Finances, in the Light of Utility and Equity Criteria

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Abstract. *This article intends to present tendencies in the regional industry and specialisation in Romania during the transition period. The authors present the main tendencies in the analysis of concentration and specialisation and industry developments by electoral cycles as well. An econometric model for the analysis in time of concentration and specialisation in Romania during the transition period is also presented.*

Key words: utility; equity; preference for risk; risk-averse.

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JEL Codes: D01, D81.
REL Codes: 7J, 11B.

The theory of utility can be successfully used in order to emphasize the principles of fiscal equity which are the basis of every modern fiscal system. Adam Smith identified at the end of the XIX-th century the following tax principles: rightness, certainty, comfort and efficiency. The fiscal equity is defined by financial theory as social justice and its existence is presumed by the observance of the following conditions:

- differentiated taxation of the revenues and fortunes;
- non-taxable minimum income;
- the correlation of fiscal duties;
- the generality of taxation.

The fundamental objective of the utility theory under uncertainty conditions is given by the rationalization of the choices made by the individuals in risky situation on the financial markets and not only. Each individual has an utility function and its form depends on the individual's attitude towards risk. The utility is defined separately for each individual in relation to his subjective perception.

Depending on individuals' attitude towards risk, there are three categories of utility functions:

- concave, specific to those individuals who are risk-averse;
- convex, specific to those individuals who have preference for risk;
- linear, specific to individuals that are risk-neutral.

We note with U the utility function of an individual in relation with the obtained income V. This expresses the individual behaviour with respect to taxable income and fulfills the following conditions: U'>0 and U"<0. This means that the utility of a loss in revenues is higher than the utility of an earning.

Since taxes can be considered as a sacrifice of obtained income, from the fiscal equity point of view the following concepts have been drawn:

- the absolute equal sacrifice;
- the proportional equal sacrifice;
- the marginal equal sacrifice.

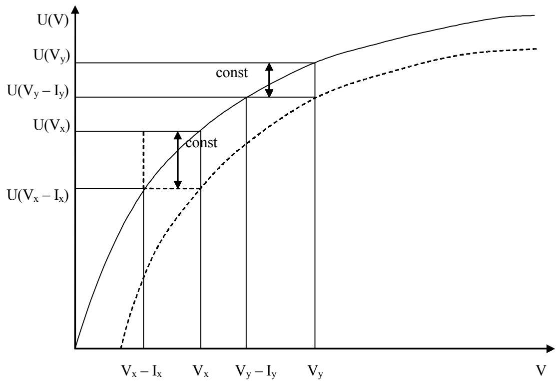
I. The absolute equal sacrifice

From the fiscal equity point of view, this concept means the loss of the utility registered by the individuals belonging to a fiscal community.

In order to develop this model, we assume that there is an individual X, considered to be a reference individual and characterized by a gross income $V\text{-}_x$ and a tax I_x. For any other individual Y, we try to determine the level of tax I_y given the income V_y and taking into account the following condition:

$$U(V_x) - U(V_x - I_x) = U(V_y) - U(V_y - I_y)$$

The absolute equal sacrifice can be represented as follows:



The tax due by contributor Y can be evaluated taking into account the bijectivity of the utility function:

$$U(V_y - I_y) = -U(V_x) + U(V_x - I_x) + U(V_y)$$

$$V_y - I_y = U^{-1}[U(V_y) + U(V_x - I_x) - U(V_x)]$$

$$I_y = V_y - U^{-1}[U(V_y) + U(V_x - I_x) - U(V_x)]$$

Interesting results can be obtained by observing the absolute equal sacrifice in case of a logarithmic utility function.

Let U(V) = lnV. The absolute equal sacrifice condition requires that

$$\ln(V_x) - \ln(V_x - I_x) = \ln(V_y) - \ln(V_y - I_y)$$

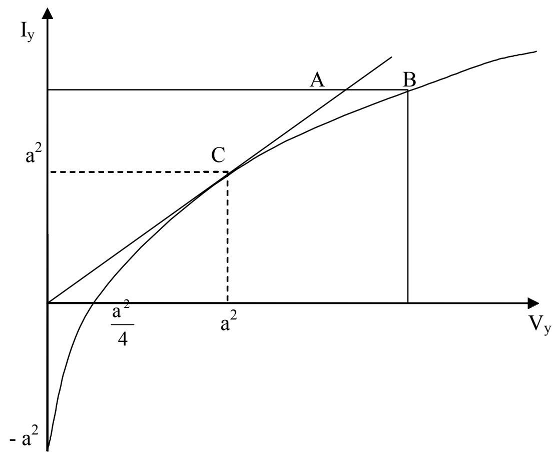
$$\ln\frac{V_x}{V_x - I_x} = \ln\frac{V_y}{V_y - I_y} \Rightarrow \frac{V_x}{V_x - I_x} =$$
$$= \frac{V_y}{V_y - I_y} \Leftrightarrow \frac{I_x}{V_x} = \frac{I_y}{V_y}$$

As we can see, the ratio of tax value to obtained income is constant regardless of contributor and this means that the fiscal system uses proportional taxation. This principle represents a direct expression of equality before taxation. In case the power to contribute changes, contributors with lower power to contribute shall harder stand the payment of the taxes, instead of those with higher power to contribute.

In case the utility function is of radical type ($U(V) = \sqrt{V}$) we have:

$$\sqrt{V_x} - \sqrt{V_x - I_x} =$$
$$= \sqrt{V_y} - \sqrt{V_y - I_y} = a = \text{const}$$
$$\Rightarrow \sqrt{V_y - I_y} = \sqrt{V_y} - a$$
$$V_y - I_y = V_y + a^2 - 2a\sqrt{V_y}$$
$$\Rightarrow I_y = 2a\sqrt{V_y} - a^2$$

The chart of the tax paid by a contributor in relation to his taxable income is presented below:



It can be noticed that the chart is located under the first bisectrix and this is tangent in the point C, where the taxable income is equal to the tax. Segment AB represents the net income. In case the taxable income decreases under $\frac{a^2}{4}$ it can be noticed that the tax is negative and this means that there should be a nontaxable minimum greater than or equal to $\frac{a^2}{4}$.

Let us now consider the following utility function(1):

$$U(V) = -e^{-\eta V}, \eta = const, \eta > 0$$

The absolute equal sacrifice condition requires that

$$-e^{-\eta V_x} + e^{-\eta(V_x - I_x)} = -e^{-\eta V_y} + e^{-\eta(V_y - I_y)} =$$
$$= a = const, a > 0, a < 1$$

$$-e^{-\eta V_y} + e^{-\eta(V_y - I_y)} = a \Rightarrow$$
$$-\eta \times (V_y - I_y) = \ln(a + e^{-\eta V_y})$$

$$V_y - I_y = -\frac{1}{\eta} \times \ln(a + e^{-\eta V_y}) \Rightarrow$$
$$I_y = V_y + \frac{1}{\eta} \times \ln(a + e^{-\eta V_y})$$

Assume further that $a + e^{-\eta V_y} < 1$ (otherwise, the tax would be greater than or equal to the contributor's income).

The tax rate is:

$$\frac{I_y}{V_y} = 1 + \frac{\frac{1}{\eta} \times \ln(a + e^{-\eta V_y})}{V_y}$$

It can be shown that the tax rate is an increasing function of the contributor's taxable income.

II. The proportional equal sacrifice

The proportional equal sacrifice means that a constant ratio is maintained between the utility of net income and the utility of gross income:

$$\frac{U(V_x - I_x)}{U(V_x)} = \frac{U(V_y - I_y)}{U(V_y)}$$

If the utility function is logarithmic we have:

$$\frac{\ln(V_x - I_x)}{\ln V_x} = \frac{\ln(V_y - I_y)}{\ln V_y} = a = const,$$
$$a < 1$$

$$\ln(V_y - I_y) = a \ln V_y \Rightarrow V_y - I_y = V_y^a$$
$$\Rightarrow I_y = V_y - V_y^a = V_y(1 - V_y^{a-1})$$

$$\frac{I_y}{V_y} = 1 - \frac{1}{V_y^{1-a}}, 1 - a > 0$$

It can be noticed that the tax rate $\frac{I_y}{V_y}$ is an increasing function of taxable income and this means that the tax rate grows as taxable income grows.

In case the utility function is a radical function ($U(V) = \sqrt{V}$) we have:

$$\sqrt{\frac{V_x - I_x}{V_x}} = \sqrt{\frac{V_y - I_y}{V_y}} = a = const, a < 1$$

$$\frac{V_y - I_y}{V_y} = a^2 \Rightarrow V_y - I_y = a^2 V_y \Rightarrow$$
$$I_y = V_y(1 - a^2) \Rightarrow \frac{I_y}{V_y} = 1 - a^2 = const$$

In this case the same tax rate is used for all individuals, with the advantages and disadvantages resulted thereof.

For a CARA utility function ($U(V) = -e^{-\eta V}, \eta = const, \eta > 0$) we have

$$\frac{e^{-\eta(V_x - I_x)}}{e^{-\eta V_x}} = \frac{e^{-\eta(V_y - I_y)}}{e^{-\eta V_y}} = a = const, a > 1$$

$$\frac{e^{-\eta(V_y - I_y)}}{e^{-\eta V_y}} = a \Rightarrow e^{-\eta(V_y - I_y)} = a \times e^{-\eta V_y} \Rightarrow$$
$$-\eta \times (V_y - I_y) = -\eta V_y + \ln a$$

$$I_y = \frac{1}{\eta} \ln a.$$

The tax is the same for all contributors, regardless of their taxable income. Obviously, this principle of taxation is unfair, as it does not take into account the power to contribute of tax payers.

III. The marginal equal sacrifice

Marginal equal sacrifice means that the loss of marginal utility between net income and gross income is constant:

$$U'(V_x - I_x) - U'(V_x) =$$
$$= U'(V_y - I_y) - U'(V_y)$$

In case of a logarithmic utility function we have:

$$\frac{1}{V_x - I_x} - \frac{1}{V_x} = \frac{1}{V_y - T_y} - \frac{1}{V_y} =$$
$$= a = \text{const}, a > 0$$

$$\frac{1}{V_y - I_y} = a + \frac{1}{V_y} = \frac{aV_y + 1}{V_y}$$

$$V_Y - I_y = \frac{V_y}{aV_y + 1} \Rightarrow I_y = V_y - \frac{V_y}{aV_y + 1}$$

$$I_y = V_y \left(1 - \frac{1}{aV_y + 1}\right)$$

$$I_y = \frac{aV_y^2}{aV_y + 1}$$

$$\frac{I_y}{V_y} = \frac{aV_y}{aV_y + 1} = 1 - \frac{1}{aV_y + 1}$$

The tax rate is an increasing function of taxable income: the greater the taxable income of contributor Y, the greater the tax due.

If we consider ($U(V) = \sqrt{V}$) we have:

$$\frac{1}{2\sqrt{V_x - I_x}} - \frac{1}{2\sqrt{V_x}} =$$
$$= \frac{1}{2\sqrt{V_y - I_y}} - \frac{1}{2\sqrt{V_y}} =$$
$$= a = \text{const}, a > 0$$

$$\frac{1}{2\sqrt{V_y - I_y}} - \frac{1}{2\sqrt{V_y}} = a \Rightarrow$$

$$\frac{1}{V_y - I_y} = \left(2a + \frac{1}{\sqrt{V_y}}\right)^2 =$$

$$4a^2 + \frac{4a}{\sqrt{V_y}} + \frac{1}{V_y} =$$

$$\frac{4a^2 V_y + 4a\sqrt{V_y} + 1}{V_y}$$

$$V_y - I_y = \frac{V_y}{4a^2 V_y + 4a\sqrt{V_y} + 1} \Rightarrow$$

$$I_y = \frac{4a^2 V_y^2 + 4aV_y\sqrt{V_y}}{4a^2 V_y + 4a\sqrt{V_y} + 1}$$

$$\frac{I_y}{V_y} = \frac{4a^2 V_y + 4a\sqrt{V_y}}{4a^2 V_y + 4a\sqrt{V_y} + 1} =$$
$$= 1 - \frac{1}{4a^2 V_y + 4a\sqrt{V_y} + 1}$$

The tax rate is an increasing function of taxable income.

Setting CARA

($U(V) = -e^{-\eta V}, \eta = \text{const}, \eta > 0$)

we have:

$$\eta e^{-\eta(V_x - I_x)} - \eta e^{-\eta V_x} =$$
$$= \eta e^{-\eta(V_y - I_y)} - \eta e^{-\eta V_y} = a = \text{const}, a > 0$$

$$\eta e^{-\eta(V_y - I_y)} - \eta e^{-\eta V_y} = a \Rightarrow$$

$$e^{-\eta(V_y - I_y)} = \frac{1}{\eta} a + e^{-\eta V_y}$$

$$-\eta \times (V_y - I_y) = \ln\left(\frac{1}{\eta} a + e^{-\eta V_y}\right) \Rightarrow$$

$$V_y - I_y = -\frac{1}{\eta} \times \ln\left(\frac{1}{\eta} a + e^{-\eta V_y}\right)$$

$$I_y = V_y + \frac{1}{\eta} \times \ln\left(\frac{1}{\eta} a + e^{-\eta V_y}\right)$$

We must assume further that $\frac{1}{\eta}a + e^{-\eta V_y} < 1$ (otherwise, the tax would be greater than or equal to the taxable income).

The tax rate is:

$$\frac{I_y}{V_y} = 1 + \frac{\frac{1}{\eta} \times \ln\left(\frac{1}{\eta}a + e^{-\eta V_y}\right)}{V_y}$$

The tax rate is an increasing function of the contributor's taxable income.

Note

(1) These functions are constant absolute risk aversion (CARA) utility functions.

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