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Bayes-Nash Equilibrium and Game Theory in Public Expenditure Management

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Abstract. *Auctions purchases represent an effective mechanism for public authorities, designed to provide the buyer (public authority), products and services with a convenient time to complete the transaction and the option to set a minimum price. In this paper we will study the problem of the optimal public expenditure rules using Bayes-Nash equilibrium in an symmetrical auction with knowledge of independent value, meaning each bidder knows only his own information. After setting the function for optimum balance to profit for the bidders by minimizing this function (the derivation of I order) and maximize it (the derivation of II order), it has to identify the optimal range where the offer of a bidder for products and services will stand.*

Keywords: auction, Bayes-Nash equilibrium, optimization, public authorities, public expenditure.

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1. Introduction

The today science of public finance is required in our days, through its specialists to resolve many of the problems that the financial economy of any country has. Chronic budget deficits and public debt are just two examples of indicators which are taken under observation constantly by international financial institutions. Of course, to these indicators there are added others that are given by sciences like macroeconomics, namely: GDP, inflation rate, unemployment rate or monetary matters such as foreign exchange or interest rate.

All these indicators provide financial institutions and central or local governments information about the evolution of the financial health of the country in general. They are not only diagnostic indicators. They are also signaling indicators, under which certain measures to rationalize public spending or increase revenues to the public budget may be taken to mitigate the adverse effect of chronic budget deficits.

The global economic crisis has led governments, for not getting out of control the excessive budget deficits, to intervene with crisis action plans, in hopes of identifying the necessary antidote to neutralize and remove the negative impact that the global economic crisis has had and has upon the economic development.

What can the science of public finances do under these circumstances? Or what can the public finance specialists do to help reduce, if not remove, the negative impact of the global economic crisis that has spread with great intensity at all levels of economic and social life?

The answer is difficult to formulate in only a few rows. This is because the science of public finances, as any science of complexity, is satisfying in this period a number of functions that do not remove the "template" of a scientific science. On the contrary, the science of public finances, thus strengthening its credibility in the face of those who believed the power of information and decision impaired by frequent university essays from pulpits, they have no immediate applicability, even practical.

We say, without fear of being wrong, that the first service of the public finances science is the information. This is because the science of public finance has a tool that large corporations seem to use it more and more, known for specialists as "budget execution account". An attempt to define it, beyond the scientific rigor required of specialists in the field, would lead us to the idea that the budget execution account is the mirror image of fiscal discipline imposed by budgetary appropriations, but also a way of measuring the degree of collection of funds. The game of financial resources dynamics, combined with budgetary discipline imposed by the rules given by the budgetary credits offer, in the end, budgetary excess or deficit, which can often be dangerous. Beyond the figures recorded in the statement of budget execution, sometimes hard to decipher, is the financial dimension of budgetary resources by the classic and modern rules used in estimating budgets. Specialists using these method lead ultimately to the development of another document of financial summary, known as the budget of income and expenditure.

A second function of the science of public finances in a period of global economic crisis is the signaling, in general, a very important role in preventing risks. It's like a driver without signaling, runs the risk of injury, which can have a fatal impact on it. The science of public finances, in a coded language, can indicate such risks, even in real time, to avoid financial collapse, state that may intervene in the management of public resources. This is because information from the budget of income and expenses are translated into the budgetary execution account, whose harsh rules of are to be followed almost daily by specialists in the field. What information does the budgetary execution account provide us? It is a first category of information related to public expenditure. We can use the financial resources to finance expenditure than within the approved budget appropriations. And we all know this. A simple rule that signals when certain categories of public spending have exhausted the funds approved. A second category of information concerns government revenue. All financial resources budgeted in the form of taxes, coded with indicators of budgetary classification, are followed up after their receipt by determining the degree of collection. So we will identify the areas of budgetary execution account which surplus of the collections, marking them as areas with (+) or the same account areas of budget execution are with deficit, marking them as areas with (–). The frequency of areas with (+) and areas with (–) can be the signal of a cash deficit for which preventive measures should be taken immediately by those responsible (Boloş, 2002, p. 284).

A third function of public finance science is to rationalize public spending. Usually in times of economic crisis there are constraints imposed to operational expenditures. Wage reductions or reductions in operating costs are famous examples in this regard. Protected in times of crisis are capital expenditures. By increasing their attempts to send a pulse in the economy, hoping that the engine will restart the stopped economies, fueled with public investments.

Finally the last service of the public finance science is the function of fiscal rigor. Are the public finances able to provide models for calculating taxes and duties to meet the requirements of fairness and rational tax bases for taxation? An answer difficult and quite complicated. This is because not anyone is able to generate principles of taxation on ficality, when the number of taxpayers exceeds hundreds of thousands or even millions of taxpayers. Either tax equity requirements and imposing rational can not be neglected. In lieu of taxes or tax increases is preferred a measure, also with fiscal origins, leading to an increased tax base or, why not, to diminish the phenomenon of tax evasion. But all these are measures to be extremely well planned, so as not to disturb the natural course of the economy. Or not to transform a tax measure in a measure of negative impact on national economies.

All four of these functions, which are not limited to, can transform the science of public finances, not necessarily overnight, into a useful tool, invested with credibility, to base decisions in the public sector.

2. Auction theory – step prior to the introduction of game theory in public finances

Why are auctions important for the theory of public finances and why acutions represent the prior step to the introduction of game theory in the science of public finances. The answer to this questions is not simple.

First of all we have to clarify the link between public finances and the theory of auctions. To understand this link we need to identify the impact they have on the management of livestock auctions in public financial resources sector.

It is already known that through auctions are purchased goods and services required for the functioning of public authorities and services. Auctions, throughout the Community, is the generic name that is assigned to the procurement procedures, represented by direct purchase, direct negotiation, request for bids and the actual auctions, through which goods are procured in the public sector.

For the science of public finance, auctions is the key element through which is formed the value of public assets to be acquired. And this value is the result of competition between suppliers of goods and services, which intends to provide public assets, at the lowest price and best quality.

Auction rules are established by public authorities, which set the values of public assets that are expected to purchase (V_{est}). These estimated values of public assets are seen as the market values or fair values, that values that will be allowed by the public authority. Over any of these estimated values, the bidders can not deliver the goods and services in the auction organized.

Hence the first rule of auctions, namely that the values of goods and services provided for the needs of public authorities, can not exceed their estimated value.

At the same time, each of the bidders participating in the auction have a specific cost (C_{ef}) that is determined by direct and indirect costs of goods and service providers have. None of the bidders who comply with legitimate rules of operation on the market will not make a bid (V_{ofi}) that is lower than its actual cost (C_{ef}).

In these circumstances the second condition is formed, that is characteristic for the behavior of the suppliers of goods and services market, namely that the amount of assets is greater than or equal to the cost of each of the bidders.

The conclusion is that any of the bidders will have a value of its offer (V_{ofi}) that is lower or at most equal to the estimated value (V_{est}), which in turn is greater than or equal to the cost of each of the bidders (C_{ef}) after a relationship of the form: $V_{est} \geq V_{ofi} \geq C_{efi}$.

These are financial rules that follow the rules of economic efficiency for each of the two parties: the public authority and the provider of goods and services. These rules are added, practical and specific tenders held by public authorities and certain eligibility criteria and criteria for awarding tenders.

The eligibility criteria for public authorities is a safety feature, because with this organization of the auction bidders that not have the technical and financial capacity to deliver goods or complex cases will not show up such as auctions to make the public investment. Criteria, although they are quite diverse in practice, require a certain financial level for bidders, performance but also some equipment carrying. The combination of financial and technical criteria make it possible to prevent specific risks of high complexity public investment (highways, airports, etc.).

Criteria for the award of tenders are rules established and known by the participants in the auction before the organization. Such criterias, although very diverse in practice, are enabling the optimal combination between the price of public assets that are to be provided and quality. Any successful bid that meets the criteria for the award is declared the winner.

Therefore any auction of public assets must abide by the rules of economic efficiency, legitimate of public authority and the bidder that when tendering to the conditions imposed by the eligibility criteria and the criteria for the award of tenders for an offer to be declared the winner.

Why science is important for public finances auctions? It is easy to see how the value of public assets is formed is in the bidding process. The value of any offer is merely a confirmation that the asset values are formed in the public bidding process.

But between the amount bid and the terms that are used in the science of public finances is a direct link. This connection is made through budget appropriations. The value of public assets acquired may be greater than the approved budget appropriations. The rule is compulsory for most public assets that the public authorities acquire. The exception is public investment, for which a solution was multi-annual budget appropriations.

So auctions are those that generate competition among bidders of public assets. Auction rules are established by public authority, which sets the estimated value, eligibility criteria and the criteria for awarding tenders. Estimated value is often, market value, which the public authority considers it can pay for goods and services that are expected to be delivered. No bidder may bid values are over the estimated value. Estimated value in turn can not be higher than the approved budget appropriations. This rules of the game but also the created competition among bidders, are leading eventually to the purchase of public assets whose value is between the estimated and the specific cost of each bidder.

Auction theory is quite old. Pioneering work in the theory of auctions was written by Vickery, in 1960. Currently there are a number of five types of auctions that international practiced whose characteristics are presented in the table below:

Table 1

Types of auctions and their rules

Tipe of auction	Rules of the auction
English auction	The auction price starts at safety, starting at a low price. Tenders increase sequentially, with a growth step, the winner being the highest price. Players can adjust their offer made during the auction.
Dutch auction	The auction starts at a high price, after which the price is gradually reduced until the first offer is made. This type of auction is frequently applied to exchanges with flowers.
First price auction	This type of auction is characterized by the fact that the bid is submitted in a sealed envelope. Bidders do not know what do other bidders offer. It is declared winner if its offer is the highest price. If it comes to buying a property is the winning bid with the lowest price.
Vickery auction	It is characterized by the fact that each bidder submitted its bid in a sealed envelope, containing a maximum and a minimum amount. None of the bidders do not know about other offer. Successful bidder pays the second sum in decreasing order.
Japanese auction	The auction is characterized by the fact that it starts at a reserve price, then just as the English auction, the price increases sequentially. The difference between the two types of auction is that bidders for the tender must remain at the price bid for each auction closes when the competition has remained a single bidder

Source: processed after Stelian Stancu, Nora Mihail (2007), *Metode şi tehnici de conducere. Teorie şi aplicaţii*, Editura Economică, Bucureşti.

Auctions, regardless of type, is for the science of public finance a topic of debate among experts from at least two reasons: first, that the auctions, which are generated by the competitions we enter the exciting world of game theory, and the second is that through their form value of public assets represent a crucial step to engage in public spending.

3. Game theory. Description of the games with incomplete information. Bayes-Nash equilibrium in public finances science

In many cases economically important, the game can begin with the player that has some private information about something relevant to his decisions. These are called games with incomplete information, or Bayesian games (Incomplete information should not be confused with imperfect information in which players do not observe perfectly the actions of other players).

Although no player knows the private information of an opponent, he will have some beliefs about what the opponent knows, and we assume that these beliefs are common assumptions.

In many situations of interest, we will be able to model information asymmetry by specifying for each player position for their own gain, but each player is not sure what their opponents gain functions are.

We use the term "type" of a player to describe his private information. The type fully describe for a player any information that is not public information. Each player knows his own type with complete confidence. His beliefs about other players are comprised of one common level of knowledge about the probability distribution over others' "types".

We can imagine a game as starting with a random motion, which assigns each player a specific type. This movement is imperfectly observed, however each player type information indicates that the movement has marked on it, but no player has direct knowledge of the type awarded to any other player. We can think of as game as a strategic game being played by a single type of each player, provided that at least one player does not know what type is assigned to another player he faced. The notion of "type" indicates the private information that a player holds.

A strategy for a player in the game with incomplete information is one strategy for each of them by that player to formulate a strategic game, with that form.

A Bayesian equilibrium in a static game with incomplete information is a strategy profile, so that each type of each player seeks to maximize expected utility, given the strategies of his opponents and the type of probability distributions for the types of each player.

Information asymmetry and, in particular, playing with incomplete information are extremely important in game theory. This is especially true for dynamic games in which players have more decisions to make in a row. Their previous moves may send private information that are relevant to the decisions of later moving players. Revealing and concealing information are the basis for much of the analysis of strategic behavior and are especially useful as a means to explain the irrational actions that could become part of a nonstrategic world.

A second approach to the game with asymmetric information has been brought by the work of Kreps and Wilson (1982) and provides a game that starts prior convictions common to all players, which specifies the probability that random movement of players to assign types beginning of the game. Some players see movement and update their beliefs, while other players can update their beliefs only through deductions during the follow-up action game for the players that already know.

We analyze in this paper the optimum problem (maximizing profits) in an auction in which the symmetrical independent value is known, meaning each bidder knows only his own information. After the vendor, which has an indivisible object, announces his bidding mechanism, independent bidders decide whether to participate in it or not. Bidders know their assessments before deciding on the amount to bid. But the particularity of the work is the fact that we track the sellers interest, which in our case is the public authority that auctions the purchase of a public good, so it is going to incur costs, and its interest is to make it at the smallest price.

4. Assumptions of the dynamic game with incomplete information in auctions of public assets

We show that the optimal outcome of a tender offer shall not involve stochastic bidding. Each bidder will participate in the auction if his assessment on the cost implications may provide a profit margin. Given an arbitrary distribution of such profit margins depending on the characteristics of bidders, the optimal rule is a known one: the bidder with the higher (or lower, depending on the nature of the tender) evaluation from the participants will be the winner. So the problem of the auctioneer (seller) is reduced to finding the optimal level for reducing the margin by the bidders.

Consider a symmetric independent private value framework. There is a seller (public authorities) characterized by risk-neutral, whom wants to conclude a contract for the purchase of a public good, indivisible, his assessment for the good being estimated V (Vest). There are n ≥ 2 potential risk-neutral buyers (bidders). May Cef_i be a vector representing the actual costs of the tenderers on their assessments, with i Є {1, ..., n}. Bidders have independent evaluations, distributed according to the distribution function F(x), a positive density continuos function. Bidders know only their evaluations. The problem is that the seller should choose an auction mechanism that maximizes her expected profit in one of the Bayesian-Nash equilibrium points (Milgrom, Weber, 1982, p. 114).

A few remarks are needed here. In our analysis we exclude the agent-agent communication. It is noted that the lack of communication between agents is a sufficient condition for our hypothesis that bidders in the auction behave noncooperative, which is a standard assumption in the optimal auctions literature.

We consider any Bayes-Nash equilibrium of any auction where the seller can choose. Since the bidder is characterized as risk-neutral, he cares only to his expected probability to win the bid, denoted by Fi (being a function uniformly distributed on the interval [0,1]) and its expected cost, denoted Cef_i. The expected equilibrium from an offer Vofi can be written as follows:

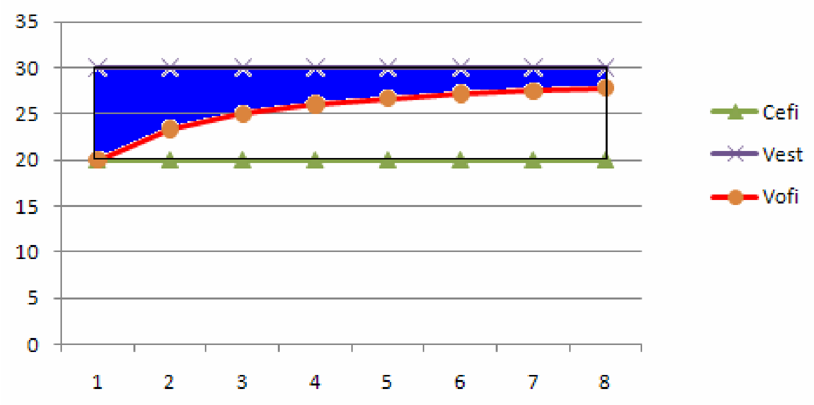
$$\Pi i(Vofi) = (Vofi - Cefi) \times F(Vofi < B(Cef1)\ ;\ Vofi < B(Cef2)\ ;\ \ldots\ ;\ Vofi < B(Cefn)), \quad (1)$$

where $B(C_{efi})=V_{ofi}$.

In relation (1), if we replace the probability function with the uniformly distributed probability density function $\frac{X-Xmin}{Xmax-Xmin}$, we obtain the players profit function with normally distributed probabilities between the two limits, Vest - upper limit and Cefi - lower limit.

$$\Pi i(Vofi) = (Vofi - Cefi) \times \prod_1^n \frac{Vest-Vofi}{Vest-Cefi} \quad (2)$$

Since in the game situation with the proposed auction, a player's probability of winning increases as its offer is reduced, the distribution function in our case will be written as the difference between the upper limit on the amount estimated by the public authority and the player's offer value (the blue part in the graph), reported to the difference between the two limits. Thus, the graph in Figure 1 presents the evolution of both the amount of profit a player obtains, V_{ofi} - C_{efi}, the portion below the axis of the offer, and the probability of winning the bid to Vofi, on the portion above the axis.



Source: own approach.

Figure 1. *The profits of a player in the game with the proposed tender*

To determine the expected profit of the bidders, but also to reach for the optimum function for the public authority, we determine first and second order derivative of profit function written in (2) (Bulow, Klemperer, 1996, p. 54).

Thus, the first-order derivation of the profit function we get:

$$\delta\Pi_i/\delta V_{ofi} = (V_{ofi} - C_{efi})' \text{ x} \left(\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}}\right)^{n-1} + (V_{ofi} - C_{efi}) \text{ x } \left(\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}}\right)^{n-1\,'}$$

$$\delta\Pi_i/\delta V_{ofi} = \left(\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}}\right)^{n-1} + (V_{ofi} - C_{efi}) \text{ x} (n-1) \text{x} \left[\left(\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}}\right)^{n-2}\right] \text{x} \frac{-1}{V_{est}-C_{efi}}$$

$$\delta\Pi_i/\delta V_{ofi} = \left(\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}}\right)^{n-2} \text{x} \left[\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}} + (V_{ofi} - C_{efi}) \text{ x} (n-1) \text{x } \frac{-1}{V_{est}-C_{efi}}\right]$$

In a Bayesian game, the player's potential supply will certainly be less than the value estimated by the public authority, we consider $\left(\frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}}\right)^{n-2}$ different from 0, removing it from $\delta\Pi_i/\delta V_{ofi}=0$. The maximization of a function by first-order derivation involves equating to zero.

$$\delta\Pi_i/\delta V_{ofi} = \frac{V_{est}-V_{ofi}}{V_{est}-C_{efi}} + (V_{ofi} - C_{efi}) \text{ x} (n-1) \text{x } \frac{-1}{V_{est}-C_{efi}} = 0$$

$$\delta\Pi_i/\delta V_{ofi} = V_{est} - V_{ofi} - (V_{ofi} - C_{efi}) \text{ x} (n-1) = 0$$

$$V_{ofi} = \frac{V_{est}+(n-1)C_{efi}}{n} \qquad (3)$$

The offer value from equation (3) is the value, depending on its effective cost, for a player where it minimizes its profit function with the highest probability of winning that competition, when n players are competing in the auction.

But not always the players will bid the offer with the highest probability of winning, sometimes taking a risk by choosing to have a greater profit with less chance of winning (Matthews, 1987, p. 128). Therefore for the second order derivation of the profit function we get a value for which the offer is made to maximize its profit function, reducing the corresponding probability of winning.

$$\delta^2\Pi_i/\delta^2V_{ofi}=[(\frac{Vest-Vofi}{Vest-Cefi})^{n-2}]'x[\frac{(Vest-Vofi)-(Vofi-Cefi)\,x(n-1)}{Vest-Cefi}] + [(\frac{Vest-Vofi}{Vest-Cefi})^{n-2}]x[\frac{(Vest-Vofi)-(Vofi-Cefi)\,x(n-1)}{Vest-Cefi}]'$$

$$\delta^2\Pi_i/\delta^2V_{ofi}=[(n-2)x(\frac{Vest-Vofi}{Vest-Cefi})^{n-3}x\frac{-1}{Vest-Cefi}]x[\frac{(Vest-Vofi)-(Vofi-Cefi)\,x(n-1)}{Vest-Cefi}] + [(\frac{Vest-Vofi}{Vest-Cefi})^{n-2}]x[\frac{-n}{Vest-Cefi}]$$

$$\delta^2\Pi_i/\delta^2V_{ofi}=(\frac{Vest-Vofi}{Vest-Cefi})^{n-3}[\frac{(Vest-Vofi)-(Vofi-Cefi)\,x(n-1)}{Vest-Cefi}\;x\frac{-n+2}{Vest-Cefi} + (\frac{Vest-Vofi}{Vest-Cefi})x(\frac{-n}{Vest-Cefi})$$

$$\delta^2\Pi_i/\delta^2V_{ofi}=[(Vest-Vofi)-(Vofi-Cefi)\,x(n-1)]\,x(-n+2)-(Vest-Vofi)x\,n=0$$

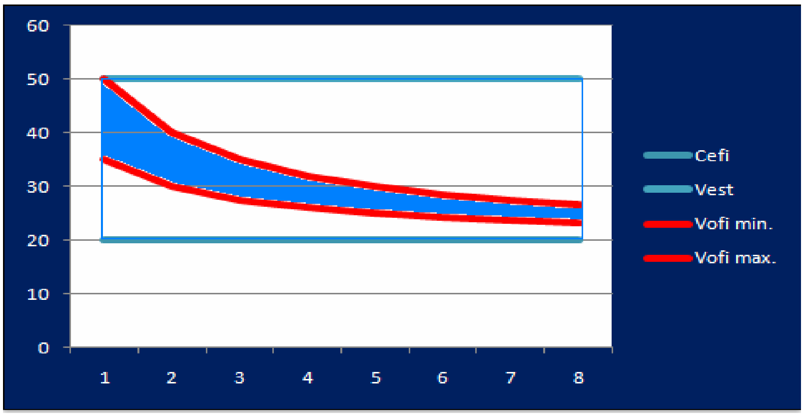
$$[(Vest-Vofi)(-n+2)-(Vofi-Cefi)\,x(n-1)(-n+2)]-(Vest-Vofi)x\,n=0$$

$$Vofi(n-2+(n-1)x(n-2)+n)+Vest(-n+2-n)-Cefi\,x(n-1)x(n-2)=0$$

$$Vofi=\frac{(n-1)(-2Vest-Cefi(n-2))}{(n-1)n}$$

$$Vofi=\frac{2Vest+Cefi(n-2)}{n} \qquad (4)$$

Thus from relations (3) and (4) we obtained two limits for the bidders offer, which can be determined by each player depending on the value estimated by the public authority, its effective cost and the number of players in the auction. When an auction with the terms of the proposed game, the public can create a “channel” of each player's bids, within which it follows to choose which offer to tender.



Source: own approach

Figure 2. *Optimal auction channel, formed by the optimization public authority – bidder relationship*

In a series of simulations carried out by generating the channel, which took into account individual and simultaneous change in the estimated value, effective cost and the number of players, we could see a number of conclusions relevant and useful to the public authority.

In figure 2, the simulation has as input information, a constant estimated value and a constant actual cost and the increase in the number of players. The conclusion that can be observed is that as the number of players increases, the player is forced to reduce it's bided amount, the auction channel thus narrows, but also closes to the actual cost value of the player. This means that an increase in the number of players in a bid determines any player to cut its bidded value, fact that benefits the public authority, leading to the idea of accepting in public auctions a larger number of bidders.

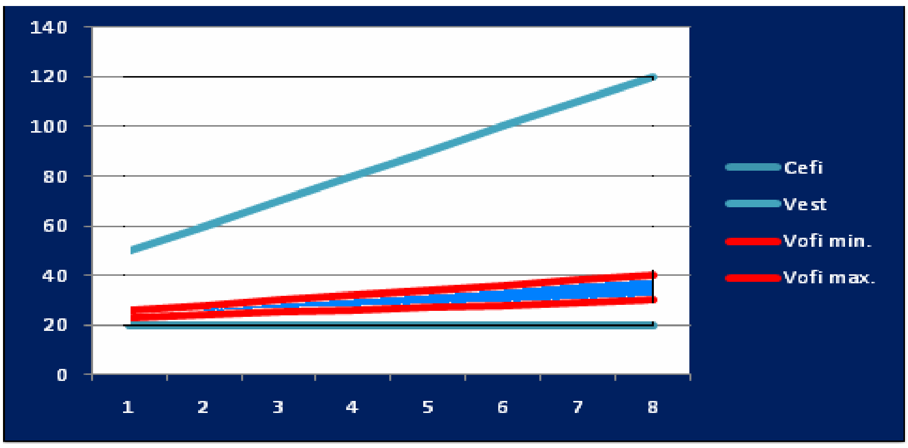


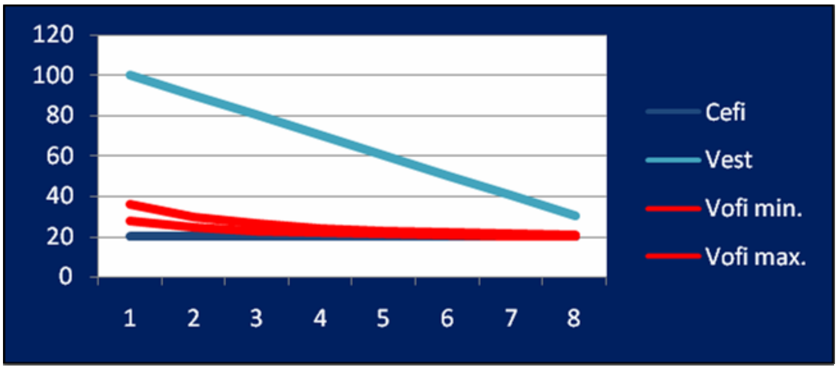
Figure 3. *Simulation game: the actual cost kept constant, number of players constant, the estimated value changed Source: own approach*

Figure 3 is playing back a simulation where a total of 10 players, and the effective cost is kept constant, the estimated value is changed. As the estimated value established by the public is a big difference to the lower limit given by the actual cost, the offer channel of the auction widens and the bidder will have a greater range from which to choose what offer value will bid. On the left side of the graph, where the estimated value is reduced, the range is narrower, and the auction channel is closer to the actual cost of the players.

Table 2

Simulation game by changing the bidding variables

N	10	15	20	25	30	35	40	45
Cefi	20	20	20	20	20	20	20	20
Vest	100	90	80	70	60	50	40	30
Vofi min.	28	24.66667	23	22	21.33333	20.85714	20.5	20.22222
Vofi max.	36	29.33333	26	24	22.66667	21.71429	21	20.44444



Source: own approach.

Figure 4. *Simulation - increase the number of players, with concomitant reduction of the estimated value*

5. Conclusions and proposals

Following the above two simulations generated by increasing the number of players and decreasing the estimated value, we can conclude that this favors the approach of the tendered value to the actual cost of the player, as shown in Figure 4. Thus, the auction channel becomes increasingly narrow and closer to the actual cost of the player's value, reaching nearly to coincide with this value. Basically these two conditions, the increase of the number of players at the same time with establishing an estimated value closer to the actual cost of the potential winner, not restricting the entry of other players in the game, are two objectives that public authorities should consider them a priority in organizing an auction of a public expenditure commitment.

It is obvious that these proposed findings fit in a game where there is no transmission of information. This is one of the main limits of the model, but to some extent they are reflected in an asymmetric information game. Another limit that we try to adjust in future research is the character of neutral-risk players, but also the little amount of variables taken into consideration in calculating the amount of players offer.

Game theory provides a broad framework for further research on many economic issues, including financial resources and improving management of public authorities in tendering for public goods. This area of research has a direct impact on the public budget. Identification of an optimal auction model or draw conclusions relevant to this form of signing contracts of public spending should be a priority for every public authority, in order to make management more efficient and fair for public funds.

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