The Models of Inter-temporal Consume

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Abstract. *The articol presents a category of consumption models which shows the manner how the expenses of consume in an economy are related to the available income achieved by this economy and the interest rate from the financial market. Since the income as well as the expenses of consume are realized in time, such dynamic models of consume are also referred to as models of inter-temporal consume, emphasizing therefore the fact that the available income achieved at a certain moment may be used for consume at a future moment, whereas the decision of consume taken at a current moment may consider the income that is to be achieved in the future.*

Key words: model of inter-temporal consume; budget restriction; transversality condition; synergic effect; systemic approach.

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1. The structure of the models of inter-temporal consume

First of all, we shall present the general structure of the models of inter-temporal consume and we shall present *a model of inter-temporal consume with contributions at the fund of pensions*

In order to consume, an individual needs a certain income which may derive from the wealth previously acquired (incomes from property) or from wage incomes that he obtains at present. The relation between income and consume at certain moments of time is performed with the help of the *budgetary restrictions*.

There are several forms of such budgetary restrictions that we shall analyze further on.

Let's take therefore the following *budgetary restriction*:

$$a_t = (1+r)\, a_{t-1} + y_t - c_t = (1+r)\, a_{t-1} + s_t \qquad (1)$$

where we noted with a_t the acquired wealth until the moment t, with r – interest rate (considered constant), with y_t – the available income realized at the moment t, with c_t – the consume expenses at the moment t and with $s_t = y_t - c_t$ the economy realized at the moment t. The relation (1) wants to suggest the fact that the individual begins each period having a wealth which derives from the previous period, he receives an income (from wages) equal with y_t, he consumes c_t and the rest he saves.

Keep in mind that y_t excludes the income, resulting from the holding of financial and real assets (therefore, it contains only the income generated by work) which is given by the term r a_{t-1}.

There are other possibilities too to write the budgetary restriction. Therefore, this relation may also be written as:

$$a_t = (1+r)\,(a_{t-1} + y_{t-1} - c_{t-1})$$

or

$$a_t = (1+r)\, a_{t-1} + y_{t-1} - c_{t-1}$$

each of them having an economic justification.

The static version of the budgetary restriction is:

$$y_t = c_t \qquad (2)$$

Although, a dynamic version of the relation of the type (2) may be written, in case of consume models, with two or more periods where the current value of the income is equal with the current value of consume if it is considered the duration of the entire life.

If we extend the budgetary restriction to three intervals of time, we shall obtain:

$$a_t = s_t + (1+r)\,[(1+r)\,a_{t-2} + s_{t-1}] = s_t + (1+r)\,s_{t-1} + (1+r)^2\,a_{t-2}$$

From here we may write:

$$a_t = \sum_{i=1}^{\infty} (1+r)^i\, s_{t-i} \tag{3}$$

assuming that $\lim\,(1+r)^i a_{t-i} = 0$, which is true if $a_{t-i} \rightarrow 0$. This means that after a number of years, the wealth accumulated until a given moment of time is entirely consumed.

If it is assumed, for instance, that the individual begins his life with a wealth equal to zero, this condition is necessary to accomplish.

Another means to obtain the relation between the wealth and saving is the direct solution of the equation with differences (1). This leads to:

$$a_t = a_{t+1}\,(1+r)^{-1} - s_{t+1}\,(1+r)^{-1} = s_{t+1}\,(1+r)^{-1} - s_{t+2}\,(1+r)^{-2} - \ldots$$

where:

$$a_t = -\sum_{j=1}^{\infty} (1+r)^{-j}\, s_{t+j} \tag{4}$$

assuming this time that $\lim\,(1+r)^{-j} a_{t+j} = 0$, which is also obvious.

Such a condition is determined as well by the *transversality condition.*

How we may interpret the conditions (3) and (4)? A dynamic version of the budgetary balance is:

$$VA\,(\{y_t\}) = VA\,(\{c_t\}) - a_t \tag{5}$$

where we have noted with VA the operator of current value. From the relation (5) we immediately obtain that:

$$\sum_{i=-\infty}^{\infty} (1+r)^{-i}\, y_{t+i} = \sum_{i=-\infty}^{\infty} (1+r)^{-i}\, c_{t+i} \tag{6}$$

relation which shows that the incomes realised during a cycle of life are used to support the consume during the entire life. In other words, an individual consumes all that he realised as income during his entire life. In this way, it is ensured the condition that the initial wealth of each individual to be equal with zero. Although an individual inherits a wealth, this may be considered as an income realised during a certain year of his life, for instance at the coming of age.

2. A model of inter-temporal consume with contributions at the pension's fund

We shall consider a consumer who gets an employment (begins to achieve incomes) in the year t, he retires in the year ν ($\nu > t$) and lives until the year T.

It is raised the issue of determining the optimal levels of consume in each year τ ($t < \tau < T$) so as the satisfaction of the consumer (the utility of his consume) to be maximum in the conditions when his expenses of consume do not overcome the realised incomes. During the active period, we assume that these incomes derive from the wage of which, besides consume, a part is deposited at the fund of social insurances and the fund of pensions. During the pension period, the incomes meant for the consume result from the fund of pensions and social insurances.

The issue of optimising the inter-temporal consume is written in this case as:

$$\begin{cases} \max\limits_{\{c_\tau\}} U(c_\tau) = \sum\limits_{\tau=t}^{T} (1+r)^{t-\tau} \dfrac{1}{1-\gamma} c_\tau^{1-\gamma} \\ \text{In the conditions:} \\ \sum\limits_{\tau=t}^{T} (1+r)^{t-\tau-1} c_\tau = a_t + y \sum\limits_{\tau=t}^{T} (1+r)^{t-\tau-1} \delta_\tau (1-\delta_\tau) \end{cases}$$

Here, r is the interest rate; the function of consume $\dfrac{1}{1-\gamma} c_\tau^{1-\gamma}$ is of the type Bernoulli with the parameter $0 < \gamma < 1$; a_t is the wealth held by the consumer in the moment t; y is the annual average income realised by the consumer from the wage.

With respect to the dimensions δ_τ and ρ_τ, these are defined as it follows:

$$\delta_\tau = \begin{cases} 1, & 0 < \tau \le n \\ \delta, & \nu < \tau \le T \end{cases}$$

represents the share of the annual income τ in the annual average income from the wage y. It is noticed that during the period of active life the entire annual average τ enters in the determination of the average income whereas, after retiring, this share is being reduced to δ which shows what percentage from the annual average income receives the consumer after retiring.

$$\rho_\tau = \begin{cases} \rho, & 0 < \tau \le n \\ 0, & \nu < \tau \le T \end{cases}$$

Here, ρ represents the rate of drawings from the annual income to the fund of pensions and social insurances (we shall consider it constant). During the period of active life, the consumer pays a percentage ρ from the income to the fund of pensions and social insurances, the rest of 1-ρ shall be used for consume. From here, results that:

$$1-\rho_\tau = \begin{cases} 1-\rho, & 0<\tau\le n \\ 1, & \nu<\tau\le T \end{cases}$$

shows that the consumer pays a part 1- ρ during the active life to the fund of social insurance and pensions ($0 < \tau \le \nu$) and he doesn't pay anything after retiring.

Therefore, during the active life, the individuals (consumers) pay $y \times \rho \times \nu$ from the income for social insurance and pensions. After retiring, they receive back $y \times \delta\,(T-\nu)$ from these funds.

At balance, the two sums must be equal, therefore:

$$y \times \rho \times \nu = y \times \delta\,(T-\nu)$$

where

$$\rho \times \nu = \delta T - \delta\nu \Rightarrow \delta\,(T-\nu) = \rho \times \nu$$

or:

$$\delta = \rho\frac{\upsilon}{T-\upsilon}$$

If we replace in the model the value of ä given from this relation, we shall have

$$P_1: \begin{cases} \max\limits_{\{c_\tau\}} U(c_\tau) = \sum\limits_{\tau=t}^{T} (1+r)^{t-\tau}\dfrac{1}{1-\gamma}c_\tau^{1-\gamma} \\ \text{in the condition:} \\ \sum\limits_{\tau=t}^{T} (1+r)^{t-\tau-1}c_\tau = \\ = a_t + y\sum\limits_{\tau=t}^{T}(1+r)^{t-\tau-1}(1-\rho) + y\sum\limits_{\tau=t}^{T}(1+r)^{t-\tau-1}\times\delta \end{cases}$$

Because we observe that:

$$\delta\nu(1-\rho_\tau) = \begin{cases} (1-\rho)\times 1 & 0<\tau<\nu \\ \delta\times 1 & \nu\le\tau\le T \end{cases}$$

For the problem P_1, the necessary conditions of optimization require the maximization of the Lagrangean, therefore of the function:

$$L(c_\tau;\lambda)$$

$$\triangleq \sum_{\tau=t}^{T}(1+r)^{t-\tau}\frac{1}{1+\gamma}c_\tau^{1-\gamma} -$$

$$-\lambda\left[a_t + y\sum_{\tau=t}^{\upsilon}(1+r)^{t-\tau-1}(1-\rho) + y\sum_{\tau=\upsilon+1}^{T}(1+r)^{t-\tau-1}\delta - \sum_{\tau=t}^{\upsilon}(1+r)^{t-\tau-1}c_\tau\right]$$

They are the following:

$$\frac{\partial L(c_\tau;\lambda)}{\partial c_\tau} = 0$$

$$\frac{\partial L(c_\tau;\lambda)}{\partial \lambda} = 0$$

We also notice that:

$$\sum_{\tau=t}^{\nu}(1+r)^{t-\tau-1} = \beta\frac{1-\beta^{\nu-t+1}}{1-\beta},$$

and:

$$\sum_{\tau=\nu+1}^{T}(1+r)^{t-\tau-1} = \beta^{\nu-t+2}\frac{1-\beta^{T-\nu}}{1-\beta}$$

Now we replace the sums so determined in the second condition and we obtain:

$$\beta\frac{1-\beta^{T-t+1}}{1-\beta}c_\tau = a_t + y(1-\rho)\beta\frac{1-\beta^{\nu-t+1}}{1-\beta} +$$

$$+ y\times\delta\times\beta^{\nu-t+2}\frac{1-\beta^{T-\nu}}{1-\beta}$$

From here, also considering the fact that, at balance:

$$\delta = \rho\frac{\nu}{T-\nu}$$

we obtain the expression which gives the optimal consume $c_\tau^*,\ \tau\in[t,T]$, and namely:

$$c_\tau^* = \frac{1}{1-\beta^{T-t+1}}\left\{r\times a_t + y\left[(1-\rho)-(1-\rho)\beta^{\nu-t+1} + \rho\frac{\nu}{T-\nu}\beta^{\nu-t+1} - \rho\frac{\nu}{T-\nu}\beta^{T-t+1}\right]\right\}$$

Now, we may determine the effects that different dimensions have when they interfere in the above relation differentiating totally c_δ with respect to these. We shall have:

$$dc = \frac{dc}{dr}dr + \frac{\partial c}{\partial a_t}da_t + \frac{\partial c}{\partial y}dy + \frac{\partial c}{\partial T}dT + \frac{dc}{dv}dv + \frac{\partial c}{\partial \rho}d\rho$$

Each factor from the right expresses the marginal influence that the respective factor has in its modification c with a unit.

For instance, $\dfrac{\partial c}{\partial a_t} = \dfrac{r}{1-\beta^{T-t+1}}$, shows how much the consume increases if the wealth increases with a unit. The analysis may continue in the same manner for each factor (*y*, *T*, υ, ñ).

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