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# Input-Output Analysis and Demographic Accounting: A Tool for Educational Planning

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**Abstract.** *Accounting concept induces usually the idea of transactions expressible in terms of money in a system of interlocking statements in each of which total incomings are equal to total outgoings. It is known that, in a close system, the entries are not unrelated but are connected by a number of independent accounting identities. Thus, within an accounting framework we can build models in the certain knowledge that these connections will be respected, input-output analysis being the most obvious example of this kind of model-building.*

*The application of elementary accounting ideas is not restricted to the concepts expressible in terms of money, but, for example, it can be applied to demography, with its obvious unit the individual human being, and to education, with its obvious unit the student. Such analysis can be applied in order to plan the educational system in a rational manner, based on demographic information that allows us to determine the flows of individuals with various characteristics into various activities, to show in detail the structure of the population at any time, and to design in the future how this structure may change under the impact of individual decisions or of those underwritten to economic and social aims.*

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### 1. Economic input-output model

In economic input-output analysis the economic system is divided into a number of branches or industries, each being defined as producing a particular product or group of products, which constitutes its output. Within a given time-period, this output has two main destinations: as an intermediate input, to be used in fabrication of some other products; or as a final product of the system. Final product, in its turn, has three destinations: to satisfy the demands of consumers; to replace and extend the capital equipment of industries; and a small part to form a stock of products, some unfinished, which will flow back into production for intermediate use in the future. Producing goods and services for final use is the principal/main function of the industries. In order to perform this function, each industry, beside the materials, fuels and business services (which constitute its intermediate inputs) flow into production from outside the productive system.

The purpose of this type of analysis is to study the interdependence of the industries and their connections with other parts of the economy. The whole set of input-output flows and their totals can be arranged in a table or matrix (with outputs in the rows and inputs in the columns). The entries can be expressed in terms of money, in which case each row and column pair can be regarded as an account which balances: the revenues realised from the sale of the outputs are equal to the cost of the inputs. Reduced to its simplest terms, such a table can be represented as follows (Table 1):

Table 1

**A simple economic accounting matrix**

| | Production | Non-production | Total |
|---|---|---|---|
| Production | $W$ | $f$ | $q$ |
| Non-production | $y'$ | $O$ | $y'i$ |
| Total | $q'$ | $i'f$ | |

The row and column for production are to be thought of as divided into many rows and columns, equal in number to the industries we wish to distinguish, and symbol $W$ denotes a submatrix of intermediate product-flows between the industries (for example, the element at the intersection of row $j$, say, and column $k$, say, of this submatrix shows the amount of the product of industry $j$ absorbed by industry $k$ during the period to which the analysis tables refers relates). The symbol $f$ denotes a column vector of final products (for example, the $j^{th}$ element of $f$ shows the amount of final products made by industry $j$). The symbol $q$ denotes a column vector of total outputs (the $j^{th}$ element of $q$ shows the total output of industry $j$).

The static, open input-output model for a closed economy is based on two premises:

- Total output of a period is absorbed either in intermediate or in final uses, that is:

$$q \equiv W\,i + f \tag{1}$$

- The inputs of intermediate products are related in fixed proportion to the output into which they enter, that is:

$$W = A \times \hat{q} \tag{2}$$

where $q$ form a diagonal matrix and where $A$ denotes a matrix of input-output coefficients in which the element at the intersection of row $j$ and column $k$ measures the amount of input $j$ needed to make one unit of output $k$.

Substituing for $W$ from (2) into (1) it results:

$$q = (I - A)^{-1} f \tag{3}$$

Since each industry needs primary inputs in addition to intermediate inputs, the column sums of A are less than one and $A^{\theta}$ approaches the null matrix as $\theta$ increases, with the result that $(I - A)^{-1} \equiv (I + A + A^2 + A^3 + ....)$ has finite elements.

The given model is termed "open" because it does not generate all its variables but depends for its solution on a variable, *f*, which must be estimated exogenously. Its purpouse is to enable us to calculate the amount of total output, q, which must be produced in order to satisfy a given level of final demand, *f*, and hence to work out what would happen to *q* if *f* were to change.

But this model is much too simple because it implies that all the intermediate product used this year is made in the course of the year. However, a part of the final product of any period goes to form a stock of intermediate products for use in the succeeding period; in other words, *f* includes additions to stocks and work in progress. It results that some of the intermediate product used this year must have been made last year.

If we consider a productive system in which all the intermediate product used this year was made last year and in which all the intermediate product made this year will be used next year, so that provision must be made in advance for any expected change in final demand (Table 2).

Table 2

**An economic accounting system with simple time-lags**

| | | Production | | | Non-production | Total |
|---|---|---|---|---|---|---|
| | | Last year | This year | Next year | | |
| Production | Last year | | *W* | | | |
| | This year | | | *ΛW* | *e* | *q* |
| | Next year | | | | | |
| Non-production | | | *y´* | | | |
| Total | | | *q´* | | | |

In this case production is divided into three periods, last year, this year and next year, but the table is filled in only for this year. The intermediate product absorbed this year, W, comes from last year, and intermediate product made this year, $\Lambda \times W$, is carried forward to next year. In a growing economy any difference between $\Lambda \times W$ and $W$ represents an excess of products made over products used, that is to say represents stockbuildings, and the symbol $e$ denotes the final product redefined to exclude stockbuilding.

In this case the equation (3) can be written as:

$$q = \sum_{\theta=0}^{\infty} A^{\theta} \times \Lambda^{\theta} \times e \qquad (4)$$

This year's output, $q$, is no longer given simply in terms of this year's final demand but in terms of a weighted sum of present and future demands, the weights tending to zero with time.

It can also be written:

$$q \equiv A \times q + A \times \Delta \times q + e \qquad (5)$$

where $\Delta \equiv \Lambda - 1$. Here the productive system is explicitly represented as:

- replacing, for use next year, the intermediate product, $A \times q$, which it is using up for current production and which was carried forward from last year;
- adding to stock a supplementary amount, $A \times \Delta \times q$, needed to sustain the increment of output from this year to next year;
- satisfying this year's final demand for consumption goods and capital equipment, $e$.

In the real world, a system gets this year's supplies of intermediate product partly from last year's production and partly from this year's, and produces intermediate product partly for this year and partly for the next (Table 3).

Table 3

**An economic accounting system with partial time-lags**

| | | Production | | | Non-production | Total |
|---|---|---|---|---|---|---|
| | | Last year | This year | Next year | | |
| Production | Last year | | $W^{**}$ | | | |
| | This year | | $W^{*}$ | $\Lambda\, W^{**}$ | $e$ | $q$ |
| | Next year | | | | | |
| Non-production | | | $y'$ | | | |
| Total | | | $q'$ | | | |

In this case, the intermediate product absorbed this year is divided into two parts, $W^{**}$, which was made last year, and $W^{*}$, which was made this year.

Also, the intermediate product made this year is partly absorbed this year, *W**, and partly carried forward for use next year, $\Lambda \times W^{**}$.

The model of this situation is built up the following elements:

$$q \equiv W^{*} i + \Lambda \times W^{**} i + e \tag{6}$$

$$W^{*} = A^{*} \times \hat{q} \tag{7}$$

$$W^{**} = A^{**} \times \hat{q} \tag{8}$$

where $A^{*} + A^{**} = A$.

In this case:

$$q = \left(1 - A^{*}\right)^{-1} \sum_{\theta=0}^{\infty} \left[ A^{**} \left(I - A^{*}\right)^{-1} \right]^{\theta} \Lambda^{\theta} \times e \tag{9}$$

This situation express the fact that in each year the amount of intermediate product made but not used is precisely equal to the need of the succeeding year which cannot be met form that year's production.

## 2. Demographic input-output model

For applying input-output model to the analysis of demographic flows, it is necessary to define the terms regarding the social system. In this case the unit will be the human individual and the main categories, within which these units will be grouped, instead of being industries and products, will be age-groups, and within age-groups, activities and occupations. We shall say more about these categories in the next section, but first let us be clear about flow equations of the system and their solution.

Reinterpreting the symbols from Table 2 as follows:

- total output, $q$, represents the total population of a country during a given period;
- intermediate product, $W$, represents the surviving part of this population;
- final output, $e$, represents the deaths and emigrations;
- primary inputs, $y$, represents the births and immigrations.

we can rewrite the table more appropriately as Table 4.

In this table the sources of $p'$, the population vector, are partly the survivors from last year, the elements of $\Lambda^{-1} \times S$, and partly the births and immigrations of this year, the elements of $b'$. Correspondingly, the destinations of $p$ are partly the survivors into next year, the elements of $S$, and partly the deaths and emigrations of this year, the elements of $d$. The vector of the living population at the end of the year is $Si \equiv p - d$.

Table 4

**A demographic accounting matrix**

| | | Our country | | | Elsewhere | Total |
|---|---|---|---|---|---|---|
| | | Last year | This year | Next year | | |
| Our country | Last year | | $\Lambda^{-1}S$ | | | |
| | This year | | | $S$ | $d$ | $p$ |
| | Next year | | | | | |
| Elsewhere | | | $b'$ | | | |
| Total | | | $p'$ | | | |

In order to build, based on Table 4, a demographic model analogous to the economic model as shown in Table 2, we must reverse the roles of inputs and outputs. In the economic model it was assumed that while output patterns change with changes in final demand, the input pattern in the different industries are fixed.

In the demographic case it seems more reasonable to make the opposite assumption, namely that input patterns may change with changes in the number of births and immigrations, but that the output patterns (or transition probabilities) for the different age-groups and activities are fixed.

For example, if out of 1,000 graduates aged 18, say, 500 go on to further taining, 400 get jobs in industry, 50 become civil servants and 50 emigrate or die, we assume that if the number of graduates aged 18 were 1,200, then 600 of them would go on to further training, 480 would get jobs in industry, 60 would become civil servants and 60 would emigrate or die.

This assumption means that instead of fixing the coefficients by columns, as we did in the economic model, we must fix them by rows. Also, since the determining variable is no longer the vector of final demands but the vector of primary inputs, we must take the building blocks for this model not from this year's row, as in the case of production, but from this year's columns; in other words, we must ignore $S$ and $d$ and concentrate on $\Lambda^{-1} \times S$, $b'$ and $p'$. In manipulating these variables, however, it is convenient to transpose them, that is turn $b'$ and $p'$ into columns vectors so that they become $b$ and $p$, and interchange the rows and columns of $S$ so that it becomes $S'$.

In this case we can obtain the following equations:

$$p \equiv \Lambda^{-1} \times S' i + b \qquad (10)$$

$$\Lambda^{-1} S' = C \times \Lambda^{-1} \times p \qquad (11)$$

where $\Lambda^{-1} S'i$ represents the column sums of $\Lambda^{-1} \times S$ written as a column vector, and $C$ denotes a matrix of transition probabilities.

From these equations we obtain:

$$p = C \times \Lambda^{-1} \times p + b \qquad (12)$$

Then we can determine what the composition of $p$ is likely to be in $\tau$ year's time. If we apply the operator $\Lambda$ to (12), substitute for $p$ into the new equation, carry out this operation $\tau$ - *1* times and apply $\Lambda$ to the final equation of the series, we obtain:

$$\Lambda^{\tau} \times p = \sum_{\theta=0}^{\tau-1} C^{\theta} \times \Lambda^{\tau-\theta} \times b + C^{\tau} \times p \qquad (13)$$

Comparing the two models, the equation of economic model shows that the present output depends on present and future demand, while, by contrast, demographic model shows the future output (future population) depends on present and future supply. The solution of the economic model is based on the assumption that input patterns, the elements of $A$, remain constant over time; by contrast, the solution of the demographic model is based on the assumption that the transition probabilities, or output patterns, the elements of $C$, remain constant over time.

The two points of contrast raise two important practical questions:

a) how are we estimate the future values of $e$, and of $b$?
b) how can we introduce the changes in the coefficients within the two models?

In the demographic case, if we ignore the highly erratic element of immigration and concentrate on births, we can express the future values of $p$ simply in terms of its present values, as follows.

Since it is possible to relate the number of births of either sex to the age composition of the female population, consider a vector of the female population grouped by year of age, $f^*$, ranging from birth to the end of the female reproductive span. Then consider a matrix, $H$, whose rows and columns are equal in number to the elements of $f^*$: the first row of $H$ contains the rates at which females are born to females of different ages; the diagonal below the leading diagonal contains the survival rates of females at the different ages; all the other elements of $H$ are zero. Thus we can write:

$$\Lambda^{\theta} \times f^* = H^{\theta} \times f^* \qquad (14)$$

If we consider another matrix, $J$, whose rows are equal in number to the elements of $p$ and whose columns are equal in number to the elements of $f^*$: the top left-hand corner of $J$ contains a one; the rest of the first row contains the rates at which males are born to females of different ages; all the other elements of $J$ are zero, and in this case:

$$\Lambda^{\tau-\theta} \times b = J \times H^{\tau-\theta} \times f^* \qquad (15)$$

where the first element of J picks out the first element of $H^{\tau-\theta} \times f^*$, that is, the female births in year $\tau - \theta$, and the age-specific male birthrates is the rest of the first row of J calculate and add together the male births in respective year. If we substitute for $\Lambda^{\tau-\theta}$ from (15) into (13), we obtain

$$\Lambda^{\tau} \times p = \sum_{\theta=0}^{\tau-1} C^{\theta} \times J \times H^{\tau-\theta} f^{*} + C^{\tau} \times p \qquad (16)$$

It is noticed that the demographic model, from an open model with one exogenous variable, b, has become a closed model, that is a model which generates all its variables endogenously from given initial conditions.

With regards to the second question, for the demographic model, if C changes with successive values of $\Lambda^{\tau}$, then we can rewrite (13) as:

$$\Lambda^{\tau} \times p = \Lambda^{\tau} \times b + \sum_{\theta=0}^{\tau-1} \left(\prod_{\lambda=\tau-1}^{\tau-\theta} \Lambda^{\lambda} \times C\right) \Lambda^{\tau-\theta} b + \left(\prod_{\theta=\tau-1}^{0} \Lambda^{\theta} \times C\right) p \qquad (17)$$

where Π denotes the operation of forming a product.

In the demographic case, Table 4 can be filled in because in the population case it might want to account for changes of activity within the year (for example, a person who flows in from the preceding year as a schoolboy may go to a university in the course of the year and thus flow out into the succeeding year as an undergraduated). Therefore, we must introduce a new matrix, S*, at the intersection of the row and column for this year, and replace S by S**, and the model can be depict according to Table 5.

Table 5

**A demographic accounting model with intra-year transitions**

| | | Our country | | | Elsewhere | Total |
|---|---|---|---|---|---|---|
| | | **Last year** | **This year** | **Next year** | | |
| Our country | Last year | | $\Lambda^{-1} \times S^{**}$ | | | |
| | This year | | $S^{*}$ | $S^{**}$ | $d^{*}$ | $p^{*}$ |
| | Next year | | | | | |
| Elsewhere | | | $b'$ | | | |
| Total | | | $p^{*\prime}$ | | | |

The function of the new matrix, $S^{*}$, can best be explained by an example: when an individual moves from category $j$ to category $k$ in the course of the year, this movement is represented in $S^{*}$ by a –1 at the intersection of row $j$ and column $j$, balanced by a 1 at the intersection of row $j$ and column $k$. Thus the sum of the elements of $S^{*}$ is zero, since in the aggregate all the transfers cancel out. From this it follows that the matrices $\Lambda^{-1} \times S^{**}$ and $S^{**}$ do not differ in the sum of their elements from $\Lambda^{-1} \times S$ and $S$; they do deffer from them in the arrangement of their elements, however, because each survivor now enters next year from the activity in which he leaves this year and not from the activity in which he had entered this year. For the same reason the vectors *d* and *p* now become *d* şi p*;* the vector of primary inputs, *b*, is not affected.

Similar to its economic counterpart, the model of this situation is built from three elements:

$$p \equiv \Lambda^{-1} \times S^{**'} i + S^{*'} i + b \tag{18}$$

$$S^{*'} = C^{*} \times \hat{p}^{*} \tag{19}$$

$$S^{**'} = C^{**} \times \hat{p}^{*} \tag{20}$$

from which we obtain:

$$p^{*} = C^{**} \times \Lambda^{-1} \times p^{*} + C^{*} \times p^{*} + b \tag{21}$$

whose solution for year τ is:

$$\Lambda^{\tau} p = \sum_{\theta=0}^{\tau-1} \left\{ \left[ \left( I - C^{*} \right)^{-1} C^{**} \right]^{*} \left( 1 - C^{*} \right)^{-1} \Lambda^{\tau-\theta} \times b \right\} + \left[ \left( I - C^{*} \right)^{-1} C^{**} \right]^{\tau} p^{*} \tag{22}$$

### 3. Applying of demographic matrix

Applying of demographic model, where there is statistical data, it is no more difficult than the construction of their economic counterpart. However, it is necessary to clarify the way in which we are going to define the categories and to find a means of reconciling the classifications used in the different statistical sources with each other and with our classifications.

In a demographic matrix the primary classification should be by age and the secondary classification should be by activity. Dividing all information into uniform age-groups is not quite so simple; for example, we may take into consideration the age at the end of the calendar year, but in other cases flows are recorded by the age at which they take place (first employment is recorded by age on entry, death by age at death etc.) asking for endless adjustments.

As concerns the classification by activities, it should be drawn up on the following considerations. With minor exceptions, the newly born enter the home of their parents and remain there until, at age two, a few of them begin to go to nursery school. Accordingly, at this age we must establish a new category, requiring a separate row and column in the matrix, so as to distinguish between two-year olds who go to nursery school and two-year olds who stay at home. All two-year olds come from the one-year olds of the year before. At the age three the supply comes from two sources: two-year olds who went to nursery school and two-year olds who stayed at home. With the statistical data available, it is not possible to tell how many children return to the category “home” after first year at nursery school nor how many go to school for the first time when they are three; therefore, it is necessary to make assumption that all children who were at nursery school at age two continue in it at age three, and that only the additional three-year old school-goers come directly from “home”. With increasing age more and more children go to school until, when the age of compulsory school attendance is reached, the category “home” becomes virtually empty. Even at this early age it would be possible to distinguish different administrative types of school (for example, public or private), but from an educational point of view this thing is not representative. An argument for making such distinctions would be only if there were significant differences in the economic inputs (personnel expenditure, material, and investments) used in the different types of school.

Around the age of 10 or 11 most children pass from primary education to some form of secondary education. Here again, a purely administrative classification of schools is of only minor interest, being more useful to group schools by types accordingly to curricula applied (for example, theoretical education or vocational education and training).

At the age of 16 or 17 compulsory education comes to an end and the majority of children leave school and seek employment. Even when employed, however, they may continue to receive education, mainly continuos professional training. Those who remain at school are going to continue their education for two years more, usually a specialised kind of education. So, from the age of 16 on, it becomes important to distinguish among those who have left the educational system (some of which participate in continuous professional training and some do not participate), and among those who remain in the educational system, between those who focus on the real profile (mathematics and science), those who focus on the human profile (humanities), those who continue in vocational education or those who continue in technical education.

Also, those who remain at school after the age of 18 gradually drop out of school either into employment or into some other institution of advanced education, such as post high-school education or a university. By the age of 23 to 30 even the longest type of formal education, such as medical training or postgraduate work at university, is over and virtually the whole population is engaged in gainful occupation. From this point we can follow them throughout their working life, at retiring age, their home or some institution becomes the centre of such activity. At age 100 or so the accounts close.

The structure described realtes to a cross-section of the existing human population in a given year. In economic terms, it provides a basis for cross-section analysis of the category described above “demographic input-output”.

If we could compile a set of tables as Table 5 stretching over 100 years, we could select the information relating to successive ages in each table and thus obtain the elements for a time-series analysis of a particular human population. However, it is difficult to carry out a classification of the population after completion of initial training but the model can be applied to educational planning for which data on the flow of students are available.

## Conclusions

In the usual, static accounting system all entries relate to a single time-period and the set of accounts is completely closed. In the dynamic system the inputs for a given period come, either in whole or in part, from the preceding period and the outputs go, either in whole or in part, to the succeeding period.

Two types of model can be build within the framework of this dynamic accounting structure: the conventional input-output model (appropriate to the analysis of production flows), in which the input coefficients are fixed, and allocation model (appropriate to the analysis of demographic flows), in which the outputs coefficients are fixed.

These two models provide us with the main building blocks for an educational model, because the human inputs on the educational system are simply a partition of the demographic system, while economic inputs are a partition of the productive system.

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