Econometric Modeling of GDP Time Series

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Abstract. Article aims of time series econometric model of macroeconomic variable GDP in the US economy. Because that is a nonstationary time series, there are used several statistical tests in order to turn into a stationary series. After applying these tests, the time series became stationary and integrated of order I; thus, we use Box-Jenkins procedure for the determination of ARMA. We estimate by OLS the parameters of various models. Performances chosen ARIMA model (1,1,1) are verified on the basis of classical statistical tests and forecasting.

Keywords: stationary time series; nonstationary time series; statistical tests.

JEL Codes: C22, E01. **REL Codes:** 8C, 10G.

In order to set time series, we use macroeconomic variable GDP during 1947 (first quarter) – 2010 (third quarter) in the US economy. Times series values are observed at the same frequency, namely quarterly; each of these values are random variables. Therefore, we can say that GDP is a stochastic process and the actual values observed in period mentioned above are individual achievements of this process.

Descriptive analysis time series gives us informations about the following indicators:

i) time series mean:

$$\bar{y} = \frac{\sum_{t=1}^{T} y_t}{T} = 6,372.390$$

ii) time series variance:

$$s^{2} = \frac{\sum_{t=1}^{T} \left(y_{t} - y_{t} \right)^{2}}{T - 1} = 12,735,711$$

iii) Jarque-Berra statistic value, which suggest a normal distribution of time series in terms of asymetry and flattening.

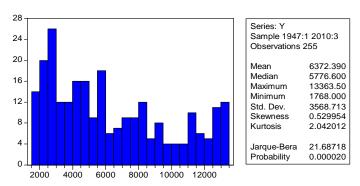


Figure 1. *Histogram and descriptive indicators*

View time series graph shows a linear path with positive slope; therefore it is *nonstationary homogenous type*, characterized by constant changes from one period to another.

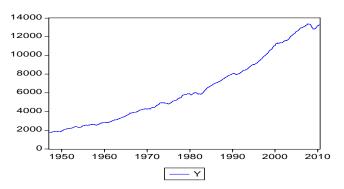


Figure 2. GDP, US, 1947-I to 2010-III

In order to check time series stationarity or nonstationarity we performed statistical tests, such as:

■ Unit root Test -Dickey – Fuller Test (DF):

$$\Delta Y_t = \alpha_0 + \gamma \times Y_t + \alpha_1 \times \Delta Y_{t-1} + e_t$$

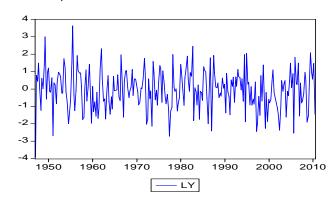


Figure 3. $\ln Y_t$ evolution

We obtain the following results:

$$\Delta Y_{t} = 0.020348 + (-0.01768) Y_{t-1} + 0.353951 \Delta Y_{t-1}$$

$$t = (2.407469) \quad (-1.817024) \quad (6.020099)$$

$$R^{2} = 0.146846 \quad d = 2.062047$$

Our primary interest is in the t (= τ) value of the Y_{t-1} coefficient. Since $\hat{\tau} = -1.82$, and critical τ value is 2.87, for the 5 percent confidence threshold ($\tau > 0.05$) we can not reject RW null hypothesis.

Therefore, GDP time series is *nonstationary*, i.e. it contain a unit root; this conclusion is supported also by the ρ -value (0.3717) associated with t-statistic, which is higher than 0.005.

Autocorrelation function (ACF) and correlogram
 Table 1a
 CDP Correlogram US 1947 I to 2010 III

GDP Correlogram, US, 1947-I to 2010-III

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The autocorrelation coefficients start at a very high value - $\hat{\rho}_1$ = 0.989, $\hat{\rho}_2$ = 0.977, ..., $\hat{\rho}_5$ = 0.943 – and decline very slowly; thus, GDP time series is *nonstationary*.

Since *statistical tests* concluded that time series is nonstationary, the solution is the first differences of GDP time series.

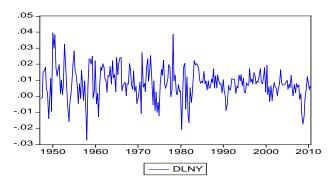


Figure 5. GDP first differences, US, 1947-I to 2010-III

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\Delta Y_t = 0.005049Y_t - 0.632433Y_{t-1}
t = (6.783151) \quad (-10.79566)
R^2 = 0.317093 \quad d = 2.068089
```

The 1 percent critical DF τ value is - 3.456093; since τ < 0.01, we conclude that we can *reject RW null hypothesis* (H₀: γ = 0). Thus, GDP first differenced is stationary, i.e. time series is I(1).

From first differences correlogram we observe that values of autocorrelation coefficients $-\hat{\rho}_1 = 0.368, \hat{\rho}_2 = 0.211, \hat{\rho}_3 = 0.000, \hat{\rho}_4 = -0.008, \hat{\rho}_5 = -0.142$, decline very quickly, so after the first lag it is likely to have an autoregressive process (AR) component of order p = 1.

Table 1b

First differences time series correlogram

Sample: 1947:1 2010:3 Included observations: 254								
Autocorrelation	Partial Correlati o n	A	\C	PAC	Q-Stat	Prob		
1		ı	.368	0.368	34.717	0.000		
' -	ייב' ו		.211	0.088	46.185	0.000		
'] '				-0.120	46.185	0.000		
<u>"</u> " '	"5"			-0.083	48.180	0.000		
<u>.</u> !	'\.'			-0.075	53.440	0.000		
<u>"</u> !	<u> </u>		0.061	0.044	54.431	0.000		
 	'}'			-0.016	55.051	0.000		
'1'	' <u>L</u> '	1		-0.023	55.184	0.000		
, <u> </u>			0.070	0.087	56.494	0.000		
' . .	'. '		0.071	0.019	57.851	0.000		
'	<u>"</u> !			-0.041	58.031	0.000		
5 '	- '			-0.175	62.626	0.000		
!				-0.018	66.532	0.000		
' 5 '	'"	14 -0		0.046	68.571	0.000		
'4.'	'4.'			-0.054	70.515	0.000		
' <u>[</u>] '	I		0.042	0.079	71.007	0.000		
<u> </u>	' '		0.053	0.000	71.782	0.000		
' 🗗	' <u> </u> '	ı	.087	0.035	73.879	0.000		
' [['	' '		0.055	-0.016	74.720	0.000		
'_"	']'		0.061	0.006	75.738	0.000		
" !	<u>"</u> !			-0.098	77.744	0.000		
' <u>"</u> '	']'	22 -0		0.019	78.855	0.000		
'	'['	23 -0		-0.022	81.422	0.000		
'"['	'['	24 -0		0.014	81.714	0.000		
''	<u> </u>		0.032	0.042	81.996	0.000		
1] 1	' '	26 0	1.021	-0.047	82.126	0.000		

Box-Jenkins procedure takes into account the value of d parameter, equal to the initial time series differences order applied to obtain a stationary time series.

Since first differences time series in stationary, $\ln Y_t$ time series is I(1), i.e. d = 1.

To calculate p and q we estimate model parameters by OLS; estimation results are presented in the table bellow.

Table 2a US GDP time series estimated models results

	Eq1	Eq2	Eq3	Eq4
С	0.007984	0.008025	0.007999	0.008012
ar(1)	0.367567	0.332860	0.485481	-0.218832
ar(2)		0.088608		0.302587
ma(1)			-0.134935	0.557360
AIC	-6.520910	-6.518205	-6.51786	-6.520263
SCI	-6.492978	-6.476188	-6.47596	-6.464240
DW	2.068089	1.967281	2.020407	1.981934

We choose between competing models Eq3 model, ARIMA (1,1,1), which admits the following representation:

$$Y_t = 0.007999 + 0.485481Y_{t-1} - 0.134935e_t$$

ARIMA model selection was done taking into account: a) the classical criteria (*adjusted R-square*, *F test*); b) indicators based on information theory (*Akaike and Schwartz statistics*).

For diagnostic checking we analysed:

• residual correlogram: autoregressive model defined by Eq3 suggest that residuals series do not present autocorrelation, i.e. is no need to look for another ARIMA model.

Table 2b AC, ACP and O-stat for Eq3

AC, ACP and Q-stat for Eq3
Sample: 1947:3 2010:3
Included observations: 253
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
111	l di	1 -0.011	-0.011	0.0289	
ı 🛅	<u> </u>	2 0.089	0.089	2.0836	
10 1	ng -	3 -0.064	-0.063	3.1501	0.076
10 1	ng -	4 -0.068	-0.078	4.3537	0.113
= 1	<u> </u>	5 -0.131	-0.123	8.8242	0.032
1 1	1 1	6 -0.005	0.001	8.8321	0.065
101	1 (1)	7 -0.036	-0.023	9.1683	0.103
101	' ['	8 -0.038	-0.061	9.5468	0.145
ı b ı	<u> </u>	9 0.079	0.066	11.215	0.130
ı b ı		10 0.060	0.054	12.179	0.143
ı b ı	<u> </u>	11 0.079	0.060	13.824	0.129
= +	<u> </u>	12 -0.128	-0.148	18.179	0.052
10 1	10 1	13 -0.062	-0.079	19.205	0.058
10(1		14 -0.031	0.025	19.471	0.078
i	' ['	15 -0.090	-0.076	21.645	0.061
ı b ı	<u> </u>	16 0.072	0.064	23.038	0.060
1)1		17 0.025	0.010	23.203	0.080
1 🛍		18 0.064	0.039	24.323	0.083
1)1	1 1	19 0.018	0.003	24.411	0.109
ı b ı		20 0.087	0.038	26.507	0.089
4	i = i	21 -0.107	-0.086	29.674	0.056
1 1	1 1	22 -0.003	0.004	29.677	0.075
1	141	23 -0.086	-0.029	31.745	0.062
141	10	24 -0.018	-0.010	31.841	0.080
1 10 1	<u> </u>	25 0.040	0.048	32.292	0.094
- 4	101	26 -0.017	-0.043	32.372	0.118
· þ ·	1(1	27 0.038	-0.009	32.785	0.137
, þ i	<u> </u>	28 0.053	0.051	33.580	0.146
· þ.	' <mark> </mark>	29 0.081	0.073	35.469	0.127
<u> </u>		30 -0.141	-0.151	41.257	0.051

Sample: 1947:3 2010:3

 $\begin{tabular}{ll} Table 2c \\ ARIMA model residuals correlogram \end{tabular}$

Autocorrelation Partial Correlation	AC			Sample: 1947:3 2010:3 Included observations: 253 Q-statistic probabilities adjusted for 2 ARMA term(s)								
		PAC	Q-Stat	Prob								
2 3 4 4 5 6 6 7 7 8 8 9 9 10 10 10 11 12 13 14 14 15 15 16 16 16 17 17 18 18 19 19 19 19 19 19 19 19 19 19 19 19 19	-0.029 0.047 0.066 0.071 -0.035 0.013 0.039 0.024	0.156 -0.017 0.132 -0.014 -0.004 -0.004 -0.077 0.143 0.091 -0.043 0.051 0.092 -0.098 -0.013 0.007 0.046 0.054 -0.066 -0.059 0.055 0.008	22.995 28.177 32.179 32.741 33.705 37.827 37.954 38.621 38.856 39.462 40.650 42.033 42.331 42.8431 42.867	0.001 0.000 0.000 0.001 0.002 0.002 0.003 0.001 0.001 0.001 0.001 0.001 0.001 0.002 0.002 0.002 0.002 0.002 0.002 0.002 0.002 0.003								

- residuals series homoschedasticity: the phenomenon is not present at residuals series correlogram level;
- residuals series distribution: Jarque Berra statistic value suggests a normal distribution in terms of asymmetry and flattening.

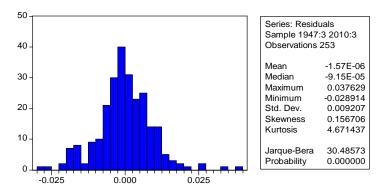


Figure 9. Jarque-Berra test for Eq3

Analysed Y_t time series, by initially nonstationary, was transformed into a stationary after applying two transformation: logarithms and first order difference. Using Box-Jenkins methodology we obtained an ARIMA (1,1,1) model. Model validity was justified on the basis of adjusted R-square, F test, Akaike and Schwartz indicators, statistical tests and forecasting.

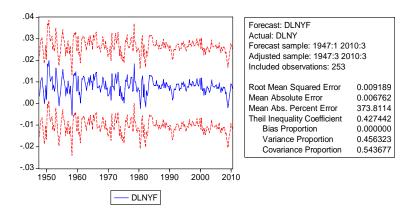


Figure 10. Forecast Eq3

Taking into account the loss function value (0.009189), time series prediction is considered accurate for 5 percent significance level and ARIMA (1,1,1) model.

Since R^2 value is very low, 13.9%, we can not successfully use the previous values of the series in order to make predictions with a high degree of accuracy.

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