_{\min}(0) = 1$, where $\tau > 0$ is the growth rate of the human capital, and noting the ratio capital on effective labour unit with k, given by:

$$k = \frac{K}{a \times k_{\min}(E) \times L} \quad (7)$$

which has the next dynamic equation:

$$\dot{k}(t) = s \times f(k(t)) - (r_1 + r_2 + \rho) \times k(t) \quad (8)$$

For the case when the production function is a Cobb-Douglas one, the relation (8) becomes:

$$\dot{k}(t) = s \times k^{1-\gamma_2}(t) - (r_1 + r_2 + \rho) \times k(t) \quad (9)$$

and the equilibrate growth trajectory is obtained for $\dot{k} = 0$, with the solution:

$$k^* = \frac{s}{(r_1 + r_2 + \rho)^{1/\gamma_2}} \quad (10)$$

Observations:

1. For the case $k = k^*$, the economy is one the equilibrate growth trajectory and $\frac{Q}{L}$ rises with rate $r_1 + r_2$;
2. Considering the R.M. Solow model without human capital, there are maintained the quantitative and qualitative of the savings rate changes s, over k and c, where c is representing the consumption for each employee;
3. The $a(t)$ trajectories and $k_{\min}(E)$ are not affected by of the savings rate changes: a rises with r_2 and $k_{\min}(\cdot)$ is independent on s.

The effect of E modification: $\dot{k}$ remains constant – not depending on E. Because $\frac{Q(t)}{L(t)} = a(t) \times k_{\min}(E)$, $q(\cdot)$ will growth in the same way with .

In conclusion, the differences from the ratio output on employee are depending on physical and human capital, too.

Note

(1) The human capital is defined as profesional abilities and employees knoledges.

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