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To Differentiate or Not to Differentiate: A Question When Some Consumers Are Loyal

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Abstract. *I analyze a duopoly that competes first in product characteristics and then in prices. I show that when there exist consumers that are loyal to specific brands with no regard for product characteristics, the second-stage price game doesn't have a pure-strategy equilibrium or symmetric mixed-strategy equilibrium and the firms choose to maximize their product differentiation in the first stage, contrary to what is usually assumed in the price dispersion literature.*

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Introduction

Since Hotelling's seminal paper in 1929, there has been a long debate over whether firms tend to minimize or maximize product differentiation at equilibrium (D'Aspremont, 1979, pp. 1145-1150). But none has studied firms' equilibrium product differentiation decisions when some consumers are loyal to specific brands with no regard to whether the products are differentiated or not. On the other hand, it has been assumed in the price-dispersion literature (Varian, 1980, pp. 651-659, Rosenthal, 1980, pp. 1575-1579, Baye, Morgan, 2001, pp. 454-474) that firms sell homogenous products but each possess a number of loyal consumers.

This paper combines the two lines of literature. It sets up a model with two firms competing in a linear city of unit length and two types of consumers: shoppers and loyals. It shows that even when some consumers have no regard for product characteristics, it pays for the two firms to differentiate their products. What is usually assumed in the price dispersion literature, that the firms sell homogeneous products, is therefore never a result of firms' equilibrium choice behavior.

Santore (1999, pp. 43-52) uses a similar model set-up and analyzes firms' pricing strategies when some consumers swithes with transportation costs. It shows that a mixed-strategy equilibrium generally exists and a pure-strategy equilibrium exists if and only if the number of swithers is small.

I'm able to algebraically solve for the firms' equilibrium profits using mixed strategy in the second stage, when there are both shoppers and loyals, and compare them with the case of homogeneous products to show that the firms would choose to maximize product differentiation and that selling homogeneous products, as is assumed in the price dispersion literature, is not an equilibrium choice.

The model

There are two firms located on a linear city of unit length. The two firms sell identical products with constant marginal cost of production, which is normalized to zero. Firm 1 is located at distance a from one end of the city and Firm 2 is located at distance b from the other end of the city. Without loss of generality, I restrict a and b to be between $\left[0, \frac{1}{2}\right]$. That is, the two firms are located on different halves of the line city. The two firms charge prices of respectively p_1 and p_2 for their products. I assume both firms earn strictly increasing profits up to a unique monopoly price r.

There are two types of consumers: shoppers and loyals, both of whom have unit demand. That is, they buy exactly one unit. There are S shoppers uniformly distributed along the linear city. Shoppers incur unit transportation cost c to buy from the firms. I assume the transportation cost is proportional to the square of distance. That is, a shopper located at x will incur transporation cost of $c(x-a)^2$ to buy from firm 1 and transportation cost of $c(x+b-1)^2$ to buy from firm 2. Shoppers buy from the firm that offers the lower delivery price. Delivery price is the sum of mill price (p_1 or p_2) and transportation cost. There are L loyals who buy from one and only one firm. I assume half of the loyals buy from firm 1 and the other half buy from firm 2. Loyals don't pay for transportation. Or, they are located at the same place on the linear city as their prefered firm. For them, delivery price is the same as mill price.

At the first stage of the game, the two firms choose their respective locations on the line city and commit. At the second stage, given their respective locations, the two firms compete in prices. I consider only the case when r is large enough and/or c is small enough for the model to be interesting. I analyze the case by backward induction. That is, I work out the price equilibria for the second stage of the game first and then go back to the first stage of location choices.

Note that when $S=0$ the model is trivial, with both firms serving their respective loyal consumers at the monopoly price. When $L=0$, the model degenerates into the Hotelling (1929) model, with equilibrium prices $p_1^* = \frac{c\left[(b-2)^2-(a+1)^2\right]}{3}$, $p_2^* = \frac{c\left[(a-2)^2-(b+1)^2\right]}{3}$ and equilibrium profits $\Pi_1^* = \frac{c(1-b-a)(3+a-b)^2}{18}S$, $\Pi_2^* = \frac{c(1-b-a)(3+b-a)^2}{18}S$. It can be checked that $\frac{\partial \Pi_1^*}{\partial a} = -\frac{c(3+a-b)(b+3a+1)}{18}S < 0$, $\frac{\partial \Pi_2^*}{\partial b} = -\frac{c(3+b-a)(a+3b+1)}{18}S < 0$.

This shows that the firms tend to maximize product differentiation and would choose to locate at the two ends of the city at equilibrium.

For the rest of the paper, I'll assume neither S nor L is zero. Actually, I'll assume there are a large number of both loyals and shoppers in the paper.

I first look at the case when $a=b=\frac{1}{2}$. That is, when the two firms both choose to locate at the center of the city and to sell homogeneous products. It can immediately be seen that this is a copy of the Varian (1980) model.

The demand function for firm i in this case is then

$$D_i = \begin{cases} \dfrac{L}{2} & \text{if } p_i > p_j \\ \dfrac{L+S}{2} & \text{if } p_i = p_j \\ \dfrac{L}{2}+S & \text{if } p_i < p_j \end{cases} \quad \text{where } i = 1, 2;\ j = 1, 2 \text{ but } i \neq j .$$

This price game doesn't have a pure-strategy equilibrium but mixed-strategy equilibria generally exist. I only look at the symmetric mixed-strategy equilibrium where both firms set their prices according to the same density function. Let $f(p)$ be the density function from which both firms draw their prices and $F(p)$ the correspondent cumulative distribution function.

Proposition 1: There exists a unique symmetric mixed-strategy equilibrium for the game where both firms' pricing strategy is given by the atomless, strictly increasing cumulative distribution function $F(p) = \frac{1}{S}\left(\frac{L}{2} + S - \frac{Lr}{2p}\right)$ with support on $\left[\frac{L}{L+2S}r, r\right]$ and the equilibrium expected profits for both firms are $\Pi_1^{**} = \Pi_2^{**} = \frac{L}{2}r$.

Proof: First, the two firms would never price the same at equilibrium. This is true because one firm could price an ε lower, capture all of the shoppers and make a larger profit.

Second, the two firms would never price above the monopoly price r or below $\frac{L}{L+2S}r$ at equilibrium. The latter is also true because at equilibrium the firms can earn at least their monopoly profit of $\frac{L}{2}r$. Therefore, for the firms to serve both shoppers and loyals, we must have $p\left(\frac{L}{2} + S\right) \geq \frac{L}{2}r$, or $p \geq \frac{L}{L+2S}r$.

When firm i sets its price at p, with probability $F(p)$, it will get only its loyal consumers, and with probability $1 - F(p)$ it will also get all of the shoppers. Therefore the firm's expected profit is $E\Pi_i = p\left[\frac{L}{2}F(p) + \left(\frac{L}{2} + S\right)(1 - F(p))\right] = p\left(\frac{L}{2} + S - SF(p)\right)$. For it to be a stable equilibrium, it must be true that the firm makes the same expected profit everywhere on the support of $F(p)$. In particular, this is true when $p = r$. Substituting into the above equation, I have $\Pi_i^{**} = \frac{L}{2} r$ and $F(p) = \frac{1}{S}\left(\frac{L}{2} + S - \frac{Lr}{2p}\right)$.

I prove the existence and uniqueness of the symmetric mixed-strategy equilibrium by stating that $F(p)$ is an atomless function which is strictly increasing in p and which is the unique solution to the equation $E\Pi_i = \Pi_i^{**}$.

Next, I look at the case when the two firms do not choose to locate at the center of the city at the same time. A consumer located at x is indifferent to buy from either firm 1 or firm 2 when $p_1 + c(x-a)^2$ is the same as $p_2 + c(x-1+b)^2$. Setting the two equal and solving for x, we get

$$\bar{x} = \frac{p_2 - p_1}{2c(1-b-a)} + \frac{1+a-b}{2} \tag{1}$$

Therefore the demand function for the two firms are respectively

$$D_1 = \begin{cases} \frac{L}{2} & if \;\; p_1 > p_2 + c\left[(1-b)^2 - a^2\right] \\ \frac{L}{2} + S\left[\frac{1+a-b}{2} + \frac{p_2 - p_1}{2c(1-b-a)}\right] & if \;\; p_2 + c\left[(1-b)^2 - a^2\right] - 2c(1-b-a) \le p_1 \le p_2 + c\left[(1-b)^2 - a^2\right] \\ \frac{L}{2} + S & if \;\; p_1 < p_2 + c\left[(1-b)^2 - a^2\right] - 2c(1-b-a) \end{cases}$$

$$D_2 = \begin{cases} \frac{L}{2} & if \;\; p_2 > p_1 - c\left[(1-b)^2 - a^2\right] + 2c(1-b-a) \\ \frac{L}{2} + S\left[\frac{1+b-a}{2} + \frac{p_1 - p_2}{2c(1-b-a)}\right] & if \;\; p_1 - c\left[(1-b)^2 - a^2\right] \le p_2 \le p_1 - c\left[(1-b)^2 - a^2\right] + 2c(1-b-a) \\ \frac{L}{2} + S & if \;\; p_2 < p_1 - c\left[(1-b)^2 - a^2\right] \end{cases}$$

This game doesn't have a pure-strategy equilibrium but mixed-strategy equilibria exist. I first look at the symmetric mixed-strategy equilibrium where both firms set their prices according to the same density function.

Proposition 2: There doesn't exist a symmetric mixed-strategy equilibrium for the game.

Proof: I assume there exists a symmetric mixed-strategy equilibrium for the game. Let $g(p)$ be the density function from which both firms draw their prices and $G(p)$ the correspondent cumulative distribution function.

When firm 1 sets its price at p, with probability $G\left(p-c\left[(1-b)^2-a^2\right]\right)$, it gets only its share of loyal consumers; with probability $1-G\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right)$ it gets all of the shoppers as well as its share of the loyal consumers, and with probability $G\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right)-G\left(p-c\left[(1-b)^2-a^2\right]\right)$ the firm gets some shoppers as well as its share of the loyal consumers. Therefore, we could write the equilibrium expected profit of firm 1 as

$$
\begin{aligned}
E\Pi_1^* &= p\frac{L}{2}G\left(p-c\left[(1-b)^2-a^2\right]\right) \\
&\quad + p\left(\frac{L}{2}+S\right)\left[1-G\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right)\right] \\
&\quad + p\left\{\frac{L}{2}+S\left[\frac{1+a-b}{2}+\frac{E(p)-p}{2c(1-b-a)}\right]\right\} \\
&\quad \left[G\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right)-G\left(p-c\left[(1-b)^2-a^2\right]\right)\right] \\
&= p\left(\frac{L}{2}+S\right)-pS\left[\frac{1+b-a}{2}-\frac{E(p_2)-p}{2c(1-b-a)}\right]G\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right) \\
&\quad - pS\left[\frac{1+a-b}{2}+\frac{E(p_2)-p}{2c(1-b-a)}\right]G\left(p-c\left[(1-b)^2-a^2\right]\right) \\
&= k
\end{aligned}
$$

where k is a constant and $E(p_2)$ is the expected value of the other firm's price. The last equality is true because at equilibrium the expected profit of firm 1 must be constant everywhere on the support of the density function.

Since $p+2c(1-b-a)\geq p$, $G(p)$ is an atomless and increasing function by assumption and that $G(r)=1$. Subsituting $p=r$ into the above equation, I have

$$k=\frac{L}{2}\left\{r+c\left[(1-b)^2-a^2\right]\right\}$$

Analogously, I have

$$
\begin{aligned}
E\Pi_2^* = {} & p\frac{L}{2}G\left(p + c\left[(1-b)^2 - a^2\right] - 2c(1-b-a)\right) \\
& + p\left(\frac{L}{2} + S\right)\left[1 - G\left(p + c\left[(1-b)^2 - a^2\right]\right)\right] \\
& + p\left\{\frac{L}{2} + S\left[\frac{1+b-a}{2} + \frac{E(p_1) - p}{2c(1-b-a)}\right]\right\} \\
& \left[G\left(p + c\left[(1-b)^2 - a^2\right]\right) - G\left(p + c\left[(1-b)^2 - a^2\right] - 2c(1-b-a)\right)\right] \\
= {} & p\left(\frac{L}{2} + S\right) - pS\left[\frac{1+a-b}{2} - \frac{E(p_1) - p}{2c(1-b-a)}\right]G\left(p + c\left[(1-b)^2 - a^2\right]\right) \\
& - pS\left[\frac{1+b-a}{2} + \frac{E(p_1) - p}{2c(1-b-a)}\right]G\left(p + c\left[(1-b)^2 - a^2\right] - 2c(1-b-a)\right) \\
= {} & l
\end{aligned}
$$

where l is a constant and $E(p_1)$ is the expected price of the other firm.

Substituting $p = r$ into the above equation, I have

$$l = \frac{L}{2}\left\{r + c\left[(1-a)^2 - b^2\right]\right\}.$$

Lemma 1: If the game has a symmetric mixed-strategy equilibrium, it must be true that the two firms are located at the two ends of the city with a = 0, b = 0.

Proof: This is true because $\frac{\partial k}{\partial a} < 0$, $\frac{\partial 1}{\partial b} < 0$, where k and l are expected profits of the two firms if a symmetric mixed strategy equilibrium exists.

Substituting a = 0, b = 0 into equations for the expected equilibrium profits, we have

G (p + c) = G(p – c),

which contradicts our assumption that G(p) is an atomless and strictly increasing function.

This completes the proof that the game doesn't have a symmetric mixed-strategy equilibrium.

Proposition 3: There exist asymmetric mixed strategy equilibria for the game.

Proof: It suffices to show that the game has mixed strategy equilibria.

According to Glicksberg (1952, pp. 170-174) the game has a continuous pay-off function and compact action sets, and therefore has a mixed strategy equilibrium.

Let h(p) and i(p) be the respective density function from which firm 1 and firm 2 draw their prices and H(p) and I(p) be the cumulative density function respectively. Then we have

$$\begin{aligned}
E\Pi_1^* &= p\frac{L}{2}H\left(p-c\left[(1-b)^2-a^2\right]\right) \\
&\quad + p\left(\frac{L}{2}+S\right)\left[1-H\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right)\right] \\
&\quad + p\left\{\frac{L}{2}+S\left[\frac{1+a-b}{2}+\frac{E(p)-p}{2c(1-b-a)}\right]\right\} \\
&\quad \left[H\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right)-H\left(p-c\left[(1-b)^2-a^2\right]\right)\right] \\
&= p\left(\frac{L}{2}+S\right)-pS\left[\frac{1+b-a}{2}-\frac{E(p_2)-p}{2c(1-b-a)}\right]H\left(p-c\left[(1-b)^2-a^2\right]+2c(1-b-a)\right) \\
&\quad - pS\left[\frac{1+a-b}{2}+\frac{E(p_2)-p}{2c(1-b-a)}\right]H\left(p-c\left[(1-b)^2-a^2\right]\right) \\
&= h
\end{aligned}$$

for firm 1’s expected equilibrium profit and

$$\begin{aligned}
E\Pi_2^* &= p\frac{L}{2}I\left(p+c\left[(1-b)^2-a^2\right]-2c(1-b-a)\right) \\
&\quad + p\left(\frac{L}{2}+S\right)\left[1-I\left(p+c\left[(1-b)^2-a^2\right]\right)\right] \\
&\quad + p\left\{\frac{L}{2}+S\left[\frac{1+b-a}{2}+\frac{E(p_1)-p}{2c(1-b-a)}\right]\right\} \\
&\quad \left[I\left(p+c\left[(1-b)^2-a^2\right]\right)-I\left(p+c\left[(1-b)^2-a^2\right]-2c(1-b-a)\right)\right] \\
&= p\left(\frac{L}{2}+S\right)-pS\left[\frac{1+a-b}{2}-\frac{E(p_1)-p}{2c(1-b-a)}\right]I\left(p+c\left[(1-b)^2-a^2\right]\right) \\
&\quad - pS\left[\frac{1+b-a}{2}+\frac{E(p_1)-p}{2c(1-b-a)}\right]I\left(p+c\left[(1-b)^2-a^2\right]-2c(1-b-a)\right) \\
&= i
\end{aligned}$$

for firm 2’s expected equilibrium profit.

Substituting $p = r + c[(1 - b)^2 - a^2]$ into the equation of firm 1's expected equilibrium profit, we have $\Pi_1^{***} = h = \frac{L}{2}\left\{r + c\left[(1-b)^2 - a^2\right]\right\}$ as the expected equilibrium profit for firm 1.

Analogously, we have $\Pi_2^{***} = i = \frac{L}{2}\left\{r + c\left[(1-a)^2 - b^2\right]\right\}$ as the expected equilibrium profit for firm 2.

Proposition 4: When there are both shoppers and loyals, the firm will choose to maximize product differentiation and locate at the two ends of the city.

Proof: This is true by comparing the two firms' equilibrium profits when the two firms choose to locate at the center of the city at the same time and when they don't and to note that $\Pi_i^{**} < \Pi_i^{***}$, $\frac{\partial \Pi_1^{***}}{\partial a} < 0$, $\frac{\partial \Pi_2^{***}}{\partial b} < 0$.

Subsituting a=0 and b=0 into the equation for both firms' expected equilibrium profit, we have

$$p\left(\frac{L}{2} + S\right) - pS\left[\frac{1}{2} - \frac{E(p_2) - p}{2c}\right] H(p + c) - pS\left[\frac{1}{2} + \frac{E(p_2) - p}{2c}\right] H(p - c) = \frac{L}{2}(r + c) \quad (2)$$

and

$$p\left(\frac{L}{2} + S\right) - pS\left[\frac{1}{2} - \frac{E(p_1) - p}{2c}\right] I(p + c) - pS\left[\frac{1}{2} + \frac{E(p_1) - p}{2c}\right] I(p - c) = \frac{L}{2}(r + c) \quad (3).$$

Let $\underline{p}_1$ be the lower support of firm 1's pricing strategy and let $p+c=\underline{p}_1$,

we have $\underline{p}_1 = c + \frac{L(r+c)}{L+2S}$.

Similarly, we have $\underline{p}_2 = c + \frac{L(r+c)}{L+2S}$.

Although I cannot algebraically solve for H(p) and I(p), they are determined by equation (2) and equation (3) respectively, with support $\left[\underline{p}_1, r\right]$ for firm 1 and support $\left[\underline{p}_2, r\right]$ for firm 2.

Conclusion

The above analysis shows firms will choose to maximize product differentiation even when there exist loyal customers who don't care whether the two firms are differentiated or not. An important result of the paper is that when firms are allowed to differentiate their products, they would always do so and it's not an equilibrium for the firms to sell homogeneous products, as is assumed in most of the price dispersion papers.

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