Abstract. In the paper we analyze an adverse selection model with three states of nature, where both the Principal and the Agent are risk neutral. When solving the model, we use the informational rents and the efforts as variables. We derive the optimal contract in the situation of asymmetric information. The paper ends with the characteristics of the optimal contract and the main conclusions of the model.

Keywords: adverse selection; asymmetric information; informational rent; optimal contract.

JEL Codes: C61, D82, D86.
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Introduction

One of the most important researches of economic theory from the last 40 years is represented by the theory of incentives. This theory has been developed also as a necessity to explain some failures of the classical models from microeconomic theory. The information became an extremely costly good in real economies and the asymmetric information between contractual partners may generate some distortion of the results with respect to optimal Pareto results. Asymmetric information is present in almost all economic activities: there are incentives to work hard, to produce high quality goods, incentives to invest or to save, to reveal the correct characteristics to other partners. The adverse selection models were built in this context; they model situations where the decisions of player (named the Agent) having private information depend on this private information and could negatively affect others participants; the party making the contractual offer and who doesn’t know the Agent’ private information is attempting to reduce this informational disadvantage.

The pioneering article studying the classical approach to the adverse selection problem is Mirrlees (1971). After this article, the initial adverse selection models were developed in many theoretical and empirical ways. Some important contributions belong to Green and Laffont (1987), Myerson (1979), Dasgupta, Hammond and Maskin (1979), who studied the Revelation Principle; the problem with a continuum of types has been studied by Baron and Myerson (1982). A general characterization of necessary and sufficient conditions for the existence of a second best optimum was given by Guesnerie and Laffont (1984); Jullien (2000) and Rochet and Stole (2002) extended the classical models and introduced the type-dependent and random participation constraints; Armstrong and Rochet (1999) analyzed aspects regarding multidimensional asymmetric information.


Recent research shows that the models became more and more complex, most of them being mixed models with moral hazard, signalling or screening. Fudenberg and Tirole (1990) proposed a mixed model where the Agent’s actual
choice regarding the effort is an endogenous adverse selection variable at the renegotiation stage and this aspect generates inefficiency. This problem was partially solved by Matthews (1995) and Ma (1994). Page (1991, 1997) presented a mixed model with moral hazard and adverse selection, and Jullien and Salanie (2007) extended the moral hazard model for the situation where the Agent’s risk aversion constitutes his private information, such that the model presents also an adverse selection problem. Such approach was also used by Reichlin and Siconolfi (2004); they generalized the pure adverse selection model of Rotschild and Stiglitz, including some moral hazard variables. Mylovanov and Schmitz (2008) studied a two period moral hazard model, where the Agents are risk neutral, with limited liability and three identical activities.

In the paper we present an adverse selection problem with three states of natures, where the partners from the Principal-Agent model are both risk neutral. The Principal’s profit depends on Agent’s costly effort and also on some exogenous factors, not correlated with Agent’s behaviour; these factors are referred as “nature states”. This problem was also studied by Marinescu, Miron and Marin (2010) from the perspective of a hazard moral model followed by adverse selection with three states of natures.

The paper is organized as follows. In Section 1 we present the main assumptions of the model and the optimization problem that must be solved for deriving the optimal contract. In Section 2 we use a transformation of variables and we obtain an equivalent model with informational rents-effort as variables. Section 3 contains the analysis of the model in the situation of asymmetric information and in the last part of the paper we summarize the main features of the optimal contract and we propose some possible extensions of the model.

1. The model in the situation of asymmetric information

Consider a Principal-Agent model and suppose that, after signing the contract, the Agent observes (knows) the market conditions – if these conditions are good or bad. We denote by $\theta$ the parameter that characterizes the market conditions, with the possible values $\theta \in \{\theta^G, \theta^M, \theta^B\}$. A high level of this variable, $\theta = \theta^G$, indicates a favourable situation for the business (for Agent’s activity), while $\theta = \theta^M$ corresponds to a medium situation (a medium state of nature), and $\theta = \theta^B$ (bad situation of unfavorable situation on the market) implies some decisions regarding the effort with a higher cost than the other ones. It is obvious then that $\theta^G > \theta^M > \theta^B$.

The Agent will exert a level of total effort denoted by $E$, but this effort level cost more when the market conditions are bad.
Therefore, $E = \theta + e$, where the Agent’s decision regarding the effort level $e$ is costly, but $\theta$ doesn’t. The Agent will choose the costly effort $e$, with respect to the information he gets from $\theta$.

The Agent, after signing the contract, observes the true value of the variable $\theta$ ($\theta^B$, $\theta^M$ or $\theta^G$). The Principal observes the total decision $E$. Because he cannot distinguish between the market conditions, the Principal doesn’t know the effort level exerted by the Agent. This means that the later could exert a high level of effort or a medium or low level of effort.

With adverse selection, the Principal has to offer the same menu of contracts to all types of Agents and has to anticipate that each type of Agent chooses the contract he prefers. Without loss of generality, the Revelation Principle says that the Principal can restrict the menu to a set of direct revelation mechanisms so that the optimization problem is significantly reduced.

A contract is represented by the couple 
$(w,e) \equiv (w,E)$,
where:
- $w$ – is the Agent’s wage;
- $e$ – is the costly effort for the Agent;
- $E$ – is the total effort, $E = e + \theta$;
- $\theta$ - represents the market conditions.

In such a situation with asymmetry of information, we denote the menu of contracts by \( \{(w^G,e^G),(w^M,e^M),(w^B,e^B)\} \). These contracts correspond to the three states of nature (market conditions), with the respective probabilities $\pi^G$, $\pi^M$ and $\pi^B$ from the interval (0,1) and with $\pi^G + \pi^M + \pi^B = 1$.

The menu of optimal contracts is derived solving a nonlinear optimization problem that corresponds to a maximization of Principal’s final expected payoff (meaning the difference between gross expected payoff and the expected wage), subject to the participation and incentive compatibility constraints.

Suppose that the Principal’s profit is equal to total effort $E$. Then, $e = E - \theta$.

We also assume that both Principal and the Agent are risk neutral.

Therefore, for a given value of $\theta$ (a given market characteristic) and for the contract denoted by $(w,e)$, the Agent’s utility function is $w - v(e)$, where:

$v(e)$ represents the disutility of effort, with the well-known properties: $v'(e) > 0$, $v''(e) > 0$, $v(e) > 0$ and $v(0) = 0$.
We can now write the Principal’s optimization problem as follows:

\[
(P) \max_{w^G, w^M, w^B, e^G, e^M, e^B} \left\{ \pi^G \left[ e^G + \theta^G - w^G \right] + \pi^M \left[ e^M + \theta^M - w^M \right] + \pi^B \left[ e^B + \theta^B - w^B \right] \right\}
\]

s.t.
\[
\begin{align*}
w^G - v(e^G) & \geq u \\
w^M - v(e^M) & \geq u \\
w^B - v(e^B) & \geq u \\
w^G - v(e^G) & \geq w^M - v(e^M + \theta^M - \theta^G) \\
w^M - v(e^M) & \geq w^B - v(e^B + \theta^B - \theta^M) \\
w^G - v(e^G) & \geq w^B - v(e^B + \theta^B - \theta^G) \\
w^M - v(e^M) & \geq w^G - v(e^G + \theta^G - \theta^M) \\
w^B - v(e^B) & \geq w^M - v(e^M + \theta^M - \theta^B) \\
w^B - v(e^B) & \geq w^G - v(e^G + \theta^G - \theta^B)
\end{align*}
\]

(with all variables satisfying the sign constraints – non negativity constraints).

2. The transformed model – using the variables: informational rents and costly effort levels

In this section we propose a way of solving the model that differs from the analysis presented by Marinescu Miron and Marin (2010).

We first introduce some other notations.

Let \( U^G, U^M, U^B \) be the Agent’s utility levels obtained in each state of nature. Therefore, we can express these informational rents as:

\[
\begin{align*}
U^G &= w^G - v(e^G) \\
U^M &= w^M - v(e^M) \\
U^B &= w^B - v(e^B)
\end{align*}
\]

In the above optimization program we replace the original variables \( (w^G, w^M, w^B, e^G, e^M) \) with the new ones \( (U^G, U^M, U^B, e^G, e^B) \).

Another assumption that doesn’t restrict the generality of the model is that the parameter values \( \theta^G, \theta^M \) and \( \theta^B \) (the states of nature) are satisfying the following condition:

\[
\Delta \theta = \theta^G - \theta^M = \theta^M - \theta^B > 0
\]

This means that the spread of uncertainty is the same between these three values.
Proposition 1. The function \( f : [0, \infty) \to \mathbb{R}, \ f(e) = v(e + \Delta \theta) - v(e) \) is a strictly positive and strictly increasing function.

Proof

We use the usual properties of the effort cost function \( v(\cdot) \), i.e. \( v(e) > 0 \), \( v'(e) > 0 \), \( v''(e) > 0 \) and \( v(0) = 0 \).

From the property of the monotonic function \( v(\cdot) \) it is obvious that \( f(e) > 0 \), and from the convexity of this function we have also:

\[
f'(e) = v'(e + \Delta \theta) - v'(e) > 0
\]

We use all these assumptions for transforming the problem (P) into an optimization problem with other variables.

With this change of variables, the Principal’s objective function can be rewritten as:

\[
\max_{U^G, U^B, M^G, M^B} \left\{ \pi^G \left[ e^G + \theta M - v(e^G) \right] + \pi^M \left[ e^M + \theta M - v(e^M) \right] + \pi^B \left[ e^B + \theta B - v(e^B) \right] - \left[ \pi^G U^G + \pi^M U^M + \pi^B U^B \right] \right\}
\]

This expression shows that the Principal’s objective is maximizing the expected social value minus the expected informational rent of the Agent.

The participation constraints are now represented by the usual sign constraints for the variables \( U^G, U^B \) and \( U^M \):

\[
U^G \geq 0
\]
\[
U^B \geq 0
\]
\[
U^M \geq 0
\]

The upward and downward (local and global) incentive compatibility constraints are written using the function \( f(\cdot) \) as follows:

\[
U^G \geq U^M + f(e^M - \Delta \theta) \quad (1)
\]
\[
U^M \geq U^B + f(e^B - \Delta \theta) \quad (2)
\]
\[
U^G \geq U^B + f(e^B - \Delta \theta) + f(e^B - 2\Delta \theta) \quad (3)
\]
\[
U^M \geq U^G - f(e^G) \quad (4)
\]
\[
U^B \geq U^M - f(e^M) \quad (5)
\]
\[
U^B \geq U^G - f(e^G) - f(e^G + \Delta \theta) \quad (6)
\]
We show how the constraint (1) is obtained from the incentive compatibility constraint:

\[ w^G - v(e^G) \geq w^M - v(e^M + \theta^M - \theta^G) \]

The right-hand side of this relation is then:

\[ w^M - v(e^M + \theta^M - \theta^G) = w^M - v(e^M - \Delta \theta) \]

\[ = w^M - v(e^M) + v(e^M) - v(e^M - \Delta \theta) \]

\[ = U^M + f(e^M - \Delta \theta) \]

The constraint (3), written in terms of informational rents and efforts, represents a transformation of the upward global constraint:

\[ w^G - v(e^G) \geq w^B - v(e^B + \theta^B - \theta^G) \]

The right-hand side of the above relation is then:

\[ w^B - v(e^B + \theta^B - \theta^G) = w^B - v(e^B) + v(e^B) - V(e^B - 2\Delta \theta) \]

\[ = U^B + v(e^B) - v(e^B - \Delta \theta) + v(e^B - \Delta \theta) - v(e^B - 2\Delta \theta) \]

\[ = U^B + f(e^B - \Delta \theta) + f(e^B - 2\Delta \theta) \]

We obtained in the same way all other constraints.

The major technical difficulty of the Principal’s program is a great number of the constraints imposed by incentive compatibility and participation of Agent. We show that the downward constraints (the last three constraints) can be ignored when solving the optimization problem. Then we check if the optimal solution satisfies these constraints.

We begin by determining which of the participation constraints are the relevant ones.

**Proposition 2.** If \( U^B \geq 0 \), then \( U^M > 0 \) if \( U^G > 0 \).

**Proof**

Using that \( f(e) > 0 \) and the first two constraints we have:

\[ U^M \geq U^B + f(e^B - \Delta \theta) > 0 \]

and

\[ U^G \geq U^M > 0 \]

Therefore, if the participation constraint for the least favourable state is satisfied, then all other participation constraints are satisfied.
3. The optimal contract in the situation of asymmetric information

We are now interested in deriving the optimal solution for the program presented in the above section, assuming that this program has feasible solutions. Before we proceed, we need to prove one more proposition. The following Proposition describes the well-known condition from the literature “the implementation condition or the monotonicity condition”.

**Proposition 3.** If the set of feasible solutions (the set of incentive feasible contracts) of the program (P) is nonempty, then the following condition holds:

\[ e^G + \Theta^G \geq e^M + \Theta^M \geq e^B + \Theta^B \]

_(implementability condition)._

**Proof**

Adding two local (upward and downward) constraints we get:

\[ U^G \geq U^M + f(e^M - \Delta \Theta) \]
\[ U^M \geq U^G - f(e^G) \]

Then:

\[ f(e^G) \geq f(e^M - \Delta \Theta) \]

The function \( f(\cdot) \) is strictly increasing and so, the above relation yields to:

\[ e^G \geq e^M - \Delta \Theta = e^M - (\Theta^G - \Theta^M) \]
\[ e^G + \Theta^G \geq e^M + \Theta^M \]

The same is done, using (2) and (5):

\[ f(e^M) \geq f(e^B - \Delta \Theta) \]

or

\[ e^M \geq e^B - \Delta \Theta = e^B - (\Theta^M - \Theta^B). \]

Then:

\[ e^M + \Theta^M \geq e^B + \Theta^B \]

We finally get:

\[ e^G + \Theta^G \geq e^M + \Theta^M \geq e^B + \Theta^B \]

This condition shows that, the higher value of the parameter characterising the state of market is, the higher the final payoff is.

More than this, if the conditions (1) and (2) are satisfied, then the constraint (3) also holds.

Indeed, adding (1) and (2) yields:

\[ U^G \geq U^B + f(e^M - \Delta \Theta) + f(e^B - \Delta \Theta) \]
But, from the implementability condition $e^M + \theta^M \geq e^B + \theta^B$ and then:

$$U^G \geq U^B + f\left(e^B - \Delta \theta\right) + f\left(e^B - 2\Delta \theta\right)$$

The results derived above allow us to rewrite the reduced optimization program as follows:

$$\begin{align*}
\max_{\theta^G, \theta^M, \theta^B, \theta^G, \theta^M, \theta^B} & \left\{ \pi^G \left[ e^G + \theta^G - v(e^G) \right] + \pi^M \left[ e^M + \theta^M - v(e^M) \right] + \pi^B \left[ e^B + \theta^B - v(e^B) \right] - \\
& \left[ \pi^G U^G + \pi^M U^M + \pi^B U^B \right] \right\}
\end{align*}$$

s.t.

$$\begin{align*}
U^G & \geq U^M + f\left(e^M - \Delta \theta\right) \quad (1) \\
U^M & \geq U^B + f\left(e^B - \Delta \theta\right) \quad (2) \\
U^B & \geq 0
\end{align*}$$

This is the final form of the program we solve further.

First, we show that all the remaining constraints are binding at the optimum.

**Proposition 4.** At the optimum, the informational rent corresponding to the least favourable state satisfies $U^B = 0$.

**Proof**

Suppose that this is not true. Assume that $U^B > 0$. Let $\varepsilon > 0$ be an arbitrary small scalar such that $U^B - \varepsilon \geq 0$.

We have $U^G \geq U^M \geq U^B > 0$ and then we could reduce the informational rents of the other types of Agent such that the relevant constraints continue to hold:

$$\begin{align*}
U^G - \varepsilon & \geq U^M - \varepsilon + f\left(e^M - \Delta \theta\right) \\
U^M - \varepsilon & \geq U^B - \varepsilon + f\left(e^B - \Delta \theta\right) \\
U^B - \varepsilon & \geq 0
\end{align*}$$

In this case, if $(U^G, U^M, U^B, e^G, e^M, e^B)$ represents the optimal solution, then the feasible solution $(U^G - \varepsilon, U^M - \varepsilon, U^B - \varepsilon, e^G, e^M, e^B)$ is better than the optimal solution (in the sense that the corresponding value of the objective function is higher than the corresponding value of the supposed optimal solution) and gives the Principal an extra payoff equal to $\varepsilon$. And this is a contradiction.

So $U^B = 0$ it is optimal.
Therefore, in the least favourable state of nature the participation constraint is binding, the Agent obtains the outside opportunity level of utility exactly.

A similar result is true for the incentive compatibility constraint (2).

Proposition 5. The incentive compatibility constraint given by (2) is binding at the optimum.

Proof
Suppose it is not. Assume that \( U^M > f(e^B - \Delta \theta) \).

Therefore, we can decrease by \( \varepsilon > 0 \) the informational rents \( U^M \) and \( U^G \); the program’s constraints are indeed satisfied, but the objective function’s value is increased by \( (1 - \pi^B)\varepsilon \). But this is again a contradiction.

Hence, \( U^M = f(e^B - \Delta \theta) \). This means that the informational rent associated with the parameter value \( \theta^M \) is strictly positive and higher than the corresponding rent associated with \( \theta^B \).

Proposition 6. The constraint \( U^G \geq f(e^B - \Delta \theta) + f(e^M - \Delta \theta) \) is binding at the optimum.

Proof
Suppose it is not, i.e. \( U^G > f(e^B - \Delta \theta) + f(e^M - \Delta \theta) \)

Then, the feasible solution \( (U^G - \varepsilon, U^M = f(e^B - \Delta \theta), U^B = 0, e^G, e^M, e^B) \) (the optimal solution is modified) is better than the optimal solution in the sense that the objective function’s value is increased by \( \pi^G \varepsilon \). But this is a contradiction.

Hence, for the state of nature that corresponds to \( \theta^G \), the Agent gets a strictly positive informational rent, given by:

\( U^G = f(e^B - \Delta \theta) + f(e^M - \Delta \theta) \), with \( U^G > U^M = f(e^B - \Delta \theta) \).

Substituting the results given by the last three Propositions, \( U^B = 0 \), \( U^M = f(e^B - \Delta \theta) \) and \( U^G = f(e^B - \Delta \theta) + f(e^M - \Delta \theta) \) into the reduced program \( (P') \), this problem is transformed into an unconstrained maximization problem, with concave obtive function; we denote this new problem by \( (P'') \):
The necessary and sufficient conditions are:

\[
\frac{\partial F}{\partial e^G} = 0 \quad \text{or} \quad \pi^G \left[ 1 - v'(e^G) \right] = 0 \tag{7}
\]

\[
\frac{\partial F}{\partial e^M} = 0 \quad \text{or} \quad -\pi^G f'(e^M - \Delta \theta) + \pi^M \left[ 1 - v'(e^M) \right] = 0 \tag{8}
\]

\[
\frac{\partial F}{\partial e^B} = 0
\]

or

\[
-\pi^G f'(e^B - \Delta \theta) - \pi^M f'(e^B - \Delta \theta) + \pi^B \left[ 1 - v'(e^B) \right] = 0 \tag{9}
\]

From (7) it follows immediately that:

\[
v'(e^G) = 1 \tag{10}
\]

Compared with the full information setting, asymmetric information doesn’t affect the effort level corresponding for the characteristic \( \theta^G \). So the second best effort is efficient for this parameter value.

The first order condition (8) yields to:

\[
v'(e^M) = 1 - \frac{\pi^G}{\pi^M} f'(e^M - \Delta \theta) < 1 \tag{11}
\]

This equation gives the second best effort \( e^M \) associated with the state of nature \( \theta^M \).

The relation (9) can be rewritten as:

\[
v'(e^B) = 1 - \frac{\pi^G}{\pi^M} f'(e^B - \Delta \theta) - \frac{\pi^M}{\pi^B} f'(e^B - \Delta \theta) < 1 \tag{12}
\]

This last equation yields to the second best effort level \( e^B \) in the case of the least unfavourable state of nature.

We can now check that the optimal solution derived above satisfies also the ignored constraints.
Proposition 7. The second best optimal informational rents:

\[ U^G = f(e^B - \Delta \theta) + f(e^M - \Delta \theta), \]
\[ U^M = f(e^B - \Delta \theta), \]
\[ U^B = 0, \]
satisfy also the downward (local and global) incentive constraints.

Proof

We substitute the above expressions of the informational rents into each ignored incentive constraint. We then have:

i) The constraint (4):

\[ U^M \geq U^G - f(e^G) \]
becomes:

\[ f(e^B - \Delta \theta) \geq f(e^B - \Delta \theta) + f(e^M - \Delta \theta) - f(e^G) \]
or

\[ f(e^G) \geq f(e^M - \Delta \theta) \]

The function \( f(\cdot) \) being strictly increasing, it follows that:

\[ e^G \geq e^M - \Delta \theta \]

But this is true, from the implementability condition \( e^G + \theta^G \geq e^M + \theta^M \).

ii) The constraint (5):

\[ U^B \geq U^M - f(e^M) \]
becomes:

\[ 0 \geq f(e^B - \Delta \theta) - f(e^M) \]
or

\[ f(e^M) \geq f(e^B - \Delta \theta) \]

The function \( f(\cdot) \) being strictly increasing, it follows that:

\[ e^M \geq e^B - \Delta \theta \]

and this is an equivalent relation to the implementability condition:

\[ e^M + \theta^M \geq e^B + \theta^B \]

iii) The constraint (6) is a consequence of (4) and (5). Adding (4) and (5) we get:

\[ U^B \geq U^G - f(e^G) - f(e^M) \geq U^G - f(e^G) - f(e^G + \Delta \theta), \]
where the last inequality was written using the implementability condition and the monotonic property of $f(\cdot)$.

Hence, we obtain:
\[ U^B \geq U^G - f(e^G) - f(e^G + \Delta \theta), \]
and this is exactly the constraint (6).

4. Conclusions

We derived in the last section the necessary and sufficient condition for the optimal solution in the situation of asymmetric information. We can now summarize the main features of the optimal menu of contracts in the following theorem:

Theorem. Under adverse selection with three states of nature, the optimal menu of contracts entails:

A. No effort distortion for the most favourable state of nature ($\theta^G$) with respect to the first best level $\bar{e}^G = \bar{e}^G$. This efficient level is given by (10):
\[ v'(\bar{e}^G) = 1 \]

B. A downward distortion for the characteristics $\theta^M$ and $\theta^B$. The equations (11) and (12) yield to the second best effort levels $\bar{e}^M$ and $\bar{e}^B$, where:
\[ \bar{e}^M < \bar{e}^M \quad \text{and} \quad \bar{e}^B < \bar{e}^B \]

These effort levels are inefficient with respect to the corresponding first best levels, for both states of nature (the medium one and the least favourable one).

C. The optimal (second best) informational rents are given by:
\[ \bar{U}^G = f(\bar{e}^G - \Delta \theta) + f(\bar{e}^M - \Delta \theta) \]
\[ \bar{U}^M = f(\bar{e}^B - \Delta \theta) < \bar{U}^G \]
\[ \bar{U}^B = 0 \]

The Agents acting in good or medium market conditions get strictly informational rents, while the Agent who acts in an unfavourable state of nature gets no informational rent

D. The second best optimal transfers are respectively given by:
\[ \bar{w}^G = \bar{U}^G + v(\bar{e}^G) \]
\[ \bar{w}^M = \bar{U}^M + v(\bar{e}^M) \]
\[ \bar{w}^B = v(\bar{e}^B) \]
While the Agents acting in a good or medium market conditions get a level of transfer higher than their disutility of costly effort, the Agent acting in an unfavourable state of nature gets a transfer exactly equal to his effort disutility.

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