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Negative Income Tax and Labor Market Participation. A Short Run Analysis

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Abstract. *This article examines the effects of the negative income tax, in a matching model, on labor market participation. We show that the introduction of such instrument reduces unemployment and improves the situation of the poorest. But, amazingly, it provokes a fall on labor market participation principally because the agents are then less selective. We find another surprising result: despite the rise on participation, the increasing of unemployment benefits improves the situation of the firms at the expense of workers.*

Keywords: matching; participation; negative income tax.

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1. Introduction

To answer the perspectives of demographic evolution and more particularly the problem of pension financing, the European councils of Lisbon and Stockholm already fixed ambitious objectives concerning the rates of employment in the European Union before 2011. It is wished that the rate of employment be then equal to 70 % of the whole population old enough to work, with at least 60 % as regards the women and 50 % for the oldest workers (from 55 to 64 years). Now, the realization of these objectives obviously implies an improvement of the labor market conditions and, possibly, a revision of the redistributive systems in order to increase substantially the participation rate. In order to reduce the disincentive effects of the going back to work, measures consisting in the preservation of the allowances after the return to activity were imagined. The NIT was imagined by M. Friedman in 1962 and resumed by neo-keynesians such as J. Tobin to avoid the traps of assistance favouring an encouragement of employment.

In this article, we develop an analysis on the effects of the NIT in a matching model with horizontal differentiation of the workers and jobs (Marimon, Zilibotti, 1999, pp. 266-291). However, we consider, here, the labor supply at the extensive margin so as to study the effects of a policy based on a NIT scheme on the labor market participation. Authors such as Pissarides (1990) or Garibaldi and Wasmer (2003) have already introduced an endogenous participation into standard job search models. As in these articles, we are interested in the relations between frictions on the labor market and the labor supply. However, none of these works considers, as we do here, the implications on the decision of participation linked to externalities inferred by the meeting process between firms and workers. In other words, we point out the very particular interactions between employment policy, selectivity of the agents, productivity and participation, engendering all the more interesting results as they are *a priori* unexpected. Indeed, we verify that the implementation of a tax credit allows to reduce inequalities for the benefit of the poorest and to increase employment. However, contrary to what we could expect, the tax credit can lead to a decrease of the participation essentially cause of a lesser selectivity of the agents. Furthermore, the results present another unexpected effect concerning the unemployment compensation system. Indeed, in this framework, the increase in unemployment benefits favors firms and provokes a degradation of the situation of the workers. This article gets organized in the following way. In the section 2, we present the model. We solve it in a section 3. Then, we specify and confirm the results of the analysis by proceeding to quantitative exercise in a fourth section. Finally, we conclude our study in a fifth and last section.

2. The model

We consider an economy including two risk neutral agents: the population susceptible to work and firms. Among all the people "capable" of working (N), some integrate the labor market ("active persons", N_A) and the others prefer to stay outside ("non-working - inactive - population", N_I). At each period, the agents capable of working, heterogeneous and having an infinite horizon, decide according to their utility in each situation (that we shall define later) if they participate to the labor market (by trying to find a job) either if they stay "inactive" taking advantage of social-security benefits and of their household production. Besides, firms, in number K, produce the same good and offer each a single job. These jobs are also heterogeneous and we suppose that, at each period, filled jobs can become vacant with a probability s. Besides, among active people (N_A), some will have a job (L) while the others will be unemployed persons (U) and among jobs offered by firms (K), some will be filled (L) while the others will be vacant (V). Consequently, we have: $N_A - U = K - V$.

All the agents have the same discount rate r and R represent the sum $(1+r)$. To describe the differentiation of the workers and the jobs, we use the analytical framework of Salop (1979). We consider that workers ("active people") and firms are uniformly distributed on a circle of circumference equal to 2. This distribution is exogenous. The position of a worker on the circle represents his "type" of qualification while that of the firm represents the exact "type" of qualification whom it looks for. The distance l (between 0 and 1) separating a worker of a firm measures the adequacy between the profiles of each. The adequacy is completed when l=0 and the mismatch is maximum for l=1. The productivity of a worker is then a decreasing function of this distance l noted $y(l)$ with $y'(l)<0$ and $y''(l)\leq 0$. Let us remind that every firm employs only a single worker and its production is determined by the productivity of this one.

Concerning the meeting process (Petrongolo, Pissarides, 2001, pp. 390-431), we consider that the firm which an unemployed worker is going to meet is taken at random among all the firms. Let us note U the number of unemployed workers and V the number of vacant jobs. The labor market tightness is then noted $\theta = V/U$. Let us suppose that λ represent the maximal distance which can separate an employee of his employer. To provide a vacant job, the firm needs to meet only a single worker filling the requirements, that is a worker whose "type" is at a distance not exceeding this mismatch threshold λ. The association employer/employee is then productive enough and thus practicable. We show (appendix 1) that the probability to fill a vacant job, noted q, is determined by:

$$q = 1 - e^{-\lambda/\theta} \qquad (1)$$

We notice that a greater selectivity of firms and workers (*i.e.* a decline of λ) has for consequence a decrease of the probability to fill a vacant job. The probability to be hired, noted p, satisfies:

$$p = (1 - e^{-\lambda/\theta})\theta \tag{2}$$

This probability p is an increasing function of the threshold λ. We show that p is also an increasing function of the labor market tightness θ (appendix 2).

2.1. Intertemporal utilities and profits

Every agent arbitrates between two choices: participate to the labor market by becoming an unemployed worker susceptible to reach employment or stay out of this market and benefit from the return on its household production and on the social-security benefits. Let us suppose that z represents the value of the household production of the "inactive" people and m all the social-security benefits which he/she perceives. His/her intertemporal utility is then written as follows:

$$W_I = z + m + R^{-1}\hat{W} \quad with \quad \hat{W} = max\{W_U; W_I\} \tag{3}$$

The greater is the amount of the social-security benefits from which benefits an inactive worker, the greater is the proportion of those who decide to stay out of the labor market. In the same way, the more an inactive people benefits from his/her household production, the more it is attractive to keep the "status" of "inactive". On the other hand, if the unemployed worker's situation tends to become more interesting (thanks to, for example, an increase of the amount of unemployment benefits or of the probability of hiring), the participation rate will be higher. When a worker obtains a job, his/her productivity, $y(l)$, and thus his/her (gross) salary, $w(l)$, is going to depend on the distance l which separates his/her "type" of that of the firm which hired him/her. We note $W_E(l)$ the intertemporal utility of such a worker. As regards unemployed workers, we consider that they benefit from unemployment benefits, noted b. Their intertemporal utility W_U also depends on the distance λ, which affects the rate of hiring (p) and the expected utility of an employee $\overline{W}_E$. As the distributions on the circle of workers and jobs are supposed uniform, the expected value of a variable x is written as follows:

$$E\,[x(l)] = \bar{x} = \frac{1}{\lambda}\int_0^{\lambda} x(l)dl \tag{4}$$

We introduce a linear taxation scheme such as the Negative Income Tax schematizing the taxation progressiveness. We assume a tax function as follows: $t(w) = -\alpha + \gamma w$. The amount of the tax $t(w)$ paid by each employee depends on the level of his/her income. The peculiarity of the fiscal table holds in the fact that only the workers whose income exceeds a certain threshold (the average wage) pay a tax, while those who earn low incomes benefit from a tax credit. Besides, the workers who earn the average wage are tax-exempt ($t(w(0))= \bar{t}$: the highest tax paid by the worker perfectly adapted to his/her job; $t(w(\lambda))=\underline{t}$: tax credit perceived by the least productive employee. The budget constraint satisfies then:

$$\int_{w(\lambda)}^{w(0)} t(w)dw=0 \tag{5}$$

In the stationary state, the intertemporal utilities $W_E(l)$ and W_U satisfy:

$$W_E(l) = w(l) - t[w(l)] + R^{-1}[sW_U + (1 - s)W_E(l)] \tag{6}$$

$$W_U = b + R^{-1}[pW_E + (1 - p)W_U] \tag{7}$$

The jobs which firms have are vacant or filled. Let us note $J_F(l)$ the value of a filled job:

$$J_F(l) = y(l) - w(l) + R^{-1}[sJ_V + (1 - s)J_F(l)] \tag{8}$$

This value of a filled job depends on the immediate net gain and the future profits dependent on a possible separation between employer and employee. The value of a vacant job J_V is a function of the mismatch threshold λ. This threshold indeed affects the probability q to provide this job as well as the expected value of a filled job $\bar{J}_F$. We have then:

$$J_V = -c + R^{-1}[q\bar{J}_F + (1 - q)J_V] \tag{9}$$

As long as it is not filled, the job costs c to the firm (*i.e.* the employer has to invest to create this job and "to look for" an employee).

2.2. The surplus sharing

According to the generalized Nash rule, the surplus created by a couple employer/employee is distributed between both agents according to their respective bargaining strength. We shall note β ($0<\beta<1$) the workers bargaining strength. The maximization program of the surplus verifies then:

$$Max\beta \ln[W_E(l) - W_U] + (1 - \beta)[J_F(l) - J_V] \tag{10}$$

Then, the following first order condition satisfies:

$$\beta[1 - t'(w(l))][J_F(l) - J_V] = (1 - \beta)[W_E(l) - W_U] \tag{11}$$

The tax schedule gives a constant marginal tax rate ($t'(w(l))$) that we shall note γ. The previous equation can be rewritten in the following way:

$$\beta(1 - \gamma)[J_F(l) - J_V] = (1 - \beta)[W_E(l) - W_U] \quad (12)$$

So, the surplus of the workers with a filled job is represented by:

$$W_E(l) - W_U = \beta[W_E(l) - W_U + J_F(l) - J_V] - \beta\gamma[J_F(l) - J_V] \quad (13)$$

It seems that the proportion of the total surplus got by a worker is lower than his/her bargaining strength (β). Indeed, considering the tax scheme, the average tax rate is increasing with regard to the wage. Consequently, firms take advantage of the fact that workers are incited to negotiate lower wages to get a greater part of the collective surplus. The association employer-employee is practicable only if it generates a positive total surplus. Consequently, the threshold λ, which corresponds to the couple employer/employee the least effective possible (beyond λ, the association does not engender a positive surplus), satisfies:

$$W_E(l) - W_U + J_F(l) - J_V = 0 \Rightarrow W_E(\lambda) = W_U \Leftrightarrow J_F(\lambda) = J_V \quad (14)$$

3. Model equilibrium

3.1. Optimal selectivity and labor market tightness

Using the relation defining the mismatch threshold and the surplus sharing process, we deduce the following relation between the labor market tightness (θ) and the mismatch threshold (λ) (see appendix 3):

$$(r+s)c = \frac{1-\beta}{1-\beta\gamma}q\left[\bar{y} - y(\lambda) + \underline{t}\right] - \left[y(\lambda) - \underline{t} - z - m\right](r+s) \quad (15)$$

In space (λ; θ), this relation ($\lambda \equiv JC(\theta; .)$) is represented by an increasing curve (JC) (Figure 1). Since, on the one hand, [$\bar{y}$ - $y(\lambda)$], [$w(\lambda)$ - $y(\lambda)$] and the probability for a firm to meet a worker are increasing in λ and, on the other hand, q is decreasing in θ, the equation (15) implies that any increase in labor market tightness causes a rise of the mismatch threshold. When θ increases, the probability for firms to meet workers decreases. Therefore, in order to compensate for this effect, they are less selective in the hiring process (λ increases). Moreover, for a given level λ, this relation implies that an increase in a maximum tax credit causes a reduction in labor market tightness (the curve (JC) moves (JC')). Furthermore, if $W_U > W_I$, then the whole population wants to participate to the labor market and if $W_U < W_I$, everyone prefers to stay inactive. Therefore, at the equilibrium, the labor market participation satisfies:

$$W_U = W_I \quad (16)$$

The utility of an unemployed worker is, at the migration equilibrium, equal to that of an inactive. Therefore, from equations (14) and (16), we deduce:

$$W_U = W_E(\lambda) = W_I \quad (17)$$

Note that the expected utility of an employee depends on his/her decision to participate or not to the labor market. Therefore, for there to be trade-off between participation to the labor market and stay out of it, the agents must undergo a loss of instant gain ($z + m - b$) by participating and becoming unemployed. Otherwise, everyone would participate. With this migration equilibrium condition, we obtain (see appendix 3) a decreasing relationship between tightness θ and the mismatch threshold λ:

$$\beta\, p\left[\bar{y} - y(\lambda) + \underline{t}\right] = \frac{(1-\beta\gamma)(r+s)}{1-\gamma}(z + m - b) \quad (18)$$

In space ($\lambda;\theta$), this relationship ($\theta \equiv CW(\lambda; .)$) is represented by a downward curve denoted *CW* (Figure 1). Given that the probability of finding a job p and the difference [$\bar{y}$ - $y(\lambda)$] are increasing in λ and p is increasing in θ, the equation (18) implies a decreasing relationship between the mismatch threshold and the labor market tightness. The intuition behind this relationship is quite simple. For arbitrating agents between participate or not to the labor market, an increase in λ means a greater probability of hiring. Consequently, unemployment is made more attractive by increasing the number of unemployed people and thus reduces the labor market tightness. It may be noted also that λ given, increasing the amount of tax credit awarded to the least productive employee means an increase in θ (the curve (*CW*) moves (*CW'*)) while a lower welfare benefits or higher unemployment benefits causes a decrease in θ.

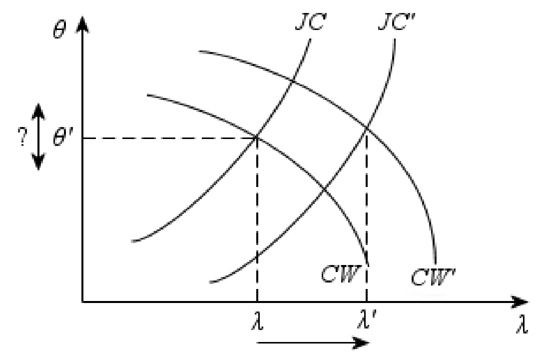

Figure 1. *Negative income tax and selectivity*

Proposition 1. *In a matching model with differentiated skills, the introduction of a tax credit makes agents less selective reducing the matching quality and the average productivity.*

3.2. Labor market participation

At the stationary equilibrium, the number of workers who lose their job must equal the number of unemployed workers who find a job. Therefore, we consider L the employment level. This equilibrium condition implies:

$$pU = sL = s(N_A - U) \text{ and } qV = sL = s(K - V) \tag{19}$$

Therefore, combining the two previous equations, we obtain the expression of the active population N_A as a function of θ and of λ:

$$N_A = \frac{K[s + p(\lambda;\theta)]}{\theta[s + q(\lambda;\theta)]} \tag{20}$$

Equation (20) corresponds to a simple accounting relationship satisfied at the flows equilibrium. Indeed, an increase in the active population means an influx of unemployed people into the labor market which, given the number of jobs, makes the market tightness lower. Similarly, an increasing number of firms means a higher number of vacancies and thus a rise in labor market tightness.

Therefore, with a variable participation, the model equilibrium satisfies the following definition:

Definition 1. *The labour market equilibrium is a set of variables (λ^*; θ^*; N_A^*) which jointly satisfy equations (15), (18) and (20).*

4. Quantitative analysis

We consider an explicit function of productivity, linear form, depending on the “distance” separating the employee from his/her firm, such that: $y(l) = y_0 - \psi l$. We retain the starting values of the following parameters: β= 0,5; ψ= 5; s = 0,02; c = 3; r = 0; N = 2; b = 2; m = 2; z = 2; K = 1 and y_0 = 16. Moreover, in the tables, SB represents the budgetary balance, $W_E(\lambda_i)$, the utility of the poorest employee in the initial simulation and SG, the collective surplus.

Table 1

Negative income tax

			N_A	U	$\bar{y}$	$\bar{w}$	$W_E(0)$	$W_E(\lambda_i)$	$\bar{W}_E$	J_V	$\bar{J}_F$	SG	
$-\underline{t}$	+	+	-	-	-	-	-	+	-	+	+	+	

As showed in *Proposition 1*, it appears (Table 1) that the introduction of a Negative Income Tax makes firms and workers less selective. However, this increase in the mismatch threshold λ causes a decrease in the average productivity. Indeed, the tax credit enjoyed by "low wages" encourages workers to lower their reservation wage, their income remains unchanged, and therefore to accept jobs farther from the type that would suit them perfectly. These jobs are then less effective and therefore tend to have lower average productivity.

Moreover, this lower selectivity of agents tends to increase the probability of filling jobs for firms and the probability of being hired for unemployed people. In fact, since the number of firms is constant, increasing λ simultaneously causes an increase in employment and a decline in the number of vacancies. Therefore, the greater decrease in unemployment causes a rise of the labor market tightness. But this decline of the number of unemployed people (and also of the unemployment rate) is explained by two simultaneous "phenomena": the increase of the hiring probability p (due to higher threshold λ) and the decrease of participation. The decline of the active population, which is verified in *Table 1*, is due to the fall of the expected utility of an employee $\overline{W_E}$ following the increase λ (because of lower average wage). Indeed, the future as employee appears less attractive, then a larger proportion of the unemployed people withdraws from the labor market leading to an increase of the hiring probability for those who remain. Therefore, given that the utility of an unemployed worker, W_U, is determined by the migration equilibrium condition ($W_U = W_I$), increasing p can then compensate for the decline of $\overline{W_E}$. But this observation does not match what is expected from the introduction of a Negative Income Tax. Indeed, while his/her real goal is *a priori* to encourage unemployed to return to work by encouraging them to accept jobs barely interesting (that is indeed the case), it appears that this public policy reduces the attractiveness of the activity comparatively to inactivity.

The Table 1 shows the effects of a NIT on utilities and profits. Firstly, in terms of incomes, it appears that the net wage of the lowest paid employee remains constant despite the decrease in $w(\lambda)$. This result was expected since we have seen previously that at the equilibrium, $W_E(\lambda)=W_I$. Indeed, the tax credit offsets the decrease in his/her direct income, but, then, worker supports an increasing mismatch compared to firm's needs and consequently, the minimum productivity ($y(\lambda)$) and the current profit ($y(\lambda)$ - $w(\lambda)$) decrease. However, we show that the intertemporal utility of the marginal worker, "the lowest paid" at the beginning (which corresponds in the simulation at λ_i), increases. Indeed, this worker, through the tax credit, earned a net income higher than what he had been receiving previously. So, the introduction of a NIT favors the most disadvantaged workers since, whatever the situation of the other employees, the situation of the poorest improves. Nevertheless, the highest net wage ($w(0) - \bar{t}$) decreases. Indeed, the utility of the richest workers declines because of a lower selectivity and their contribution to the cost of the NIT. However, the decline in average productivity is lower than the average wage. Therefore, the average values of filled jobs $\bar{J}_F$ and of vacancies J_V increase.

Remark 1. *Even if it may cause a negative effect on the labor market participation, the introduction of a NIT looks interesting as a redistributive policy in favor of the poorest but also as a policy of reducing the unemployment rate. In addition, it appears that this policy may even lead to an increase of the collective surplus since the expected profits of firms increase though the introduction of the NIT causes in average a loss of job productivity.*

Table 2 presents the unemployment benefits effects on different variables of the economy.

Table 2

Unemployment benefits

			N_A	U	u	$\bar{y}$	$\bar{w}$	$W_E(0)$	$\bar{W}_E$	J_V	$\bar{J}_F$	SG
b	-	-	+	+	+	+	-	-	-	+	+	-

It appears that rising unemployment benefits causes an increase in labor market participation (since the utility of being unemployed is therefore more attractive) leading to a decrease in the probability of hiring p (with an increase in the number of unemployed and in the unemployment rate). The process stops when the migration equilibrium condition ($W_U = W_I$) is respected again. Note that since the number of firms is considered constant, new jobs can not be opened so as to increase competition among firms and to absorb the influx of inactive into the labor market.

Therefore, the labor market tightness tends to decline and it appears that increasing the probability q to fill vacancies increased firms requirements. Firms become more selective (decrease in λ) and that improves the matching quality and thus the average productivity. However, an unusual and unexpected effect is the decline of the average wage w. Indeed, one might expect that the increase in unemployment benefits leads to increase wage demands of workers and thus to increase average productivity. However, given that the unemployed workers are more numerous and that the unemployment rate rises sharply, competition between workers is such that the average wage goes down. Therefore, the utility of all the workers decreases so that some of them (the employees initially the poorest) are advised to leave their job (to be unemployed).

Remark 2. *An increasing in unemployment benefits takes advantage only to firms. Even if this measure can increase the labor market participation, it appears that the competition between workers is then such that the average wage tends to decrease*.

5. Final comments

These results should obviously be considered in light of the different assumptions. In particular, given the rigidity of firms number, we can consider that our approach is rather short-term course. That is precisely what makes it particularly interesting here. Indeed, it is clear that the analysis of long period is only of interest if economic policy has the opportunity to continue. Therefore, to consider for example that the introduction of a NIT may, in the short run, reduce the labor market participation tends to relativize the interest of the implementation of the policy and that, regardless effects that are expected in the long term.

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Appendix

1. Matching function

We assume that U unemployed know exactly the location of the V vacancies and that each unemployed apply in each period. The probability that a vacancy receives a given application is then equal to $1/V$ and, consequently, the probability that it does not receive is equal to $(1 - 1/V)$. However, a vacancy may receive several applications including some that are not suitable (the hiring of these workers there would not in fact sufficient productivity). This assumes that firms are able to identify all applications. In the model used here, the proportion of applications that may be suitable for a particular job is equal to λU (λ is the maximum distance (mismatch threshold) can separate the qualification held by an employee and one required by an employer. Therefore, the probability that a given vacancy receives no suitable application is equal to $(1\text{-}1/V)^{\lambda U}$. The number of hires in each period is then given by:

$$H = V\left[1 - \left(1 - \frac{1}{V}\right)^{\lambda U}\right]$$

However, we have:

$$1 - \left(1 - \frac{1}{V}\right)^{\lambda U} = \exp\left[\lambda U \ln\left(1 - \frac{1}{V}\right)\right]$$

Therefore, assuming a high number of unemployed and job vacancies:

$$1 - \left(1 - \frac{1}{V}\right)^{\lambda U} = \exp\left(-\frac{\lambda U}{V}\right)$$

If we denote $\theta = V/U$, the number of hires, the probability of filling a vacancy and the probability of finding a job for the unemployed are given by:

$$H = (1 - e^{-\frac{\lambda}{\theta}})V \qquad q = 1 - e^{-\frac{\lambda}{\theta}} \qquad p = (1 - e^{-\frac{\lambda}{\theta}})\theta$$

2. Analysis function *p*(*θ*; *λ*)

We have:

$p(\theta; \lambda) = \theta q(\theta; \lambda) = \theta(1 - e^{-\lambda/\theta})$

The derivative of *p*(.) with respect to *θ* is given by:

$$\frac{\partial p}{\partial \theta} = 1 - e^{-\lambda/\theta} - \frac{\lambda}{\theta} e^{-\lambda/\theta}$$

We show that this derivative is defined on the interval [0; 1]. Therefore, the probability *p* is an increasing function of *θ*.

3. Selectivity and job creation process

Using equation (6), the intertemporal utilities of workers satisfy:

$$r\overline{W}_E = R\overline{w} - s(\overline{W}_E - W_U) \quad (21)$$

$$rW_E(\lambda) = R\left[w(\lambda) - \underline{t}\right] - s\left[W_E(\lambda) - W_U\right] \quad (22)$$

Can then be determined through equations (7), (21) and (22) the income expressions of the less productive employee and of an employee:

$$W_E(\lambda) - W_U = \frac{R\left[w(\lambda) - \underline{t} - b\right]}{r+s} - p\frac{\overline{W}_E - W_U}{r+s} \quad (23)$$

$$\overline{W}_E - W_U = \frac{R(\overline{w} - b)}{r+s+p} \quad (24)$$

Therefore, in terms of incomes, it appears, using equations (14), (23) and (24), that:

$$w(\lambda) = b + \underline{t} + p\frac{(\overline{w} - b)}{r+s+p} \quad (25)$$

As workers, we can rewrite equation (8) as follows:

$$rJ_F(\lambda) = R[y(\lambda) - w(\lambda)] - s[J_F(\lambda) - J_V] \quad (26)$$

$$r\overline{J}_F = R(\overline{y} - \overline{w}) - s(\overline{J}_F - J_V) \quad (27)$$

Using equations (25), (26) and (9), we establish the surplus generated by the least productive job, relative to a vacancy, and the average surplus generated by a filled job:

$$J_F(\lambda) - J_V = \frac{R\left[y(\lambda) - w(\lambda) + c\right]}{r+s} - q\frac{\overline{J}_F - J_V}{r+s} \quad (28)$$

$$\overline{J}_F - J_V = \frac{R\left[\overline{y} - \overline{w} + c\right]}{r+s+q} \quad (29)$$

Therefore, if we integrate the equations (23), (24), (28) and (29) in equation (18), the surplus sharing implies:

$$y(\lambda) + c - b - \underline{t} - p\frac{\overline{w} - b}{r+s+p} - q\frac{\overline{y} - \overline{w} + c}{r+s+q} = 0 \quad (30)$$

Equations (12) and (13) show that employers and employees share the surplus resulting from their collaboration based on their respective bargaining strength:

$$\overline{J}_F - J_V = \frac{1-\beta}{\beta(1-\gamma)}\left[\overline{W}_E - W_U\right] \quad (31)$$

If we take the equations (24) and (29), we have:

$$(1-\beta)\frac{\overline{w} - b}{r+s+p} = \beta(1-\gamma)\frac{\overline{y} - \overline{w} + c}{r+s+q} \quad (32)$$

So, with equation (30):

$$y(\lambda) - \underline{t} + c - b - \frac{\left[\beta(1-\gamma)\,p + (1-\beta)q\right]\left[\overline{w} - b\right]}{\beta\,(1-\gamma)\,(r+s+p)} = 0 \quad (33)$$

According to equations (8), (14) and (24):

$$\overline{J}_F - J_F(\lambda) = \frac{R\left[\overline{y} - y(\lambda)\right]}{r+s} - \frac{R\left[\overline{w} - w(\lambda)\right]}{r+s} \quad (34)$$

But, using equations (6) and (21), we deduce:

$$\overline{W}_E - W_E(\lambda) = \frac{R\left[\overline{w} - w(\lambda) + \underline{t}\right]}{r+s} \quad (35)$$

We show then:

$$\bar{J}_F - J_F(\lambda) = \frac{R\left[\bar{y} - y(\lambda) + \underline{t}\right]}{r+s} - \left[\overline{W_E} - W_E(\lambda)\right] \quad (36)$$

Therefore, equations (14), (31) and (36) give:

$$\overline{W_E} - W_U = \frac{\beta(1-\gamma)R\left[\bar{y} - y(\lambda) + \underline{t}\right]}{(1-\beta\gamma)(r+s)} \quad (37)$$

Therefore, substituting the expressions (37) and (24), we establish:

$$\frac{\overline{w} - b}{r+s+p} = \frac{\beta(1-\gamma)}{1-\beta\gamma}\frac{\left[\bar{y} - y(\lambda) + \underline{t}\right]}{r+s} \quad (38)$$

Finally, combining equations (33) and (38), we obtain:

$$(1-\beta\gamma)(r+s)\left[y(\lambda) - \underline{t} + c - b\right] = \left[\bar{y} - y(\lambda) + \underline{t}\right]\left[(1-\beta)q + \beta(1-\gamma)p\right] \quad (39)$$

Using equations (25) and (38), we get:

$$w(\lambda) - b - \underline{t} = \frac{\beta p(1-\gamma)}{1-\beta\gamma}\frac{\left[\bar{y} - y(\lambda) + \underline{t}\right]}{r+s} \quad (40)$$

Equation (39) can be written as follows:

$$(r+s)c = \frac{1-\beta}{1-\beta\gamma}q\left[\bar{y} - y(\lambda) + \underline{t}\right] - \left[y(\lambda) - w(\lambda)\right](r+s) \quad (41)$$

In the equilibrium, the labor market participation implies:

$$W_U = W_I \quad (42)$$

Using equations (14) and (42), we deduce in the equilibrium:

$$W_U = W_E(\lambda) = W_I \quad (43)$$

Combining equations (3), (7), (21) and (42), we establish a third expression of (W_E - W_U):

$$\overline{W_E} - W_U = \frac{R(z+m-b)}{p} \quad (44)$$

Using equations (37) and (44), we obtain a decreasing relationship between λ and θ:

$$\beta\, p\left[\bar{y}-y(\lambda)+\underline{t}\right]=\frac{(1-\beta\gamma)(r+s)}{1-\gamma}(z+m-b) \quad (18)$$

Using equations (40) and (18), we obtain the reservation wage expression:

$$w(\lambda) = \underline{t} + z + m \quad (45)$$

Therefore, equations (41) and (45) give:

$$(r+s)\,c=\frac{1-\beta}{1-\beta\gamma}q\left[\bar{y}-y(\lambda)+\underline{t}\right]-\left[y(\lambda)-\underline{t}-z-m\right](r+s) \quad (15)$$

The equilibrium values(λ^*;θ^*) are given by:

$$(r+s)\,c=\frac{1-\beta}{1-\beta\gamma}q\left[\bar{y}-y(\lambda)+\underline{t}\right]-\left[y(\lambda)-\underline{t}-z-m\right](r+s) \quad (15)$$

$$\beta\, p\left[\bar{y}-y(\lambda)+\underline{t}\right]=\frac{(1-\beta\gamma)(r+s)}{1-\gamma}(z+m-b) \quad (18)$$