Theoretical and Applied Economics
Volume XIX (2012), No. 7(572), pp. 5-12

Estimating the probability of stock market crashes for Bucharest Stock Exchange using stable distributions

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Abstract. *In this study we analyse the evolution of BET Bucharest Stock Exchange through an AR-GARCH model and we estimate the likelihood of extreme events using stable distributions.*

Using the time series of the Bucharest Stock Exchange main index BET we argue that stable distributions can significantly improve the prediction of an extreme event.

Keywords: stable distribution; financial crisis; stock market.

JEL Codes: G01, C10.
REL Code: 11B.

Introduction

Knowing the probability distribution of returns is essential for any statistical inference made about the stock market. In general, it is considered that major distribution characterizing the evolution of returns is the normal distribution (Gaussian) or its derivatives (e.g. log-normal distribution).

More recent papers (Rachev, 2007) show that stable distributions are a much better approach than classical distributions in financial modelling. The fact that the observed distribution of returns is heavy-tailed cannot be explained by a normal distribution.

The relationship between stable distributions and financial crisis has been addressed by Barunik, Vacha and Vosvrda (2010). In this study, they are estimating the parameters of stable distributions for US and Central Europe stock markets, using daily and intraday data. Analysing the distribution of returns for 2005-2009, and separately for the periods 2005-2007 (before the financial crisis) and 2007-2009 (the crisis), the authors conclude that there is a significant difference between the probability distribution of the returns before and during the financial crisis. Thus, the pre-financial crisis period has a small deviation from normal distribution, while the crisis period is characterized by a significant deviation from normality.

In our study we investigate the behaviour of Bucharest Stock Exchange index BET during 2000-2010, using daily data.

The study is structured as follows: first section contains a theoretical presentation of the stable distributions and their estimation methods, in the second section is estimated a model for prediction of extreme negative returns of BET and the last section is for conclusions.

1. Stable distributions

Stable distributions are a class of distributions which have the property of being invariant under linear combinations; Gaussian distribution is a special case of stable distributions.

The difficulty that occurs for stable distributions is that in most cases is not known an explicit form of the probability density function, but only the expression of the characteristic function. Thus, a random variable X follows a stable distribution with parameters $(\alpha, \beta, \gamma, \delta)$ (Nolan, 2011) if exists $\gamma > 0, \delta \in \Re$ such as X and $\gamma \times Z + \delta$ have the same distribution, where Z is a random variable with the characteristic function

$$\phi(t) = E[e^{itZ}] = \begin{cases} \exp(-|t|^{\alpha}[1 - i\beta\tan(\frac{\pi\alpha}{2})\mathrm{sign}(t)]), \alpha \neq 1 \\ \exp(-|t|[1 + i\beta t\frac{2}{\pi}\mathrm{sign}(t)(\ln(|t|)]), \alpha = 1 \end{cases}.$$

In the above notations $\alpha \in (0.2]$ is the stability index, controlling for probability in the tails (for Gaussian distribution $\alpha = 2$), $\beta \in [-1,1]$ is the skewness parameter, $\gamma \in (0,\infty)$ is the scale parameter and $\delta \in R$ is the location parameter.

A random variable X follows a stable distribution $S(\alpha,\beta,\gamma,\delta;0)$ if his characteristic function has the form

$$\phi(t) = E[e^{itX}] = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 + i\beta\tan(\frac{\pi\alpha}{2})\mathrm{sign}(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t), \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t\frac{2}{\pi}\mathrm{sign}(t)(\ln(|\gamma t|)] + i\delta t), \alpha = 1 \end{cases}.$$

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The parametrisation $S(\alpha,\beta,\gamma,\delta;1)$ has the advantage that it is more suitable for algebraic manipulations, although his characteristic function is not continuous for all parameters.

The parametrisation $S(\alpha,\beta,\gamma,\delta;0)$ is suitable for numerical simulations and statistical inference, although the expression of characteristic function is more difficult to utilise in algebraic calculus.

Nolan (2011) shows that the two parametrisations are equivalent; if $X \sim S(\alpha,\beta,\gamma,\delta_1;1)$ and $X \sim S(\alpha,\beta,\gamma,\delta_0;0)$, then $\delta_0 = \begin{cases} \delta_1 + \beta\gamma\tan\frac{\pi\alpha}{2}, \alpha \neq 1 \\ \delta_1 + \beta\frac{2}{\pi}\gamma\ln\gamma, \alpha = 1 \end{cases}$.

The behavior of stable distributions is driven by the values of stability index α: small values are associated to higher probabilities in the tails of the distribution.

2. AR-GARCH model for BET returns

It is generally known that the time series of financial asset returns exhibit special characteristics such as volatility clustering phenomenon, the presence of heavy tails, deviation from normality, stochastic volatility.

Stochastic volatility models could capture the real properties of time series of returns, particularly the phenomenon of stochastic volatility and volatility clustering.

Also, many empirical research shows that there is a correlation structure in the time series of returns, meaning that the daily returns have a memory of at least one day.

To analyse the impact of financial crisis on Bucharest Stock Exchange, we used daily data of BET Index, for the period 2000-2010.

We have computed the logreturns $r_t = \ln P_t - \ln P_{t-1}$, P_t being the closing price of BET index at time *t*.

To model the evolution of BET returns, we have used an AR(1)-GARCH(1,1) model, reflecting both the temporal memory and the stochastic volatility.

Thus, let's assume that the logreturns r_t have the following expression:

$$\begin{cases} r_t = \mu + \phi_1 r_{t-1} + \varepsilon_t \\ \varepsilon_t = \sigma_t z_t \\ \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{cases}.$$

In the above model, z_t is a sequence of *iid* zero mean random variables.

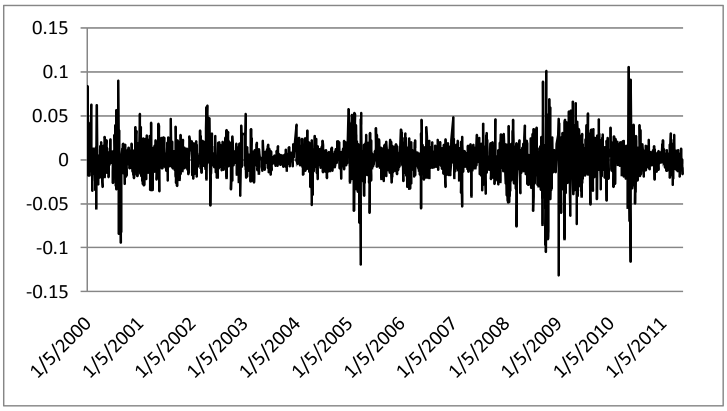

Figure 1. *Daily logreturns of BET Index*

The AR-GARCH model could be a tool for estimating the probability of extreme negative values of returns.

In the following we consider as negative extreme value of daily return any value less than -10%; from 2000 to 2011 were four such extreme events.

Table 1

Extreme negative values of BET returns (2000-2011)

Date	BET Closing Price	Logreturn r_t
1/7/2009	2741.46	-0.13117
3/28/2005	4504.4702	-0.11902
5/25/2010	4365.9902	-0.11612
10/10/2008	3187.77	-0.10454

In order to estimate the probability of a stock market crash at the time t^*, we are using the following strategy (Kim, Rachev et al., 2010):

- Estimate the parameters of AR-GARCH model with Maximum Likelihood, using data for period $[1, t^* - 1]$, and assuming that ε_t follows a normal distribution;
- Estimate the parameters of AR-GARCH model with Maximum Likelihood, using data for period $[1, t^* - 1]$, and assuming that ε_t follows a *t* distribution;
- Residuals from the model with Student distribution are used to estimate the parameters of a stable distribution $S(\alpha, \beta, \gamma, \delta; 0)$;
- Probability of a stock market crash at moment t^* is computed as: $P(\varepsilon_t \le \varepsilon_{t^*-1})$;

- The recurrence interval of such extreme events is estimated as $\dfrac{1}{250P(\varepsilon_t \le \varepsilon_{t^*-1})}$.

Table 2

Parameters of the stable distribution estimated from AR-GARCH residuals

Date	BET Index	BET return	α	β	γ	δ
1/7/2009	2741.46	-0.13117	1.5969	-0.0202	8.72E-03	5.43E-06
3/28/2005	4504.47	-0.11902	1.5385	0.1069	7.64E-03	-1.71E-04
5/25/2010	4365.99	-0.11612	1.5776	-0.0189	9.32E-03	2.98E-05
10/10/2008	3187.77	-0.10454	1.6348	-0.0169	8.65E-03	-1.29E-05

The parameters of the estimated stable distributions for the four extreme events show extreme deviations from normal distribution.

Also, the estimated volatility of the model AR-GARCH with stable innovations has a more significant temporal memory than the model with Gaussian innovations.

Table 3

AR-GARCH parameters

Date	Model	μ	ϕ_1	ω	α_1	β_1
1/7/2009	Normal	0.001	0.159	0.00001	0.189	0.783
	Stable	0.001	0.135	0.00001	0.252	0.719
3/28/2005	Normal	0.001	0.170	0.00001	0.142	0.839
	Stable	0.001	0.147	0.00001	0.228	0.755
5/25/2010	Normal	0.001	0.132	0.00001	0.204	0.780
	Stable	0.001	0.117	0.00001	0.235	0.753
10/10/2008	Normal	0.001	0.158	0.00001	0.176	0.800
	Stable	0.001	0.133	0.000	0.239	0.733

Table 4

Probability of stock market crash

Date	r_{t^*}	Model	$P(r_t < r_{t^*})$	Recurrence interval (years)
1/7/2009	-0.13117	Normal	1.210E-16	3.31E+13
		Stable	0.0021	1.910
3/28/2005	-0.11902	Normal	5.402E-14	7.40E+10
		Stable	0.0025	1.618
5/25/2010	-0.11612	Normal	2.154E-10	1.86E+07
		Stable	3.283E-03	1.218
10/10/2008	-0.10454	Normal	1.953E-10	2.05E+07
		Stable	0.0022	1.804

Estimating the probability of an extreme negative return, it appears that the normal distribution model gives overly optimistic predictions. Thus, on 10 October 2008, when BET had a negative correction of 10% and the Stock Exchange activity was stopped, the estimated probability of Gaussian model is $1.953 \cdot 10^{-10}$, much smaller than the probability of winning the lottery (at Lotto 6/49 this probability is $7.151 \cdot 10^{-8}$). In addition, the recurrence interval of such an event is estimated with normal distribution in $2.05 \cdot 10^{7}$ years, while, for comparison, the age of the universe is estimated to be around $13.75 \cdot 10^{9}$ years.

The models based on stable distributions of these extreme events gives a more realistic probability of occurrence and the recurrence interval is about 1-2 years, which gives a better fit to the empirical reality.

Conclusions

Modelling the stock market has long been based on Gaussian paradigm, that returns follow a normal distribution. Although it has many useful properties, the normal distribution severely underestimates the probability of extreme events.

In this study we have shown, using the time series the BET Index from Bucharest Stock Exchange that stable distributions can significantly improve the prediction of an extreme event.

Acknowledgements

This paper was cofinanced from the European Social Fund, through the Human Resources Development Operational Sectorial Program 2007-2013, project number POSDRU/89/1.5/S/59184, “Performance and excellence in the post-doctoral economic research in Romania”.

References

Barunik, J., Vacha, L., Vosvrda, M., „Tail Behavior of the Central European Stock Markets during the Financial Crisis”, *Working Papers IES* 2010/04, Charles University Prague, Faculty of Social Sciences, Institute of Economic Studies

Kim, Y., Rachev, S., Bianchi, M., Mitov, I. Fabozzi, F., „Time Series Analysis For Financial Market Meltdowns”, No 2, *Working Paper Series In Economics,* 2010, Karlsruhe

Institute Of Technology (KIT), Department Of Economics And Business Engineering, http://Econpapers.Repec.Org/Repec:Zbw:Kitwps:2

Nolan, J.P., „Stable Distributions - Models For Heavy Tailed Data”, 2011, Boston: Birkhauser. Unfinished Manuscript, Chapter 1 Online At Academic2.American.Edu/_Jpnolan

Rachev, S., Mitnik, S., „Stable Paretian Models In Finance”, John Wiley, *Series In Financial Economics And Quantitative Analysis*, 2000, Chechester, New York