The Penrose's Law and Decision-Making Processes in the Council of the European Union. Case Study: the Impact of the Square Root' Rule on Formation of Romania's Coalitions in the Council of Ministers

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Abstract. *Enlargement of the European Union has led to much discussion on the need to reform institutions so that the various inequalities between the member states to be removed. Such Treaties of Nice and Lisbon do not grant to citizens of member the same influence in the decision-making in the Council of Ministers. One of the solutions proposed to remedy this problem is to establish a voting scheme that gives each country a share of votes proportional to the square root of the population and determining the optimum of a threshold, known as the Jagiellonian Compromise. Starting from the analysis of Penrose square root law in the decision-making Council of Ministers, the article presents a case study on the influence of approximation by rounding the share of votes on the coalition's formation in the case of Romania.*

Keywords: Council of Ministers; decision-making processes; Jagiellonian Compromise; coalition's formation.

JEL Codes: C70, D70, D72, D74.
REL Codes: 20C, 7L, 9A.

1. Jagiellonian Compromise. Criticism treaties of Nice and Lisbon

In 2007, researchers from 10 countries formed a group called SDE (Scientist for a Democratic Europe), and have sent a joint letter to EU Member States governments, recommending the use of square root of Penrose's method of allocating votes in the Council of Ministers. They draw attention to the shortcomings of the voting system of the Treaty of Nice and the Treaty of Lisbon. Alternative voting system that they suggested was based on the square root of Penrose, considering that this system is more suitable to give to every citizen the same influence on decisions of the Council of Ministers, regardless of country of origin. The system proposed would be a fair compromise between the Nice and Lisbon systems. An institution such as the EU representative must deal with the situation of Malta, with 400,000 inhabitants, which "is at the same table" with Germany, which has 82,000,000 inhabitants, that is 211 times higher. It is as if we compare China to Malta at the United Nations.

Following criticism of Nice' system, Council members agreed that the decision should be changed in a system with qualified majority double that to have two criteria: the number of Member States and the population. This system leads to loss of power for some countries (Poland), which has generated much tension in the adoption. Although, finally, in 2004, Poland has accepted the new system, it began a systematic campaign in favor of another decision-making system within the Council, to ensure equality in representation for all countries. Such a system is based on square root law of Penrose with one majority, and the number of votes allocated to be proportional to the square root of the population of the Member States.

Table 1 shows the power of the vote compared to the system adopted in the case Jagiellonian compromise.

Qualified majority voting system for the Council of Ministers: Citizen-based Jagiellonian Compromise "JC" and negotiated Double Majority "DM"

Table 1

Member States	Population (mil.) 2007	Weight JC	Power JC[1]	Power Double Majority	Deviation %
Germany	82.438	9080	9.46	11.66	+23
France	62.886	7930	8.27	9.02	+9
UK	60.421	7773	8.10	8.69	+7
Italy	58.751	7665	7.99	8.49	+6
Spain	43.758	6615	6.90	6.55	-5
Poland	38.157	6177	6.44	5.71	-11
Romania	21.610	4649	4.85	4.15	-14
Netherlands	16.334	4042	4.21	3.50	-17
Greece	11.125	3335	3.48	2.88	-17
Portugal	10.569	3251	3.39	2.80	-17
Belgium	10.511	3242	3.38	2.80	-17

Member States	Population (mil.) 2007	Weight JC	Power JC[1]	Power Double Majority	Deviation %
Czech	10.251	3202	3.34	2.77	-17
Hungary	10.076	3174	3.31	2.74	-17
Sweden	9.047	3008	3.14	2.63	-16
Austria	8.265	2875	3.00	2.53	-16
Bulgaria	7.718	2778	2.90	2.47	-15
Denmark	5.427	2330	2.43	2.19	-10
Slovakia	5.389	2321	2.42	2.18	-10
Finland	5.255	2293	2.39	2.17	-9
Ireland	4.209	2052	2.14	2.04	-5
Lithuania	3.403	1845	1.92	1.95	+2
Latvia	2.294	1515	1.58	1.81	+15
Slovenia	2.003	1415	1.47	1.78	+21
Estonia	1.344	1160	1.21	1.69	+40
Cyprus	0.766	875	0.91	1.63	+79
Luxembourg	0.459	678	0.71	1.59	+124
Malta	0.404	636	0.66	1.58	+139
TOTAL	492.881	95.916	100	100	
Quota		59.058	61.57		

Source: Pukelsheim, 2007, p. 15.

Under the Nice Treaty, each Member State is assigned a number of votes that reflect the population. A legislative proposal passes the Council, in the Treaty of Nice, if three conditions are accomplished: number of votes, the number and percentage of state population, the system being called triple majority.

Lisbon Treaty states that a legislative proposal pass if only two criteria are met: 55% of Member States "for" representing 65% of the EU population. Additionally, a minority block includes at least four members who could block a proposal that a majority may agree. Most experts believed that the main drawback of the Treaty of Nice was the low efficiency of decision making. This performance, measured by Coleman power to act index, calculate the probability that the Council approved various issues, selected randomly, and is equal to the share of possible winning coalitions.

For the Lisbon Treaty, approximately 12.9% out of all possible coalitions are leading to a condition regarding the approval of randomly chosen legislative proposals, and for the Treaty of Nice this index is equal to 2.1%. Calculations show that by introducing the double majority decision-making reduce the danger of blocking process from the EU levels. On the other hand, the figures show that the Nice system, which is currently, extension to the EU-27 was not fear and confirming all that the decision system will be blocked (Kurpas, Schonlau, 2006, pp. 2-5). Another shortcoming of the system from Nice is that it is necessary to apply simultaneously the three criteria in calculating the qualified majority, replaced by the Treaty of Lisbon, which uses only two criteria.

All voting systems in the EU were a compromise between two fundamental principles: the principle of equality of Member States and the principle of equal citizens. Double majority brought by the Lisbon Treaty apparently reflects only those two desires. In this system the large states will gain more from direct contact with the population, while the small states will have a disproportionate power provided that the number of states supporting a proposal to increase. The combined effects of these consequences lead to decreased power of medium sized Member States. In an ironic manner, the same conclusion was also drawn by Penrose, who discovered this deficiency by analyzing double majority system in 1952. His findings can be regarded as a visionary critique of the Treaty of Lisbon. The fact that this system uses only two criteria fixes another basic flaw: the system is not more transparent because a simple citizen is not able to calculate the potential voting power of each Member State, complex mathematical calculations being required as in case the Treaty of Nice. These calculations show that the basic principle of democracy, through which the voting power of a citizen of any state is of equal value, is violated in both systems.

2. Is square root solution for optimal decision-making for the Council of Ministers?

One may be designed a voting system in which every citizen of any Member State to have the same power to influence decision-making processes? Can be done in a transparent way easily implemented efficiently and to match in future EU enlargements?

A partial answer was given by Penrose, who developed the principle of a voting system of an ideal representation in which every citizen of any state has the same voting power potential. If we consider direct elections in a state with N population, which can be gained by a simple majority of 50% +1, then the simply citizen from a country with a large population has less influence on the outcome of the election process than a citizen from a small sized country. Penrose has shown that in such elections, the voting power of a citizen drops the square root rule. Thus the indirect vote system of Council would be representative if the voting power of each country should be proportional to the square root of N (Penrose's law).

Kauppi and Widgren showed that although this law may seem strange, however, using it as a basis for voting scheme may be justified in terms of fairness (Kauppi, Widgren, 2004, pp. 221-229). It can be shown that the square root rule ensures that every citizen is equally represented in the Council regardless of country of origin. The proposal received considerable support from the academic world.

The solution of sharing votes, proportionate to the square root of population, is a mathematical application of the principle of degressive proportionality[2] and is between two extremities: „a country-one vote" (as if Europe would be a simple association of states) and "votes in proportion to the population" (as if Europe would be a single state). Note that a degressive

system is used in German Parliament, by which is designated a number of representatives for each Land. But the assertion that "each country's voting power should be proportional to the square root of the population" does not solve the problem entirely. Kirsh showed that the square root tells us just how power should be distributed between countries (2004, pp. 2-3). It is not clear how this law should be implemented in practice in terms of share of the vote.

The problem is how votes are allocated and how to set the threshold to achieve a certain distribution of power. One solution would be that the share of votes to be proportional to the square root and then to found the optimal threshold that will produce maximum transparency of the system, i.e. the system in which voting power of each Member State will be approximately equal to the share of votes as is observed in Figure 1. (Slomczynski, Zyczkowski).

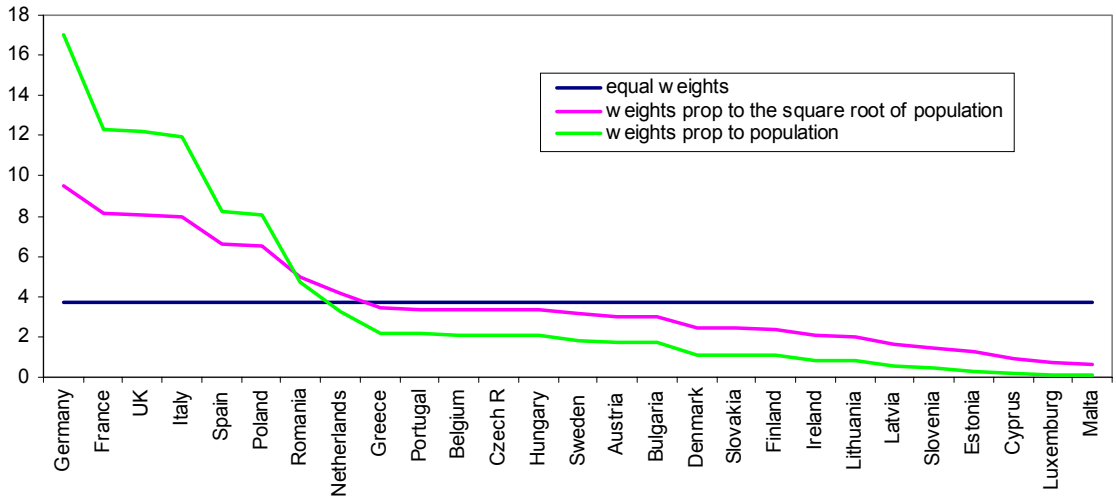

Figure 1. *The weights attributed to the Member States: proportionally to population, proportionally to the square root of population, and uniformly*

Source: Slomczynski, Zyczkowski, 2008, p. 11.

Thus Penrose's law should be respected and the potential influence of each citizen in the Council would be the same, which would lead to compliance with the principle of representativeness. Such a system should also be transparent, simple and objective and would not create any advantage or disadvantage for any country.

This system was called the Jagiellonian Compromise. Thus for the EU Council of Ministers 27 optimal threshold would be 61.57%. For an arbitrary number of Member States the best threshold q can be calculated according to the formula:

$$q = \frac{1}{2}\left(1 + \frac{\sqrt{N_1 + N_{2...} + N_m}}{\sqrt{N_1} + \sqrt{N_2} + ...\sqrt{N_m}}\right)$$

where N_i represents the population of country i.

Since the number of Member States will increase it will result in a change of the optimal threshold. Calculations show that the optimal threshold decreases with electoral body size. This shows that a threshold that change depending on future conditions is increased efficiency.

Jagiellonian Compromise is also known as the solution *Pop.-q%*. Based on the population[3] we get a threshold of q = 61.5%, with an error less than 0.00005 ‰. Analysis showed that the maximum relative deviation between β_{0i} and β_{qoi} is less than 0.14% (Langner, Werner, 2009). In addition the efficiency is 16.43%. The voting share and voting power measure by Banzhaf index (see Table 2), which is higher than the Treaty of Lisbon, not to mention the one in Treaty of Nice.

Jagiellonian Compromise. Distribution of votes and voting power in CM with threshold of 61.5%

Table 2

Distribution of votes and the voting power in the Council of Ministers.			
Jagiellonian Compromise – threshold of 61.5 %			
State	Population	Square root of population	Banzhaf index %
Germany	82.438	9079.54	9.441
France	62.886	7937.18	8.273
UK	60.421	7771.30	8.104
Italy	58.751	7664.97	7.994
Spain	43.758	6615	6.906
Poland	38.157	6177.14	6.445
Romania	21.610	4648.68	4.851
Netherlands	16.334	4041.56	4.210
Greece	11.125	3335.4	3.479
Portugal	10.569	3251.09	3.388
Belgium	10.511	3242.13	3.378
Czech	10.251	3201.73	3.338
Hungary	10.076	3174.36	3.308
Sweden	9.047	3007.95	3.138
Austria	8.265	2875.05	2.998
Bulgaria	7.718	2.77	2.898
Denmark	5.427	2329.69	2.428
Slovakia	5.389	2321.46	2.418
Finland	5.255	2292.51	2.388
Ireland	4.209	2051.59	2.138
Lithuania	3.403	1844.8	1.918
Latvia	2.294	1514.79	1.578
Slovenia	2.003	1415.40	1.478
Estonia	1.344	1159.61	1.208
Cyprus	0.766	875.45	0.909
Luxembourg	0.459	677.86	0.709
Malta	0.404	635.88	0.659

Effectiveness of the square root rule does not decrease with the number of Member States as in case of the system with a double majority. Figure 2 illustrates the dependence of the square root of residual sum of squares between σ normalized Banzhaf index and the share of the vote, the threshold R:

$$\sigma^2 = \sum_{x=1}^{N} \left(\beta_x(R) - \frac{\sqrt{N_x}}{\sum_{y=1}^{N} \sqrt{N_y}} \right)^2$$

Since the minimum value of this function is reached for R^{27}_{opt}, then one can say that the optimal value for threshold is reached when the share of the vote and the votes are equal.

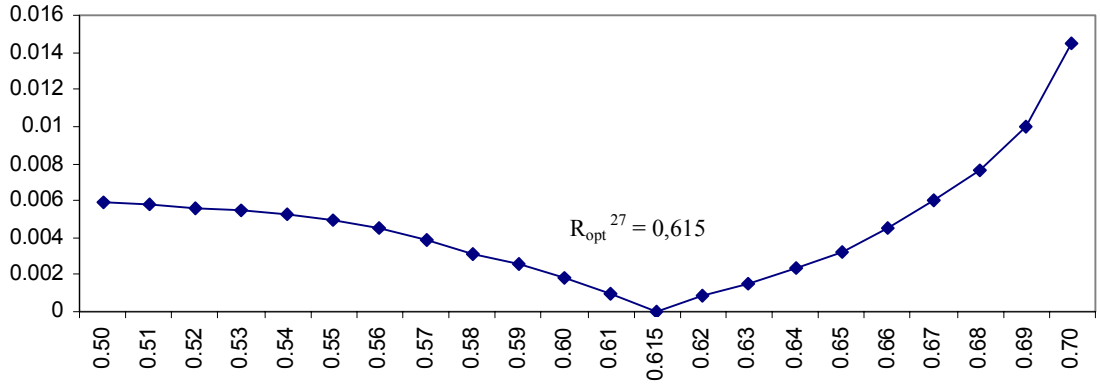

Figure 2. *The cumulative residual σ between the voting weight and power for EU-27 countries as dependent on the value of the threshold R= 0.615*

Source: Slomczynski, Zyczkowski, 2007, p. 11.

When N → ∞ then the value of critical threshold tends to 50% in accordance with Penrose's law. Table 3 shows the average critical threshold in terms of N, the number of electoral body obtained as a result of a symmetric Dirichlet distribution, with density given by the formula $P(x_1, x_2 \ldots x_N) = C_N (x_1, x_2 \ldots x_N)^{(-1/2)}$ for any $x_i \geq 0$, $\sum_{i=1}^{N} x_i = 1$ and where the normalization constant is expressed by the Euler gamma function $C_N = \Gamma(\frac{N}{2}) n^{(-\frac{N}{2})}$.

Average optimal threshold R_{opt}^{N} as a function of the number of states N

Table 3

No. of Member States N	10	12	14	16	18	20	22	24	26	27
R_{opt}^{N}	66.0%	65.8%	64.6%	64.4%	63.4%	63.1%	62.6%	62.0%	61.4%	61.57%

The results above indicate that for a given number of states N, the choice of the weighting in proportion to square root of the population and close to the threshold R^{27}_{opt} ensures optimum representativeness system of voting power as each country becomes proportional to the square root of population and voting power of individual a citizen is almost identical.

Square root is derived from the probability that an individual voter to influence the outcome of a binary vote (yes/no). A voter has some direct influence only if the rest of the people *N* voted in a proportion equal to *yes* and *no*. This approach requires that the people who voted to be in an even number, otherwise the voter will not be in such a position. The probability of compliance leaves the hypothesis that each member of the population are equally likely to vote in two ways (yes/no).

$$P_{tie} = \binom{N-1}{\frac{N-1}{2}} \frac{1}{2}^{N-1} = \frac{(N-1)!}{\left(\frac{N-1}{2}\right)^2 2^{N-1}} \approx \sqrt{\frac{2}{\pi(N-1)}} \frac{1}{\sqrt{N-1}}$$

Ease of decision-making process for various thresholds

Table 4

Majority threshold % Members - % population	UE-27	Passage Probability UE-27+Turkey
50-50	35.8	31.9
55-55	23.0	19.9
50-60	21.9	19.8
55-65	12.9	12.5
50-61.5	12.9	11.7
60-50	11.1	15.1
50-70	9.2	8.3
60-60	8.5	11.0
60-70	4.8	5.6
Nisa	2.8	1.8
70-50	2.5	1.7
70-60	2.2	1.6
70-70	1.6	1.1

Source: Baldwin, Widgren, 2007, p. 6.

The result can be viewed by considering a binomial distribution with standard deviation variation $\left(\sqrt{\frac{N-1}{4}}\right)$. Formally, the approach to the value of *N-1* represents a normal distribution and it has a fixed area: the point is the maximum height of $\frac{1}{\sqrt{N-1}}$. If the share of votes allocated to a state is $\sqrt{N}$, such a proportional distribution cancels the possibility of giving each state an equal share of votes (Ratzer, 2006).

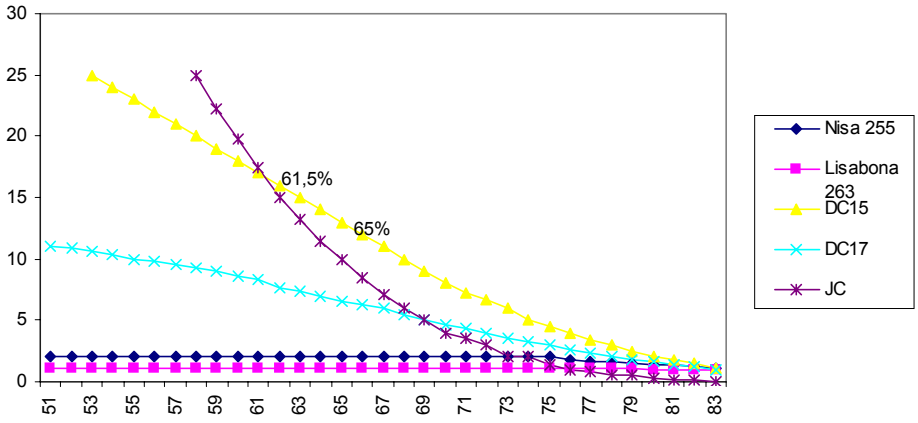

Figure 3. *Effectiveness of the Nice Treaty, the Treaty of Lisbon and the Jagiellonian Compromise*

Source: Langner, Werner, 2009, p. 15.

We conclude the Jagiellonian advantages:

- is simply, based on one criterion and may be called single majority system;
- is objective (the share of the vote and the threshold are not set arbitrarily) and hence the absence of disadvantages for different countries;
- representativeness: every citizen of the Member States has the same voting power potential;
- transparency: the voting power of each state is roughly proportional to the share of votes;
- is easily adaptable and extensible: if the number of state changes, all you have

to do is to reallocate votes in accordance with law and as the square root threshold;

- is more effective than the treaties of Nice and Lisbon;
- moderate conservative: it does not lead to a dramatic transfer of power from the current situation.

3. Approximation by rounding when applying Penrose's law in decision-making processes of the Council of Ministers and the impact on coalitions formation of Romania

The application of square root law in the EU Council decision-making processes has consequences for the coalitions formation of a Member State. We show that the approximation by rounding the share of votes, and no exact calculation, which must return a Member State, may have a major effect on the optimal threshold *R* and the minimum value of σ can be reached by the voting system of Penrose's square root law.

To make a voting system "fair" in the Council of Ministers, Banzhaf index should calculated according to the formula

$$\beta_i = \frac{\sqrt{N_i}}{\sum_{j=1}^{n}\sqrt{N_j}}$$

for all countries, where N_i is the population of country *i*. Practical implementation of this index, thus calculated, is difficult and represents a problem of multi-dimensional optimization. To estimate an ideal distribution of voting power, Penrose suggested that the share of votes to be proportional to the square root of the country's population according to the formula $v_i = \frac{\sqrt{N_i}}{\sum_{j=1}^{n}\sqrt{N_j}}$ for all *i* countries.

The number of votes obtained will not be rational numbers, but some approximations have to be made. These approximations, in the absence of precise rules, will be given to the accuracy of floating point representation, in which the performance of computer use.

Zyczkowski and Slomczynski were rounded voting weights to four decimal places obtained.

Case study will show that this rounding may have effects on the optimal threshold *R* and the minimum value of σ, which is produced by the voting system of the square root of Penrose.

Approximation of *k*-digit rounding means that the share of votes is done by *k* decimals, where the share of votes is normalized such $\sum_{i=1}^{n} v_i = 1$, where v_i is the share of votes if the country *i*. To show data rounding errors, the minimization of σ, we consider four countries as follows:

Population and share of votes for four countries

Table 5

Current no.	Country	N_i		v_i	v_i (3 decimals)
1	România	21610213	4648.678	0.4648	0.465
2	Denmark	5427459	2329.6907	0.2329	0.233
3	Ireland	4209019	2051.589	0.2051	0.205
4	Cyprus	766414	875.450	0.0875	0.088

Coalitions and the share of votes, without rounding

Table 6

Coalitions of states	$\sum v_i$	Coalitions of states	$\sum v_i$
1,2,3,4	1.000	2,3	0.438128014
1,2,3	0.902995877	1	0.464867863
1,2,4	0.785382012	2,4	0.320514149
1,3,4	0.757571875	3,4	0.292704012
1,2	0.697836938	2	0.232969075
1,3	0.670026801	3	0.205158938
2,3,4	0.525673088	4	0.087545074
1,4	0.552412937	Nici una	0.000

Using four decimal approximation for the share of votes, the share of votes for all possible coalitions could be obtained which are formed in accordance with Table 6. From this table it could be read the winning coalitions ω that have a majority for a given threshold R and the number of winning coalitions ω_i to which the country i. This means that for all thresholds $0 \le R \le 1$ and all countries i, the number of coalitions where the country's vote i is critical, η_i, and could be calculated with the formula $\eta_i = 2\omega_i - \omega$, giving the Banzhaf index and the deviation σ.

For example for $0.90299 < R \le 1$ there is only one winning coalition of countries formed from Romania, Denmark, Ireland and Cyprus, i.e. $\eta_i = 1$ for $i = 1,2,3,4$. Deviation σ is computed according to formulas:

$$B_i = \frac{\eta_i}{2^{n-1}},\ \beta_i = \frac{B_i}{\sum_{j=1}^{n} B_j} = \frac{\eta_i}{\sum_{j=1}^{n} n_j}$$

and $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(v_i - \beta_i)^2}$.

The values of σ for all the thresholds R are calculated in Table 7. Only thresholds for $R \ge 0.5$ are shown as the lower threshold of 0.5 means that a coalition can be successful even if the opposition has more votes.

Deviation from the fair voting, without rounding

Table 7

R >	R ≤	η_1	η_2	η_3	η_4	σ
0.902995877	1.000	1	1	1	1	0.126844
0.785382012	0.902995877	2	2	2	0	0.108664
0.757571875	0.785382012	3	3	1	1	0.082441
0.697836938	0.757571875	4	2	2	2	0.062765
0.670026801	0.697836938	5	3	1	1	0.060114
0.525673088	0.670026801	6	2	2	0	0.092898
0.552412937	0.525673088	5	3	3	1	0.030906
0.438128014	0.552412937	6	2	2	2	0.060498
0.464867863	0.438128014	5	3	3	1	0.030906
0.730832026	0.464867863	6	2	2	0	0.092898
0.292704012	0.730832026	5	3	1	1	0.060114
0.232969075	0.292704012	4	2	2	2	0.062765
0.205158938	0.232969075	3	3	1	1	0.082441
0.087545074	0.205158938	2	2	2	0	0.108664
0.000	0.087545074	1	1	1	1	0.126844

The lowest value of $\sigma_{min} = 0.030906$ and this happens in the $0.5256 < R \le 0.5524$.

The situation is changing very much if σ is calculated from the share of votes, with rounding. In this case the possible coalitions and the share of votes are shown in Table 8.

Coalitions and the share of votes, with 3 decimals rounding

Table 8

Coalitions of states	$\sum v_i$	Coalitions of states	$\sum v_i$
1,2,3,4	1.000	1	0.465
1,2,3	0.903	2,3	0.438
1,2,4	0.785	2,4	0.321
1,3,4	0.758	3,4	0.293
1,2	0.698	2	0.233
1,3	0.670	3	0.205
1,4	0.552	4	0.088
2,3,4	0.526	None	0.000

Comparison with Table 6 shows that the coalition consisting of Denmark, Ireland and Cyprus and coalition consisting of Romania and Cyprus has changed places. This leads to different results for and σ as shown in Table 9.

Deviation from fair voting system, with three decimal places rounding

Table 9

R >	R ≤	η_1	η_2	η_3	η_4	σ
0.903	1.000	1	1	1	1	0.126844
0.785	0.903	2	2	2	0	0.108664
0.758	0.785	3	3	1	1	0.082441
0.698	0.758	4	2	2	2	0.062765
0.670	0.698	5	3	1	1	0.060114
0.526	0.670	6	2	2	0	0.092898
0.552	0.526	7	1	1	1	0.163120
0.438	0.552	6	2	2	2	0.060498
0.465	0.438	7	1	1	1	0.163120
0.731	0.465	6	2	2	0	0.092898
0.293	0.731	5	3	1	1	0.060114
0.233	0.293	4	2	2	2	0.062765
0.205	0.233	3	3	1	1	0.082441
0.088	0.205	2	2	2	0	0.108664
0.000	0.088	1	1	1	1	0.126844

The lowest value of σ_{min} = 0.060114 and this happens in the 0.670 <R ≤ 0.698.

It is noted that the minimum value of σ has changed much from the case without rounding and also the interval it belongs to.

This shows that the rounding has a great influence on the minimum value of σ and the associated threshold.

It can happen that rounding to decrease the value of σ_{min}, which means that the approximation is improved if the Jagiellonian Compromise decision-making system based on square root.

Notes

(1) The share of the vote in the case of Jagiellonian Compromise is square root of population, rounded to the nearest number. Thus we have in Malta = 635.9 → 636. The threshold for qualified majority decisions respect the formula: (+95.916)/2=59058,47 → 59058. Since the share of Malta is 636/95916=0,006631 → 0.66%, the relative index is 0,66. Power index for the rule of double majority of 55% of Member States and 65% of its population are taken from Slomczynski and Zyczkowski, 2007, p. 12. Deviation from Jagiellonian compromise is for the Malta (1.58-0.66)/ 0.66 = +1.3939 → +139%

(2) See draft Constitution with regard to the European Parliament.

(3) See Eurostat No. 7/2008.

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