# Optimality, Rational Expectations and Time Inconsistency Applied to Inflation Targeting Strategy 

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#### Abstract

The purpose of this paper is to analyse the characteristics of an inflation targeting strategy, using the Barro-Gordon model specific tools. This paper uses the initial Barro-Gordon concepts of inflationary social costs and benefits, adding a new dimension generated by the cost of output deviating from the potential level. The main contribution of this paper is the exhaustive study of the time inconsistency problem generated by the very existence of a policymaker-established inflation rate. The mathematic simulation of a model allowed a complete analysis of several parameters' influence (parameters such as the optimal rate of inflation, the discount rate, the importance structure of inflationary social cost) on the applicable range of the target inflation rate, range that guarantees that the policymakers have no incentive to break their own rules, or at least this incentive is somewhat inferior to the future cost of doing so.


Key words: optimal inflation rate; inflation targeting strategies; the Barro-Gordon model; output gap; central bank credibility.

## Inflationary costs function

In the first chapter of this paper, we have elaborated a mathematical function for inflationary costs, altogether with a policymaker reaction function to the private agents' inflationary expectations as well as the optimal inflation rate.

Inflation is generally viewed as one negative monetary phenomenon, due to its tendency to distort the economic processes in one country by the reallocation of income (more or less arbitrary), by generating a general state of uncertainty at macroeconomic level that may limit the capital investment (both foreign and national), and finally the general state of private agents' distrust in the national currency. Through this paper, though, we will not assume that inflation is only a negative economic process, as it is generally known that zero inflation may be considered a lack of economic growth. Furthermore, as the Barro-Gordon model suggests, it is not rational for the private agents to expect a zero level of inflation.

One first inflationary cost to be analyzed is the difference between the present inflation rate $\left(r_{i}^{p}\right)$ and the optimal inflation rate $\left(r_{i}^{*}\right)$. Therefore,

$$
\begin{equation*}
C_{1}=\frac{a}{2} \times\left(r_{i}^{p}-r_{i}^{*}\right)^{2} . \tag{1}
\end{equation*}
$$

The " $a$ " parameter is greater than zero and represents the relative importance of this particular inflationary cost in the general structure of inflationary costs ${ }^{(1)}$. The higher the present inflation rate is compared to the optimal one, the higher the social cost will be.

Another inflationary cost is generated by the difference between the present unemployment rate and NAIRU, a difference
that, through Okun's law, may be correlated with the difference between the present output and the potential one, as follows:

$$
\begin{equation*}
C_{2}=b \times\left(Y_{p}-Y^{*}\right)^{2} \tag{2}
\end{equation*}
$$

(where $b$ is greater than zero, suggesting the relative importance of the output differential inflationary cost in the general structure of inflationary costs).

As the real output $\left(Y_{p}\right)$ is more different from the potential one $\left(Y^{*}\right)$, the resources will be in a greater degree misused at macroeconomic level, fact which will generate a certain variation of the inflation rate from its target. To formalize this second cost and express it as depending of the inflation rate, we have used an expectation function stated by Lucas (1981):

$$
\begin{align*}
& Y_{p}=Y^{*}+c \times\left(r_{i}^{p}-r_{i}^{a}\right) \\
& Y_{p}-Y^{*}=c \times\left(r_{i}^{p}-r_{i}^{a}\right) \tag{3}
\end{align*}
$$

Lucas considers that the output gap is directly related to the differential between the present inflation rate and the expected one. Combining equations (2) and (3) we derive a mathematical formalization of a second inflationary cost (depending on the public expectations regarding the inflation rate):

$$
\begin{equation*}
C_{2}=b \times c^{2} \times\left(r_{i}^{p}-r_{i}^{a}\right)^{2} \tag{4}
\end{equation*}
$$

Adding the (1) equation to (4) we obtain a total inflationary cost (C):

$$
\begin{align*}
& \mathrm{C}=\mathrm{C}_{1}+\mathrm{C}_{2}=\frac{a}{2} \times\left(r_{i}^{p}-r_{i}^{*}\right)^{2}+ \\
& +b \times c^{2} \times\left(r_{i}^{p}-r_{i}^{a}\right)^{2} \tag{5}
\end{align*}
$$

A potential benefit from high inflation can result from the reduction of the public debt's real value. As the present inflation rate is higher than the expected one, the public
debt instruments' yields will appear much more appealing to the private agents than they will be in reality. From this point of view, we can analyze a specific inflationary benefit gained by the government:

$$
\begin{equation*}
B=d \times\left(r_{i}^{p}-r_{i}^{a}\right), \tag{6}
\end{equation*}
$$

( $d$ is greater than zero, suggesting the relative importance of the inflationary benefit).

Costain (2004) elaborates a mathematical function of the general inflationary cost, considering that a more precise formalization will take into account the negative inflationary cost represented by the aforementioned benefit (6). Thus, we will express the general inflationary cost as in (7) for the rest of the paper:

$$
\begin{align*}
& C^{*}=C-B=\frac{a}{2} \times\left(r_{i}^{p}-r_{i}^{*}\right)^{2}+b \times c^{2} \times \\
& \times\left(r_{i}^{p}-r_{i}^{a}\right)^{2}-d \times\left(r_{i}^{p}-r_{i}^{a}\right) \tag{7}
\end{align*}
$$

## The Central Bank's reaction function

Every Central Bank's objective is to minimise the inflationary cost, by reducing the standard deviation of present inflation rate from the optimum level and from the private agents' expected inflation level. The only directly controllable parameter by a monetary authority is the present inflation rate: $\mathrm{r}_{\mathrm{i}}{ }^{\mathrm{p}}$. The expected inflation rate can only be indirectly influenced by the previous Central Bank's actions and its general level credibility (as related to the price stability goal). Minimising the total inflationary cost, we will obtain:

$$
\begin{aligned}
& \frac{\partial C^{*}}{\partial r_{i}^{p}}=0 \Rightarrow \\
& a \times\left(r_{i}^{p}-r_{i}^{*}\right)+2 \times b \times c^{2} \times\left(r_{i}^{p}-r_{i}^{a}\right)-d=0
\end{aligned}
$$

Rearranging, we obtain a Central Bank's reaction function of the present inflation rate:

$$
\begin{equation*}
r_{i}^{p}=\frac{a \times r_{i}^{*}+2 \times b \times c^{2} \times r_{i}^{a}+d}{a+2 \times b \times c^{2}} \tag{8}
\end{equation*}
$$

Thus, the inflation rate that leads to a minimal inflationary cost depends on the expected inflationary rate. Furthermore, as the optimal inflation rate is higher, the Central Bank will have incentives to set a higher inflation rate. Under these conditions, the policymaker will have incentives not to take steps in order to reduce inflation, as the real public debt will decrease. If the Central Bank's decision to set the inflation rate to zero is credible, then the inflation rate that leads to the minimum level of inflationary costs will be positive, which leads to the conclusion that it is irrational for the private agents to expect a zero level of inflation:

$$
\begin{equation*}
r_{i}^{p}=\frac{a \times r_{i}^{*}+d}{a+2 \times b \times c^{2}}>0 \tag{9}
\end{equation*}
$$

The Central Bank's reaction function demonstrates that a present inflation rate higher than the optimal one will lead to a restrictive monetary policy that will result in decreasing inflation rate expectations. Costain imagined three basic scenarios of analyzing the private agents' inflationary expectations, scenarios that we used in order to compare the inflationary costs.

## I. The Central Bank announces the optimal inflation rate to be applied

We assume that this inflation target is credible for the private agents. Under these conditions, the expectations are equal to the
optimal rate: $r^{a}{ }_{i}=r_{i}^{*}$, and the reaction function will consequently be:

$$
\begin{equation*}
r_{i}^{p}=r_{i}^{*}+\frac{d}{a+2 \times b \times c^{2}} \tag{10}
\end{equation*}
$$

When the expected inflation rate is equal to the optimal one, the present inflation rate will be greater than the optimal one. Replacing (10) in the inflationary costs equation (7) we obtain:

$$
\begin{align*}
& C^{*}(\text { scenario } 1)= \\
& =\frac{-a \times d^{2}-2 \times b \times c^{2} \times d^{2}}{2\left(a+2 \times b \times c^{2}\right)^{2}}<0 \tag{11}
\end{align*}
$$

In this case, the inflationary cost has a negative value; in fact it represents a social benefit. The reason behind this is the reduced level of taxes due to a lower level of real public debt. Furthermore, an inflation rate higher than the optimal level leads to a higher economic output than the potential one, resulting in a lower unemployment rate compared to NAIRU.

## II. The private agents have rational expectations

This scenario studies the situation where the private agents' expectations can be considered rational. If they would expect an inflation rate equal to zero, then the Central Bank will set a positive inflation rate, so this expectation level is not rational. Furthermore, if the expectations will equal the optimal rate of inflation, the Central Bank will set the inflation rate according to the first scenario, so this expectation will not be a rational choice either. To conclude, the
expected rate of inflation can be considered rational if the private agents will accurately foresee the policymaker's intention (the present inflation rate): $r_{i}^{a}=r^{p}{ }_{i}$.

Generally speaking, the economists consider that the private agents act rationally when they accurately foresee the rational choices of other economic agents. Regarding inflation, the rational expectations equilibrium occurs when the Central Bank's inflation rate choice is resulted from the reaction function and the private agents expect the very same inflation rate. $\left(r_{i}^{a}=r^{p}\right)$ :

$$
\begin{equation*}
r_{i}^{p}=\frac{a \times r_{i}^{*}+2 \times b \times c^{2} \times r_{i}^{p}+d}{a+2 \times b \times c^{2}} \tag{12}
\end{equation*}
$$

From the previous equation we obtain an expression of the inflation rate given the private agents' rational expectations:

$$
\begin{equation*}
r_{i}^{p}=r_{i}^{*}+\frac{d}{a} \tag{13}
\end{equation*}
$$

From (13) we find that the rational expectations inflation rate $\left(r_{i}^{p}\right)$ is higher than the optimal inflation rate. This is true in the hypothesis that the Central Bank will set an inflation rate higher than the expectations (the Barro-Gordon model hypothesis). Therefore, expecting the optimal inflation rate will lead to the actual inflation rate from the first scenario. Furthermore, rational anticipation of the Central Bank's behaviour will lead to second scenario's inflation rate. Consequently, the inflation rate resulting from the second scenario will be higher than the one in the first scenario, as shown in (14):

$$
\begin{align*}
& r_{i}^{p}(\text { scenario } 2)=r_{i}^{*}+\frac{d}{a}> \\
& >r_{i}^{p}(\text { scenario } 1)= \\
& =r_{i}^{*}+\frac{d}{a^{2}+2 \times b \times c} \tag{14}
\end{align*}
$$

The two inflation rates (from the first and second scenario) will be equal when the benefits of the lower public debt are very low, the case of a non-zero, low rate of inflation.

Due to rationality of expectations, private agents are as aware of the real economic evolution as the Central Bank is and consequently, if the Central Bank targets an unemployment rate different from NAIRU, while announcing a zero inflation target, the latter is not credible (as private agents expect a positive inflation rate, high enough to considerably influence the unemployment rate).

The inflationary cost function for the second scenario presents as follows:

$$
\begin{align*}
& C^{*}(\text { scenario } 2)=\frac{a}{2} \times\left(\frac{d}{a}\right)^{2}= \\
& =\frac{d^{2}}{2 \times a}>0 . \tag{15}
\end{align*}
$$

There is a social loss due to the inflation rate higher than the optimal one. Comparing the two scenarios from the inflationary costs criteria, we notice that the higher cost occurs in the rational expectations scenario when the Central Bank's targets credibility is evaluated by the private agents. One possibility to reduce this cost is the Central Bank's consistent behaviour in targeting a credible monetary policy rule.

## III. Setting an inflation rate target rule

We assume that the Central Bank targets a low differential between the present inflation rate and the optimal one. The price stability target is of uttermost importance, the Central Bank neglecting the importance of the output differential. Therefore, the Central Bank will apply an optimal inflation targeting strategy. The private agents will expect exactly the amount of inflation targeted by the Central Bank, and the present inflation rate, the expected and the optimal one will be equal: $r^{a}=r_{i}{ }_{i}=r_{i}^{*}$. One important additional condition for the existence of the aforementioned equality is represented by a fiscal policy consistent with the Central Bank's monetary policy (although the benefits from the lower public debt will decrease).

The inflationary costs function for this third scenario will consequently be:

$$
\begin{align*}
& C^{*}(\text { scenario } 3)=\frac{a}{2} \times\left(r_{i}^{p}-r_{i}^{*}\right)^{2}+ \\
& +b \times c^{2} \times\left(r_{i}^{p}-r_{i}^{a}\right)^{2}-  \tag{16}\\
& -d \times\left(r_{i}^{p}-r_{i}^{a}\right)=0 .
\end{align*}
$$

Comparing the three scenarios, we will obtain that
$r_{i}^{p}\left(\right.$ scenario 2) $>r_{i}^{p}($ scenario 1$)>r_{i}^{p}$ $($ scenario 3$)=r_{i}^{*}$.

Therefore, as long as the Central Bank has discretion over the inflation rate the differential between the present inflation rate and private agents' expectations will be positive. This is the hypothesis that the policymaker has the incentive to deceive the private agents and is higher in the
rational expectations scenario. A welfarebased comparative analysis of the inflationary costs from all three scenarios will show the following result:

$$
\begin{align*}
& C^{*}(\text { scenario } 2)>C^{*}(\text { scenario } 3)= \\
& =0>C^{*}(\text { scenario } 1)
\end{align*}
$$

The most beneficial scenario will therefore be the first one, but it is not applicable in the long run. The Central Bank may succeed in deceiving the private agents only for a limited period, but in the long run, the private agents will anticipate this behaviour. Therefore the Central Bank has only two viable options: setting an inflation rate rule or choosing the inflation rate according to the reaction function. If it chooses discretion rather than rule, then the economy will face a social loss (scenario 2 - rational expectations). Otherwise, following an inflation rate rule, there will be no social losses or no social gains.

## The problem of dynamic inconsistency

 when approaching the monetary policiesIn the previous subchapter, we have analyzed the importance of anticipating the private economic agents upon the inflation rate, which will be adopted by the monetary authorities, by analyzing the possible scenarios, which could take place in such a situation. If, on a short term, the population's misleading, through promises regarding the adoption of the optimal inflation rate, can yield a social benefit (a negative cost), on a longterm, because of the central authority's loss in reputation, only the adoption of rule of targeting the inflation can diminish the social $\operatorname{cost}^{(2)}$. However, adopting a rule represents a
disadvantage solution regarding the moral hazard. The compliance with a rule does not represent a certainty for any economic agent, especially under the terms in which its noncompliance would bring a certain short-term social benefit compared to case in which the inflation rate would be the ex-ante established one. The other side is represented by the future cost of this decision, a supplementary cost which occurs because of the authority's loss in reputation and because of the modifications in the economic agents' expectances. In case the central bank, which follows a strategy of targeting the inflation, does not obtain a legal current inflation with a target (or which could be in compliance with the foreseen fluctuation interval), then its credibility will get diminished and the result of this fact will be the intensifications in the future inflationary anticipations.

Modelling the economic agents' behaviour during the period $t$, according to the compliance or the non-compliance with the inflation targeting during the period $t-1$, the following system will be obtained:

$$
\left\{\begin{array}{l}
r_{i / t}^{a}=r_{i}^{*} \Leftrightarrow r_{i / t-1}^{p}=r_{i / t-1}^{a}  \tag{18}\\
r_{i / t}^{a}=r_{i}^{p} \Leftrightarrow r_{i / t-1}^{p} \neq r_{i / t-1}^{a}
\end{array}\right.
$$

We have noted with $\mathrm{r}_{\mathrm{i}}^{*}$ the inflation rate targeting, proposed by the monetary authority. Thus, the system (18) will be explained as it follows: if, during the previous period, the effective/current inflation rate corresponds to the population's expectances, and automatically to the target, the expectances for the current period will be equal to that target. Otherwise, the central bank loses its reputation and the anticipations will modify, following the scenario of the rational expectances.

The temptation of non-complying with the rule - mathematic formalization

We shall note the temptation of noncomplying with rule of inflation rate ( $\mathrm{r}_{\mathrm{i}}{ }^{*}$ ) with T and we shall consider a function, depending on the inflation targeting. The temptation of non-complying with the target will be equal to the social cost corresponding to applying this rule, out of which we subtract the social cost of its noncompliance. The higher this difference is, the more the stimuli to ignore the rule get increased. According to the relation (7), and taking into account the fact that the economic agents' expectations are equal to the inflation rate target, the cost of the compliance with the rule will be equal to:

$$
\begin{equation*}
C^{*}{ }_{\text {rule }}=\frac{a}{2} \times\left(r^{*}-r_{i}^{*}\right)^{2} \tag{19}
\end{equation*}
$$

On the other hand, the private economic agents' disappointment will have as a result another cost, that of the compliance with the inflation targeting. This one is formed on the basis of a current inflation rate which differs from the rule and, according to the expectations (in case the expectances correspond to the inflation targeting), according to the equation (8):

$$
\begin{align*}
& r_{i / \text { non-compliance }}^{p}= \\
& =\frac{a \times r_{i}^{*}+2 \times b \times c^{2} \times r_{i}^{*}+d}{a+2 \times b \times c^{2}} \tag{20}
\end{align*}
$$

Thus, the cost of non-compliance with the rule, under the terms of the expectances corresponding to the rule, is as it follows:

$$
\begin{align*}
& C_{\text {non-compliance }}^{*}=\frac{a}{2} \times\left(r_{i / \text { non-compliance }}^{p}-r_{i}^{*}\right)^{2}+ \\
& +b \times c^{2} \times\left(r_{i / \text { non-compliance }}^{p}-r_{i}^{*}\right)^{2}-  \tag{21}\\
& -d \times\left(r_{i / \text { non-compliance }}^{p}-r_{i}^{*}\right)
\end{align*}
$$

Accordingly, the temptation of noncomplying with the rule according to the inflation rate which represents the rule will be as it follows:

$$
\begin{equation*}
T\left(r_{i}^{\#}\right)=C_{\text {rule }}^{*}-C^{*}{ }_{\text {non-compliance }} \tag{22}
\end{equation*}
$$

## The updated future cost - mathematic

 formalizationWe shall note the updated future cost of the non-compliance with the inflation rate rule with CVS and we shall consider it, as in the temptation case, to be dependant to the inflation rate rule.

The updated future cost can be treated as a difference between the future social costs, in the case in which the rule had been complied with in the present, everything updated with an discount rate (r).

In the future period, in case the rule would have not been complied with in the present, according to the system (18) which explains the anticipation choices of the economic agents, the social cost (not updated up for the present moment) will follow the reasonability scenario presented in the previous subchapter and it will be as it follows:

$$
\begin{equation*}
C_{\text {non-compliance }}^{\text {fuutre }}=\frac{d^{2}}{2 \times a} \tag{23}
\end{equation*}
$$

The future social cost of the compliance with the rule (not updated) will be equal to the cost of the compliance with the rule calculated according to the equation (19), because of the fact that neither of the conditions modifies. Thus, updating of the future cost to the interest rate we shall obtain another function of $r_{i}^{*}$ corresponding to this value:

$$
\begin{equation*}
\operatorname{CVA}\left(r_{i}{ }^{*}\right)=\frac{1}{1+r}\left(C_{\text {non-compliance }}^{\text {future }}-C_{\text {rule }}^{*}\right) \tag{24}
\end{equation*}
$$

After defining the terms of temptation of non-complying with the rule and of future updated cost, it is relatively simple to estimate that an inflation rate targeting is applicable only in case the temptation of non-complying with it is inferior to the future updated cost of this action. Otherwise, the authorities will always be stimulated not to comply with their own decisions, because the current benefit will always be superior to the future loss, but such behaviour cannot be assimilated to a rule and the result will be that of irrecoverable credibility losses of the monetary authority. Thus, in order that an inflation rate rule ( $r_{i}^{*}$ ) could be considered as applicable, the following condition must be complied with:

$$
\begin{equation*}
\operatorname{CVA}\left(r_{i}^{*}\right) \geq T\left(r_{i}^{*}\right) \tag{25}
\end{equation*}
$$

## Is the optimal inflation rate an

 applicable rule?First of all, we shall verify if the optimal inflation rate $\left(r_{i}{ }^{*}\right)$ is applicable in an economy in case of the current temptation - future updated cost. If the rule is complied with, then the current inflation rate, as well as the population's expectances and the optimal inflation rate are equal, and this will lead to a null social cost, according to the equation (7).

$$
\begin{equation*}
r_{i}^{p}=r_{i}^{*}=r_{i}^{a} \Rightarrow C^{*} \text { rule }=0 \tag{26}
\end{equation*}
$$

If the rule is not complied with, the current inflation rate can be calculated according to the relation (20), under the terms in which the inflation targeting will be equal to the optimal inflation:

$$
\begin{align*}
& r_{i / \text { non-compliance }}{ }^{p 1}=\frac{a \times r_{i}^{*}+2 \times b \times c^{2} \times r_{i}^{*}+d}{a+2 \times b \times c^{2}}= \\
& =r_{i}^{*}+\frac{d}{a+2 \times b \times c^{2}} \tag{27}
\end{align*}
$$

The obtained result corresponds to the first scenario which has been analyzed in the previous chapter (the population's expectances were equal to the optimal inflation); the cost of the non-compliance is negative and it has the following form:

$$
\begin{equation*}
C_{\text {non-compliance }}^{*}=-\frac{d^{2}}{2\left(a+2 \times b \times c^{2}\right)} \tag{28}
\end{equation*}
$$

According to the equation (25), the temptation of not-complying with a rule of the inflation, equal to the optimal inflation, will be as it follows:

$$
\begin{align*}
& T\left(r_{i}^{*}\right)=C_{\text {rule }}^{*}-C^{*}{ }_{\text {non-compliance }}= \\
& =0-\frac{-d^{2}}{2\left(a+2 \times b \times c^{2}\right)}= \\
& =\frac{d^{2}}{2\left(a+2 \times b \times c^{2}\right)} \tag{29}
\end{align*}
$$

On the other hand, the future updated cost will be equal to:

$$
\begin{align*}
& C V A\left(r_{i}^{*}\right)=\frac{1}{1+r}\left(C_{\text {non-compliance }}^{\text {future }}-C_{\text {rule }}^{*}\right)= \\
& =\frac{1}{1+r}\left(\frac{d^{2}}{2 \times a}-0\right)=\frac{d^{2}}{2 \times a+2 \times a \times r} \tag{30}
\end{align*}
$$

The optimal inflation rate will be considered as an inflation targeting if the following inequality, deduced from the equation (25), will be complied with:

$$
\begin{equation*}
2 \times b \times c^{2}>a \times r \tag{31}
\end{equation*}
$$

Taking into account the significance of the positive parameters $a, b, c$, the previous inequality can be rearranged as it follows:

$$
\begin{equation*}
b \times c^{2}>\frac{a}{2} r>\frac{a}{2} \tag{32}
\end{equation*}
$$

In the function of the inflation social cost, $b \times c^{2}$ represents the importance of the cost of the output gap according to the Lucas function, and (a/2) represents the importance of the costs of redistributing the incomes generated by the variation in the
inflation rate compared to the optimal rate. From the previous inequality, it results that the optimal inflation rate is estimated as being higher than the importance of the costs of redistributing the incomes, supposition which is very difficult to be made in the modern economies, where the governors' attention, and especially the audience's attention, directs towards the second inflation cost factor.

## Analysis of several parameters influencing the optimal inflation rate

To discuss this economic model we have built the mathematic functions of present temptation to disregard the rule and the future costs of disregarding the inflation rate regulation (equations 29 and 30). The software application MATLAB was used in order to simulate different scenarios and to analyze the influence of several parameters: the relative importance of income redistribution cost, output gap social costs, and benefits resulting
from lower public debt; discount rates and optimal inflation rate.

## The primary model

The framework of our model consisted of Romania's monetary policy coordinates for early 2008: an optimal inflation rate $\left(r_{i}\right)$ of $5.9 \%$ and a discount rate (r) of $9.5 \%$ - the monetary policy interest rate set by NBR in March 2008. For the parameters $a$ (the relative importance of income redistribution inflationary cost), $b$ and $c$ (the relative importance of output gap inflationary cost), $d$ (the relative importance of the public debt downsizing inflationary benefit) we have chosen positive values ${ }^{(3)}$. The result of the simulation was that the enforceable rate of inflation, therefore a credible inflation rate rule should be greater than $5.47 \%$ and lower than $8.41 \%$. The size of this interval, as well as its sensitivity to different parameters variation was analyzed through further simulations.


Figure 1. The model results for $r_{i}{ }^{*}=5.9 \%, r=9.5 \%$

## Discount rate fluctuations

Increasing the discount rate from 9.5\% to $12 \%$ will result in the increase of the minimum applicable inflation rate rule by almost 0.03 percentage points. Therefore, the minimum enforceable rate is rather not sensitive to the discount rate fluctuations, even to a fluctuation relatively ample such as the one chosen in the model (more than $26 \%$ ). Also, increasing the discount rate will lead to higher updated costs according to equation (30), thus intersecting the temptation function at a higher minimum enforceable inflation rate.

On the other hand, the upper limit of the enforceable inflation rate interval remains constant at a value of $8.41 \%$ (the primary model value), which allows us to conclude
that the discount rate variations have little to no influence over the maximum level of applicable inflation rate rule. Combining the two effects depicted above, we can derive another conclusion: the downsizing of the enforceable inflation rate rule interval as the result of an discount rate increase. Therefore, a restrictive monetary policy applied by rising interest rates will diminish the policymaker's choice of enforceable inflation rate. Furthermore, as we can observe in Figure 2, the future costs of disregarding the rule will decline sharply, increasing the policymaker's incentive of not respecting the enforced rule. Even so, the maximum enforceable rate is higher than the inflation target (the optimal inflation rate).


Figure 2. Effects of discount rate increase

Fluctuations in the relative importance of income redistribution inflationary cost

This situation presents the case when, at macroeconomic level, the relative importance of income redistribution inflationary cost is significantly increased. To include this variation in the simulation, we have increased by $50 \%$
the value of parameter $a$ from the inflationary costs function. As it can be observed in Figure 3 , there is a relatively small rise in the minimum enforceable inflation rate from $5.47 \%$ in the primary model to $5.72 \%$. On the other hand, the maximum applicable inflation rate rule is more elastic to this influence, and
the relation to the importance of income redistribution is negative: the upper limit of the enforceable interval drops from an initial $8.41 \%$ in the primary model to $7.58 \%$ in this case. The tendency is natural: as inflation becomes more and more socially costly, the policymakers will have an incentive to set lower inflation targets.

Furthermore, as the decrease in the maximum enforceable rate is higher than the
increase in the minimum value of the enforceable interval, the size of the applicable inflation rates interval is reduced to a value below two percentage points, leaving little option for the policymaker to set an enforceable inflation rule. Due to the fact that inflation becomes more socially costly, the present temptation at equilibrium (Figure 3) will be considerably reduced relative to the primary model.


Figure 3. Effects of income redistribution inflationary cost increase

## Fluctuations in the relative importance of output gap inflationary cost

Analyzing the case of a fluctuation in the relative importance of the output gap inflationary cost, we notice a relatively small influence of this factor over the model results. Our simulation consisted of increasing the relative importance of output gap by $50 \%$, that meaning increasing the $b$ parameter 1.5 times. The minimum applicable inflation rate is negatively related to the output gap cost, but its elasticity is quite reduced: the minimum enforceable inflation rate drops from the primary model value only by 0.19 percentage points, meaning $3.47 \%$ of initial value.

The maximum enforceable inflation rate remains constant to the primary model: $8.41 \%$ (see Figure 4), thus resulting an increase of the enforceable inflation rate interval's size. This situation is quite the opposite of the previous one, regarding the importance of the income redistribution cost. Therefore, as the output gap becomes more important as an inflationary cost, the options a policymaker has to set an enforceable inflation rate rule increase, for the applicable interval is also larger. An increased importance of the output gap inflationary cost is helpful in analyzing the inflation targeting strategy. An excessive aggregate demand (on the background of an
already existing inflationary output gap) will enhance the inflationary pressures, which stresses the necessity of coordinating the fiscal and monetary policies in order to respect the
inflation rule. The restrictiveness of such policies will lead to disinflation, and the inflation rule/target can be set in a more credible way.


Figure 4. Effects of output gap inflationary cost increase

## Fluctuations in the relative importance

 of reduced public debt inflationary benefitThe last case analyzed in this paper is related to an increase of the relative importance of reduced public debt inflationary benefit, which is synonym at macroeconomic level with a reduced necessity for high taxes having a distorting effect over the economic activity. Our simulation consisted of increasing the relative importance of reduced public debt benefit by $50 \%$, that meaning increasing the $d$ parameter 1.5 times. The model results indicate a decrease in the minimum applicable inflation rate of $4.02 \%$ to $5.25 \%$ and an increase of the maximum applicable inflation rate to 9.66 percentage points (Figure 5). Explaining this result, we may conclude that while inflation is perceived as a "beneficial" phenomenon for a national
economy - through reduced distorting taxes - the policymaker has the option of setting a higher inflation rate and enjoy the benefits of a reduced public debt. Furthermore, the evolutions presented above lead to a consistent increase in the size of the applicable inflation rate rule interval, offering larger options for the policymakers in their attempt to set the inflation rate rule.

The Barro-Gordon model implies that a higher inflation rate leads to certain governmental benefits through the reduction of public debt. On the other hand, the government already is benefiting from high aggregate demand - and higher fiscal incomes - during the pre-election period. Therefore, if this situation happens on the background of inflationary output gap (such as in Romania), then there are high chances that the inflation rate target will not be
achieved, as long as it is not set to a higher value. Concluding, to continue the process of disinflation, the coordination of monetary and fiscal policy is a sine qua non condition.

Like in Stackelberg equilibrium, a Central Bank with an inflation targeting strategy anticipates the governmental reaction, which is adjusted to the very same monetary policy.


Figure 5. Effects of inflationary benefits increase

To summarize the model conclusions, the influence of all analyzed parameters (optimal inflation rate, discount rate, inflationary costs and benefits importance), which we have considered independent variables, over dependent variables consisting of the enforceable inflation rate interval, we will build the following table. The following table is summarizing the Figures 1 to 5: depending on
the values of present temptation and future updated cost we have shown the enforceable rate interval, which will guide a Central Bank's actions to gain credibility. This interval is significantly influenced by the increases in the income redistribution inflationary cost and the public debt reduction inflationary benefit, the first leading to a lower maximum enforceable rate, whilst the latter to a higher one.

Influence of independent parameters on the optimal inflation rate
Table 1

|  |  |  | Dependent variables |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Minimum enforceable rate | Maximum enforceable rate | Size of enforceable interval |
|  | Discount rate |  | Directly related | constant | Negatively related |
|  | ¢0 | Income redistribution importance | Directly related | Negatively related | Negatively related |
|  |  | Output gap importance | Negatively related | constant | Directly related |
|  |  | Reduced public debt importance | Negatively related | Directly related | Directly related |

## Conclusions

According to the Barro-Gordon model, adapted and tested throughout this paper,
setting an inflation rate target represents the only possible choice to minimize the long-run social costs. The only viable alternative of the
policymaker to reduce the social cost to zero is setting an inflation rate rule, which can be respected by using fiscal and monetary policy instruments. The fundamental problem of setting an inflation rate rule is respecting it, function to this compliance of the Central Bank to its own rule, there can be established a certain degree of credibility. This paper shows there is an interval of enforceable inflation rates which can be influenced both by specific inflationary costs parameters and monetary policy instruments.

The simulations realized on Romania's particular case show that the maximum enforceable inflation rate has a close value to the actual inflation rate from the first five months of 2008, but two percentage points above the inflation target set by NBR. The policymaker's credibility shall improve only if there are set inflation targets superior to the precedent levels (for example, the 2008 inflation target had been set to $3.8 \%$ eventually

## Notes

${ }^{(1)}$ Using a/2 instead of a leads to simpler further arithmetic.
${ }^{(2)}$ The inflation rate rule would suppose a cost during all the periods of time.
${ }^{(3)}$ We have chosen: $a=0.4, b=0.7, c=0.4, d=0.01$. These values comply with other policymaker's reaction

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risen to $5.9-6 \%$ ), but this behaviour will signify but an acknowledgement of the impossibility of continuing the disinflation trend. Further inflationary tendencies, like supply-side shocks at macroeconomic level will lead to failure in reaching the inflation target, negatively influencing the policymaker's credibility. The only effective actions of the Central Bank are influencing the aggregate demand, but for greater effectiveness there is required a closer coordination with the fiscal policy. As our Barro-Gordon model simulation suggests, an increase of the relative importance of the output gap as well as putting an end to the expansionary fiscal policy during elections are necessary. If a Stackelberg equilibrium is to be reached between the Central Bank (acting as leader to set the target inflation) and the Government, there will result a restrictive policy mix, leading to the reduction of inflationary anticipations in the Romanian economy.
estimates (such as the Taylor rule), the relative importance of income redistribution being greater than the output gap inflationary cost.

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