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Economy of Referential Preferences A new mathematical approach for choice theory and general equilibrium

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Abstract. *In this paper we introduce basic notions of a new economic model where preference relations on commodities set are represented by a group action on Euclidean space instead of utility function. Conditions that ensure the existence of individual demand function and a general equilibrium in the setting of exchange economy are examined.*

Keywords: preference relations; group theory; general equilibrium.

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Introduction

The mathematical modern conception of general economic equilibrium (GEE) is provided by Arrow-Debreu model developed from 1950 (Arrow, Debreu, 1954). This model pictures the economy as a collection of m economic agents who make supply and demand decisions over a finite set of l commodities in order to further their own interests. The general equilibrium research program then studies many properties of economy, particularly the price, choices of agents, individual and aggregated demand functions (Balasko, 1998).

In a pure exchange model, all agents are consumers, and each of them is provided with a preference relation represented by a utility function on R^l and an initial endowment $e \in R^l_+$ representing his supply offer in the market. Agents are assumed to take as given the market prices of goods. In exchange for his supply, each agent tries to choose the consumption bundle which maximizes his utility given his budget constraint. Such bundle represents the individual demand. Aggregated demand of an economy is the sum of all individual ones, and it is clearly a function of price.

Equilibrium is by definition the vector price $p \in \mathrm{R}^l$ which makes all markets clear (Supply = Demand). The centerpiece of the subject (GEE) deals with the existence and properties of equilibrium. To ensure an affirmative answer to that question, many conditions on preference relations, and hence on utility functions, are assumed. In summary, it is assumed that preferences are continuous, monotonic and convex, or equivalently, utility functions are differentiable and concave.

When these conditions hold for all agents, the economy is then called neoclassical, and equilibrium prices can be reached (Aliprantis et al., 1989).

The aim of this paper is to build a new general formulation of consumers' choice where rationality involves not only maximization of preference, but also a well defined reference of choice, hence our terminology of Economy of Referential Preference (ERP). Although it is clear that this approach can replace, in many instances, the conventional one based on utility function, it is not our main purpose in this paper. In some way, we prove here that the rationality of economic agents can be treated in a diffrent manner than by utility function.

In first section we treat several examples that show the consistency of the group action approach and we explicitly determine the individual demand function. In section two we give a basic definition of an ERP and we end by proving our main result (theorem 8) establishing the existence of an equilibrium in such economy.

1. Motivations and examples of referential preference

In this section it is shown by examples that preference relations on commodities set can be represented by a group action on R^l. This viewpoint sheds some new light on the economic rationality and conditions of equilibrium. In this work we will touch only a few aspects of group theory and knowledge of elementary matricial calculus is sufficient (see Roman, 2012, for details and many examples of group action).

We begin by a simple example where we can see that indifference sets of utility function may be represented, or more precisely replaced by group action on R^l. Here and subsequently R^l_+ denotes the positive cone of R^l, and $R^l_{++} = \{x \in R^l \,/\, x_i > 0,\ 1 \leq i \leq l\}$.

Example 1. The commodity space is R^2_+ and the utility function u is:
u: $\mathrm{R}^2_+ \longrightarrow \mathrm{R}$, $u(x,y)=xy$.

We choose the one-parameter's subgroup G of GL(2, R), $G=\{\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix}, a \in \mathrm{R}^*_+\}$.

The action α of G on R^2 is simply the matricial one on the Euclidean space, namely: $\alpha_g \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ y/a \end{pmatrix}$ where $g = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix}$ for some a > 0.

We assert that indifference sets of u are exactly the orbits for vectors on R^2_+. Indeed, except the trivial case (c=0) which is obviously a union of two orbits, fix c > 0 and the indifference set $I_c = \{\begin{pmatrix} x \\ y \end{pmatrix} \in R^2_+, u\begin{pmatrix} x \\ y \end{pmatrix} = c\}$. Given any commodity $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \in I_c$, his orbit is nothing but I_c itself. Actually, for any $g = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} \in G$, it is clear that $\alpha_g \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} ax_0 \\ \frac{1}{a} y_0 \end{pmatrix} \in I_c$. Conversely, any commodity $\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \in I_c$ is in the orbit $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ since $\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} \frac{\tilde{x}}{x_0} & 0 \\ 0 & \frac{\tilde{y}}{y_0} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} =$

$\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$, where $a = \frac{\tilde{x}}{x_0} = \frac{1}{\frac{\tilde{y}}{y_0}}$ which is due to the fact that $x_0 y_0 = c = \tilde{x}\tilde{y}$.

It remains to show that any orbit is an indifference set. This can be deduced from the fact that $(ax)\left(\frac{1}{a}y\right) = xy$, and for all $\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$ such that $\tilde{x}\tilde{y} = xy$ we have $= \begin{pmatrix} \frac{\tilde{x}}{x} & 0 \\ 0 & \frac{\tilde{y}}{y} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$.

As indifference set is arbitrary, this is sufficient to conclude that the description of indifference sets of consumer with given utility function u can be efficiently made by a group actions on R^2_+.

This example gains in interest only if we are able to see how group action becomes useful to define a mathematical framework of consumer's theory and general equilibrium. In other words, we have to define a complete preordering relation on R^l_+ and a consumer maximization problem in this new setting.

Actually, let be G a topological group and α a continuous action of G on R^l. Here and subsequently, O_x denotes the orbit of $x \in R^l$ under group action. It is easy to check that any group action induces an equivalence relation on R^l. Indeed, such equivalence can be obviously defined as following:

$x \sim y \ iff \ \exists\, g \in G \ st \ \alpha_g(x) = y.$

But since this is not sufficient to give a totally (complete) preorder on R^l_+, some other conditions are needed.

Axiom 1. Let X a non-empty subset of R^l. For all $x \in X$, there is a unique $v \in R^l_+$, such that $x \in O_{v \cdot I_l}$, where $I_l = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in R^l$.

We will denote by v_x the unique real v such that we have $x \in O_{v \cdot I_l}$.

Of course this implies that the quotient of X by the equivalence relation induced by the action of group is identified with R_+ .

Clearly, we can deduce a preference relation on X from a group action which verifies axiom 1. Indeed, we say that x is more desirable than y when $v_x > v_y$, and they are equivalent if $v_x = v_y$.

We simply note that $v_x = v_y \Leftrightarrow \exists\, g \in G$, such that $\alpha_g(x) = y \Leftrightarrow x \sim y$.

The above axiom is not only a simple mathematical hypothesis, but it has an evident economic meaning which asserts that consumer compares each bundle with a very simple one which is: $v \cdot I_l = v \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$.

By identifying $v \cdot I_l$ and $v \in R$ further analysis may eventually lead to interpret $v \cdot I_l$ in terms of a medium of exchange. But this is still just a mere eventuality.

In many examples, axiom 1 is available for all R_+^l and the above preference can be extended to all commodities on R_+^l. When this is not the case we assume that all $x \in R_+^l$ are preferred to anything on the boundary. Taking into account this detail, we state the following definition:

Definition 1

We say that a preference relation $\geq$ on commodity set $\mathrm{R}_+^{\mathrm{l}}$ is of reference type, or referential, whenever either:

1. It is given by a continuous and globally invariant group action on $\mathrm{R}_+^{\mathrm{l}}$ which satisfies axiom 1.
2. It is given by a continuous and globally invariant group action on $\mathrm{R}_{++}^{\mathrm{l}}$ which satisfies axiom 1, and everything in $\mathrm{R}_{++}^{\mathrm{l}}$ is preferred to anything on the boundary.

Returning to the previous example, where $u(x_1, x_2) = x_1 x_2$, we can see that $x \preccurlyeq y \Leftrightarrow u(x) \leq u(y) \Leftrightarrow v_x \leq v_y$.Actually, $u(x) \leq u(y) \Leftrightarrow x_1 x_2 \leq y_1 y_2$, but since $(v_x, v_x) \in O_x$ and $(v_y, v_y) \in O_y$, we have $v_x^2 = x_1 x_2$ and $v_y^2 = y_1 y_2$. Under the condition $v_x, v_y \geq 0$ it follows that $v_x \leq v_y$.

Now we will solve a simple problem of consumer's demand with no use of utility function. The group *G* are the same as in example 1.

Example 2. Let $p = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ the price vector and *w*=200 the budget of the consumer.

To solve the consumer's problem which is

$$\begin{cases} \text{Maximize } v_x \\ \text{subject to the constraint } p \cdot x \leq w \end{cases}$$

we set that $x = \begin{pmatrix} t & 0 \\ 0 & \frac{1}{t} \end{pmatrix} \begin{pmatrix} v_x \\ v_x \end{pmatrix}$ for some *t* and $v_x \in R_+^*$. It's not difficult to verify that *t* and v_x exist and that they are unique. Actually if $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in$

R^2_{++}, then we can see that $v_x = \sqrt{x_1 x_2}$ and $t = \sqrt{\frac{x_1}{x_2}}$. The budget constraint becomes:

$$< p, \begin{pmatrix} t & 0 \\ 0 & \frac{1}{t} \end{pmatrix} \begin{pmatrix} v_x \\ v_x \end{pmatrix} > = w \iff \frac{1}{2} t v_x + \frac{\sqrt{3}}{2t} v_x = 200 \iff \mathrm{v_x} = \frac{400t}{t^2 + \sqrt{3}}$$

We then obtain $v_x(t) = \frac{400t}{t^2+\sqrt{3}}$, that reaches its maximum at $t = \sqrt[4]{3}$, for which we have $v_x = 200\,(\sqrt[4]{3})^{-1}$. Finally, the solution of this maximization problem gives us $x = \begin{pmatrix} 200 \\ 200\frac{\sqrt{3}}{3} \end{pmatrix}$ as the consumer's demand.

To treat the general case we must give necessary and/or sufficient conditions on groups and their actions to ensure reliability and efficiency of axiom preference and so the existence of individual demand function. Indeed, under the axiom 1, we have the following theorem:

Theorem 2. Let be a consumer with referential preference on $\mathrm{R}^{\mathrm{l}}_{+}$ given by a group G. Then, the maximization problem under the budget constraint is equivalent to the minimization of a nonnegative continuous real valued function on the group G.

Proof. Since referential preferences are determined by $v = v_x$, *where* $x = v_x \cdot \alpha_g(I)$, then the demand function is given by the solution of the following problem:

$$\begin{cases} \text{Maximize } v_x \\ \text{subject to the constraint } p \cdot x = w \end{cases}$$

This maximization problem is clearly equivalent to finding the maximal value of v, such that $\langle p, v \cdot \alpha_g(I) \rangle = w$. So, we have to maximize $= v(g) = \frac{w}{\langle p, \alpha_g(I) \rangle}$.

But since $\alpha_g(I) \in R^l_+$ and $p \in R^l_{++}$, we have $\langle p, \alpha_g(I) \rangle > 0$. As $\langle p, \alpha_g(I) \rangle \neq 0 \;\forall\, g \in G$, continuity of *v(g)* follows directly from continuity of group action and scalar product on R^l. As w is fixed, and w and $\langle p, \alpha_g(I) \rangle$ are both positive, then the problem is equivalent to minimizing $\langle p, \alpha_g(I) \rangle$ for $g \in G$. □

In the remainder of this section we assume that referential preferences are given by a subgroup of GL(*l, R)* which satisfy the following axiom:

Axiom 2. For consumer $i \in \underline{I}$, $G^i \subset GL(l,R)$, and the group action's $\alpha: G^i \times R^l_+ \to R^l_+$ which defines his preference relations on the commodity space R^l_+, satisfies: there is a unique $g_i \in G^i$ such that $0 < \langle I, \alpha_{g_i}(I)\rangle \leq \langle I, \alpha_g(I)\rangle \ \forall\, g \in G^i$.

In the following theorem we can see the fundamental role of this group element g_i, which is to determine level of satisfaction v_{max} and individual demand function. Then our terminology of "referential preferences" is fully justified.

Theorem 3. Let $e_i \in R^l_+$ the initial endowment of consumer *i* whose preference is defined by a group G^i. Then its demand function is explicitly given by:

$$f_i: R^l_{++} \to R^l_+,\ f_i(p) = \frac{w_i}{v_p\langle I, \alpha_{g_i}\rangle}\alpha_{g_p^{-1}g_i} \cdot I$$

where $p = v_p\alpha_{g_p} \cdot I$, and $w_i = \langle p, e_i\rangle$ is the budget of consumer *i.*

Proof. Let be $p \in R^l_{++}$ the giving vector price. By theorem 2 the maximization problem is equivalent to minimize $\langle p, \alpha_g(I)\rangle$ for $g \in G^i$. But since $p \in R^l_{++}$, there is $G_p \in G^i$ and $v_p > 0$ such that $p = v_p(\alpha_{g_p} \cdot I)$. Then we have to minimize $\langle \alpha_{g_p} \cdot I, \alpha_g \cdot I\rangle$ for $g \in G^i$. Now $\langle \alpha_{g_p} \cdot I, \alpha_g \cdot I\rangle = \langle I,\ \alpha_{(g_p g)} \cdot I\rangle$ and by axiom 2, the minimum is given for $g_i = g_p \cdot g$ or equivalently $g = g_p^{-1} \cdot g_i$. Finally, $v_{max} = \frac{w_i}{v_p\langle I, \alpha_{g_i} \cdot I\rangle}$ and:

$$f_i(p) = v_{max}\alpha_{g_p^{-1}g_i} \cdot I = \frac{w_i}{v_p\langle I, \alpha_{g_i} \cdot I\rangle}\alpha_{g_p^{-1}g_i} \cdot I \qquad \square$$

Remark 4. Since $p \in R^l_{++}$, we can also write $p = M_p \cdot I$, where M_p is the diagonal matrix with entries $M_{ii} = p_i > 0$. In other words $M_p = v_p \cdot \alpha_{g_p} \cdot I$. And $\frac{1}{v_p}\alpha_{g_p^{-1}} \cdot I = M_p^{-1} \cdot I$, and the individual demand function for consumer *i* takes this form:

$$f_i(p) = \frac{\langle M_p \cdot I,\ e_i\rangle}{\langle I, \alpha_{g_i} \cdot I\rangle} M_p^{-1} \cdot \alpha_{g_i} \cdot I.$$

Corollary 5. The demand function is homogeneous of degree 0.

Proof. Let $\lambda \in R_+^*$, from the above expression of individual demand function $f_i(p) = \frac{\langle M_p \cdot I, e_i \rangle}{\langle I, \alpha_{g_i} \cdot I \rangle} M_p^{-1} \cdot \alpha_{g_i} \cdot I$. As M_p is a diagonal matrix form of the p vector, then $M_{\lambda p} = \lambda M_p$ and $M_{\lambda p}^{-1} = \lambda^{-1} M_p$. This clearly implies $f_i(\lambda p) = f_i(p)$. □

2. Referential preferences and conditions of equilibrium

We start with an example taken from (Aliprantis et al., 1989) to see how our groups' based approach is able to provide same results as the conventional one based on utility function.

Example 3

Let have an economy with two commodities and three agents and note that (p_1, p_2) is the vector price. Utility functions of agents are $u_1 = xy, u_2 = x^2 y$ and $u_3 = xy^2$, and their initial endowment are $e_1 = \binom{1}{2}$, $e_2 = \binom{1}{1}$ and $e_3 = \binom{2}{3}$.

These assumptions are extracted from example 1.4.10 in [Aliprantis & al, 1989, p 34-35].

For us, all preferences are given by groups and their actions on R_+^2 .

Consumer 1. The group of preference is the matricial subgroup $G_1 = \left\{ \begin{pmatrix} t & 0 \\ 0 & 1/t \end{pmatrix}, t > 0 \right\}$.

Its maximization problem

$$\begin{cases} Maximize\ v = v_x \\ subject\ to\ the\ constraint\ p_1 x + p_2 y = b_1 \end{cases}$$

where $X = (x, y)$, is equivalent to finding the greatest v such that $\langle P, v \begin{pmatrix} t & 0 \\ 0 & \frac{1}{t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle = p_1 + 2p_2 \iff$ since for each $X = (x, y)$ there is a unique $t > 0\ and\ v > 0$ with $X = v \begin{pmatrix} t & 0 \\ 0 & \frac{1}{t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Then, we have to find $Max\, v$, such that $v \langle \begin{pmatrix} p_1 \\ P_2 \end{pmatrix}, \begin{pmatrix} t \\ 1/t \end{pmatrix} \rangle = p_1 + 2p_2$ which gives: $v \left[tp_1 + \frac{1}{t} p_2 \right] = p_1 + 2p_2 \iff v = \frac{t(p_1 + 2p_2)}{t^2 p_1 + p_2}$.

Now, $v = v(t)$ reaches its optimum when $\frac{dv}{dt} = 0$, and this occurs at $t_0 = \pm\sqrt{\frac{p_2}{p_1}}$. Since for $t_0 = \sqrt{\frac{p_2}{p_1}} > 0$, we have $\frac{d^2v}{d^2t} < 0$, then we obtain $v_{max} = v\left(\sqrt{\frac{p_2}{p_1}}\right) = \frac{p_1+2p_2}{2\sqrt{p_1p_2}}$.

An easy calculation establishes the demand for the first consumer:

$$x_1 = \begin{pmatrix} \sqrt{\frac{p_2}{p_1}} & 0 \\ 0 & \frac{1}{\sqrt{\frac{p_2}{p_1}}} \end{pmatrix} \left(\frac{p_1+2p_2}{2\sqrt{p_1p_2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \left(\frac{p_1+2p_2}{2p_1}, \frac{p_1+2p_2}{2p_2}\right).$$

Same argument and relatively simple calculation gives the following results:

Consumer 2 The group is subgroup $G_2 = \left\{\begin{pmatrix} t & 0 \\ 0 & 1/t^2 \end{pmatrix}, t > 0\right\}$, $v = \frac{t^2(p_1+p_2)}{t^3p_1+p_2}$ reaches its maximum at $t_0 = \sqrt[3]{\frac{2p_2}{p_1}}$ where $v_{max} = \frac{p_1+p_2}{p_1\left(\frac{2p_2}{p_1}\right)^{1/3} + p_2\left(\frac{p_1}{2p_2}\right)^{2/3}}$. From this, we deduce that the demand of the 2nd consumer is:

$$x_2 = \left(\frac{2p_1+2p_2}{3p_1}, \frac{p_1+p_2}{3p_2}\right).$$

Consumer 3 The group is $G_3 = \left\{\begin{pmatrix} t^2 & 0 \\ 0 & 1/t \end{pmatrix}, t > 0\right\}$, the maximum of $v(t) = \frac{t(2p_1+3p_2)}{t^3p_1+p_2}$ is reached at $t_0 = \sqrt[3]{\frac{p_2}{2p_1}}$ and $v_{max} = \frac{2p_1+3p_2}{p_1\left(\frac{p_2}{2p_1}\right)^{2/3} + p_2\left(\frac{2p_1}{p_2}\right)^{1/3}}$. Then we find $x_3 = \left(\frac{2p_1+3p_2}{3p_1}, \frac{4p_1+6p_2}{3p_2}\right)$ as the demand of consumer 3.

To calculate the equilibrium price, it suffices to establish the common equilibrium condition:

$Z(p) = \sum_{i=1}^{3} x_i(p) - \sum_{i=1}^{3} e_i = 0$. It follows immediately that $\left(\frac{16p_2 - 13p_1}{6p_1}, \frac{13p_1 - 16p_2}{6p_2}\right) = 0$. The last equality gives, under the condition

$p_1 + p_2 = 1$, the value of price equilibrium, $P_{eq} = \left(\frac{16}{29}, \frac{13}{29}\right)$. All these results are exactly the same obtained by the use of utility functions.

Based on the above examples and results, we suggest to define a new mathematical framework of an exchange economy where the set *I* of agents is finite. This is to be defined as:

Definition 6

An exchange economy is said to be of referential preferences if:

1) The consumption set coincides with R_+^l;
2) Each agent *i* has a non-zero initial endowment, i.e., $e_i \in R_+^l$ and;
3) The preference relation $\succcurlyeq^i$ is referential (definition 1), and satisfies axiom 2, for all $i \in I$.

The proof of our main result (theorem 8) is based on the following mathematical result.

Theorem 7. Let $S = \{p \in R^l, p_i > 0\ for\ i = 1, 2, \dots, l,\ p_1 + p_2 + \dots + p_l = 1\}$ the set of all strictly positive prices. For a function $\zeta(\cdot) = (\zeta_1(\cdot), \zeta_2(\cdot), \dots, \zeta_l(\cdot))$ from S into R^l assume that:

1. ζ is continuous and bounded from below;
2. ζ satisfies Walra's Law, i.e., $p \cdot \zeta(p) = 0$ holds for each $p \in S$;
3. $\{p_n\} \subseteq S$, $p_n \to p = (p_1, \dots, p_l)$ and , $p_k > 0$ imply that the sequence $\zeta_k(p_n)$ of the k^{th} components of $\zeta(p_n)$ is bounded; and
4. $p_n \to p \in \partial S$ with $\{p_n\} \subseteq S$ imply $\lim_{n\to\infty} \|\zeta(p_n)\|_1 = \infty$.

Then, there exists at least one vector $p \in S$ satisfying $\zeta(p) = 0$.

For proof of theorem 7 we refer the reader to (Aliprantis et al., 1989, pp. 32-34).

The main result of this paper is provided below:

Theorem 8. Every exchange economy of referential preferences has an equilibrium price.

Proof. It is based on Theorem 7.According to theorem 3 and remark 4, the excess demand function in ERP is given by $Z: S \to R^l, Z(p) = \sum_i f_i(p) - \sum_i e_i = \sum_{i\in I} \frac{\langle M_p \cdot I, e_i\rangle}{\langle I, \alpha_{g_i} \cdot I\rangle} M_p^{-1} \cdot \alpha_{g_i} \cdot I - e$ where $e = \sum_i e_i \in R_{++}^l$.

First, continuity of Z is a consequence of continuity of application: $S \to GL(l, R), p \to M_p$, the inversion of matrix, and scalar product on R^l. And since all $f_i \in R^l_+$, then Z is clearly bounded from below.

Second, as f_i is the solution of maximization problem under the budget constraint then $\langle p, f_i(p)\rangle = \langle p, e_i\rangle$, and $\langle p, Z(p)\rangle = 0$ follows from the equality: $Z(p) = \sum_i f_i(p) - \sum_i e_i$.

Third, let now $\{p^n\} \subseteq S, p^n \to p = (p_1, \dots, p_l)$ and $, p_k > 0$. To see why the sequence $Z_k(p^n)$ of the k^th^ component of $\{Z(p^n)\}$ is bounded, we consider remark 4 and this expression of demand function: $\sum_i f_i(p) = \sum_{i \in I} \frac{\langle M_p \cdot I, e_i\rangle}{\langle I, \alpha_{g_i} \cdot I\rangle} M_p^{-1} \cdot (\alpha_{g_i} \cdot I)$. Since $p^n \in S$, then the k^th^ component of M_{p^n} *and* $M_{p^n}^{-1}$ are nonnegative for all n and tend respectively to p_k *and* $(p_k)^{-1}$, which clearly implies $f_i(p_k^n)$ is bounded for all $i \in \{1, 2, \dots, m\}$, and consequently the same holds for $\{Z_k(p^n)\}$.

Last, it remains to prove that $\lim_{n \to \infty} \|Z(p_n)\|_1 = \infty$ if $p_n \to p \in \partial S$ with $\{p_n\} \subseteq S$. Let $j \in \{1, 2, \dots, m\}$ such that $p_j = 0$. Then $p_j^n \to 0$, and $(p_j^n)^{-1} \to +\infty$ which implies that the j^th^ component of individual demand tends to infinity, namely $\left(M_{p^n}^{-1} \cdot (\alpha_{g_i} \cdot I)\right)_j \to +\infty$ for all consumer i. Since $\frac{\langle M_p \cdot I, e_i\rangle}{\langle I, \alpha_{g_i} \cdot I\rangle} > 0$ for all i, then $(f_i(p))_j \to +\infty$ and it follows immediately that $\lim_{n \to \infty} \|Z(p_n)\|_1 = \infty$. □

Conclusions

In his theory of value, Gerard Debreu wrote: "A state of the economy is a specification of the action of each agent ... But these actions are not necessarily compatible with the total resources. Can one find a price system which makes them compatible?" (Debreu, 1959, p. 74).

In this work we prove that if all agents choose their preference in some group setting, and make their choice in compliance with a simple general rule of referential nature, then we can find a system of price which makes all choices compatible.

An in depth work using additional examples will certainly allow us to come across other properties of referential preference and to better grasp its economic interpretations.

References

Aliprantis, C.D., Brown, D.J., Burkinshaw, O. (1989). *Existence and optimality of Competitive Equilibrium.* Berlin, Heidelberg, New York, Springer

Arrow, K.J., Debreu, G. (1954). "Existence of an Equilibrium for a Competitive Economy", *Econometrica*, 22, pp. 265-290

Balasko, Y. (1998). *Foundations of the Theory of General Equilibrium*. Academic Press, Boston

Debreu, G. (1959). *Theory of Value, An Axiomatic Analysis of Economic Equilibrium*, New York: Wiley

Roman, S. (2011). *Fundamentals of Group Theory, An Advanced Approach,* Boston, Birkhauser