Theoretical and Applied Economics
Volume XX (2013), No. 3(580), pp. 61-82

Forecasting the variance and return of Mexican financial series with symmetric GARCH models

Fátima Irina VILLALBA PADILLA
Escuela Superior de Economia IPN, Mexico
fatima.villalba@gmail.com
Miguel FLORES-ORTEGA
Escuela Superior de Economia IPN, Mexico
mfo@prodigy.net.mx

Abstract. *The present research shows the application of the generalized autoregresive conditional heteroskedasticity models (GARCH) in order to forecast the variance and return of the IPC, the EMBI, the weighted-average government funding rate, the fix exchange rate and the Mexican oil reference, as important tools for investment decisions. Forecasts in-sample and out-of-sample are performed. The covered period involves from 2005 to 2011.*

Keywords: volatility; variance; return; financial variables; investment decisions.

JEL Codes: C22, C52, C58.
REL Codes: 9B, 9F, 9G.

1. Introduction

The variations in the prices of any good or asset derive in the achievement of every profit or loss, to analyze and forecast the volatility of the assets has been the subject of a wide range of research, including Engle (1982), Bollerslev (1986), Nelson (1991), Bollerslev and Wooldridge (1992), Glosten, Jagannathan and Runkle (1993), Engle and Ng (1993), Rabemananjara and Zakoian (1993) and Engle and Manganelli (1999). Volatility refers to the squared root of the variance.

Engle (2001) mentions that when a data set presents heteroskedasticity, standard errors and confidence intervals are very narrow in a regression estimated by the least squares method, yielding a false sense of precision.

Autoregressive conditional heteroskedasticity models (ARCH) are due to Engle (1982) and generalized autoregressive conditional heteroskedasticity models (GARCH) are due to Bollerslev (1986), authors who inserted volatility variations to the models that describe an asset behavior; then Poon and Granger (2003) argue that from the basic ideas outlined in the preceding paragraphs there have been developed many variations of the basic models in order to improve the results or to ease the calculation by including a more refinement in the selection of the parameters. With these models the authors sought to improve the quality of the forecast of the expected volatility of financial assets.

GARCH models have the feature of analyzing the heteroskedasticity as a variance to be modeled, which corrects the deficiencies of the traditional methods such as the least squares adjustment and takes into account variations for each error term.

According to Engle and Manganelli (1999) the great financial disasters have forced the development of areas of risk analysis in the financial institutions. The use of effective tools to quantify the risks has become as important as the models that estimate the expected returns of the financial assets. It is used for decision-making and investment portfolio design, the tools are used in addition to the functions of monitoring, surveillance and control, so that all financial institutions had to modify their accounting based on historical prices to a mark-to-market valuation models which reflects the risk inherent to each of the financial assets.

According to Engle and Ng (1993), the ability to forecast the volatility in the financial markets is a requirement for the proper selection of the financial assets to structure an investment portfolio. In the literature, we find evidence that volatility is predictable in many asset markets; however it differs on how this should be modeled. Merton (1980) shows that the expected returns on the market are related to the accurate volatility forecast. Ferson and Harvey (1991) show that the forecast of the monthly returns of a portfolio is associated with the forecast of the risk premiums. Schwert and Seguin (1990) use daily forecasts of variances from financial assets to estimate monthly forecasts of the variances of a portfolio. By the other hand, Ng, Engle and Rothschild (1992) state that the risk premium of an asset is decomposed into a dynamic component and a static component and their results show that the dynamic component has a greater influence on the risk premium of the analyzed asset. Therefore, it is relevant incorporating market variations in the GARCH model.

As pointed out by Franses (1998), asset prices present in a frequent way volatility clustering, i.e. periods of disturbance where there are wide variations and periods of calm with slight variations. That is to say, with regard to asset prices, the author observed large positive and negative variations that were clustered.

Under Glosten, Jagannathan and Runkle (1993) there has always been a relationship between risk and return when dealing with fair valuation of assets. There is a consensus that in certain period of time, the investor requires a higher yield from a riskier asset, condition that is not mandatory and that is dependent upon a particular investment strategy.

GARCH models capture the volatility clustering in asset returns. As Mandelbrot (1963) sets, there are periods of time where the returns of the assets do not show high variations and certain periods in which the variation of the returns related to their mean are high. The high and low variations in asset prices are grouped in certain periods and are followed by periods with mean-reversion corresponding to the long-term volatility.

According to Taylor (1986), the prices of financial assets continuously capture our attention. In a short period of time the prices of securities may experience dramatic increases or decreases in their original level. It is essential to monitor the behavior of prices in order to try to understand the likely price performance in the future. However, forecasting financial asset prices is far from an easy task.

This paper presents the application of forecasting techniques in financial series to anticipate future fluctuations, in both volatility and future returns, and contribute to the decision making process in selecting investments. It is also presented an out-of-sample forecast in order to verify the validity of the model.

As the first stage of the study, we worked on the transformation of the financial variables in order to make them stationary, and then we conducted tests to detect autoregressive conditional heteroskedasticity effects. Regarding the analyzed stock index, the results showed a high persistence in volatility shocks and it was found that the variance converges to a stable value, as it was observed in the case of the country risk indicator, the fix exchange rate and the Mexican oil reference, achieving the same conclusions.

The study of the variance of the weighted-average government funding rate shows that the behavior of the autoregressive conditional heteroskedasticity model does not show a clear mean-reversion.

The in-sample forecast data using the estimated GARCH models threw very similar trajectories with respect to the actual data of the analyzed variables.

The out-of-sample results verify the capacity of the GARCH models to predict and capture the behavior of the analyzed financial variables.

This paper is structured as follows. Section 2 describes the theoretical framework used in the research. Section 3 points out the financial variables included in the research and the structure of the database. Section 4 presents the results of the estimation of the GARCH model and its evaluation for each of the studied variables; section 5 shows the in-sample and out-of-sample forecasting for the financial variables. Finally, we conclude the paper.

2. Theoretical framework

The GARCH model developed by Bollerslev (1986) estimated a conditional variance that is a weighted forecast that considers three different variance forecasts. One is the constant variance corresponding to the long term variance. The second one is the forecast made in the prior period and the third one involves the new information that affects the previous forecast. The equation in the simplest case is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (1)$$

Equation 1 shows the symmetric[(1)] GARCH(1,1), where σ_t^2 represents the conditional variance, due to the fact that it is calculated from past information that is considered relevant. The weights of these three forecasts determine how fast the variance changes when new information arises and how fast the variance reverts to its mean.

According to the above formulation, the behavior of the squared return in time t with regard to the conditional variance is given by:

$$\varepsilon_t = u_t^2 - \sigma_t^2 \quad (2)$$

The above equation can also be expressed as:

$$\sigma_t^2 = u_t^2 - \varepsilon_t \quad (3)$$

We use the previous equation and replace it in the conditional variance formula calculated from equation 1 and we obtain:

$$u_t^2 - \varepsilon_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta(u_{t-1}^2 - \varepsilon_{t-1}) \quad (4)$$

The prior equation leads to the following expression:

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta(u_{t-1}^2 - \varepsilon_{t-1}) + \varepsilon_t \quad (5)$$

We gather the common terms:

$$u_t^2 = \alpha_0 + (\alpha_1 + \beta)u_{t-1}^2 - \beta\varepsilon_{t-1} + \varepsilon_t \quad (6)$$

Equation 6 corresponds to an ARMA(1,1) process that contains the squared errors.

The goodness-of-fit of the GARCH model is that its use is very simple because it has a higher probability of finding non-negative constraints. The GARCH(1,1), which includes only three parameters in the conditional variance equation, is a parsimonious model since it incorporates an infinite number of squared errors in the past in order to influence the conditional variance in the present.

The GARCH(1,1) can be extended to a GARCH(p,q) where the current conditional variance is valued to depend on p lags of the squared error and q lags of the conditional variance, which is expressed in equation 7:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_p u_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2 \quad (7)$$

Generally, the above is summarized according to equation 8:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i\, u_{t-i}^2 + \sum_{j=1}^{q} \beta_j\, \sigma_{t-j}^2 \quad (8)$$

However, the GARCH(1,1) model captures data volatility clustering and we rarely use a higher superior model to estimate the financial series volatility.

The conditional variance constantly changes, nevertheless, the u_t unconditional variance is constant and is given by

$$var(u_t) = \frac{\alpha_0}{1-(\alpha_1+\beta)} \quad \text{para } \alpha_1 + \beta < 1 \quad (9)$$

If $\alpha_1 + \beta \geq 1$, then the variance is non-stationary. If $\alpha_1 + \beta = 1$ then there exists a unit root in the variance, also called integrated GARCH or

IGARCH whose analysis is beyond the scopes of this paper. In order to obtain a GARCH process with mean reversion it is required that the sum of the coefficients $\alpha_1 + \beta$ is less than one; these parameters measure the persistence of the data set volatility; therefore, the result of the sum of these parameters has a meaning, while this piece of information tends to one, the volatility will persist more while as the result tends to zero, the volatility will be closer to the long term variance with a higher speed.

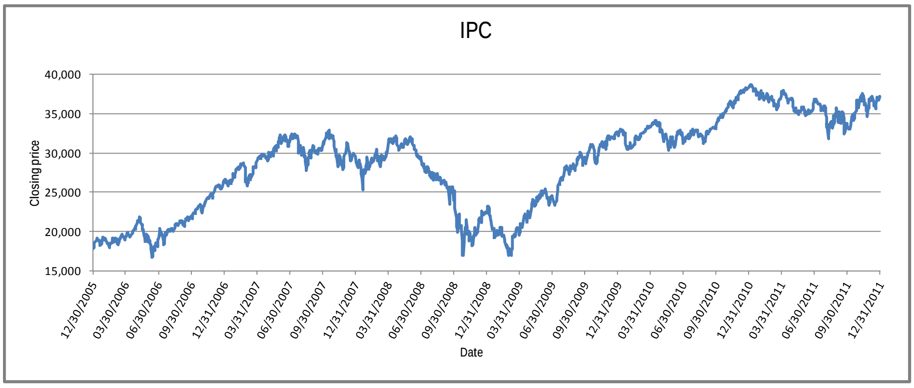
3. Database used in the research

The database that was used in the present research includes daily information that covers the period December 30, 2005 to December 30, 2011. We included 1,507 observations related to the *Mexican Stock Exchange Index* (IPC); 1,499 observations related to the *Emerging Markets Bond Index* (EMBI); 1,509 observations related to the *weighted-average government funding rate* and the *fix exchange rate*. Finally, we used 1,426 observations related to the *Mexican oil reference*. See Appendix-1 for further details.

4. GARCH model for the financial variables and evaluation of the forecasts

4.1. Symmetric GARCH model for the IPC

With regard to the IPC, we used 1,507 observations that include the daily closing price. Data are presented in Figure 1.

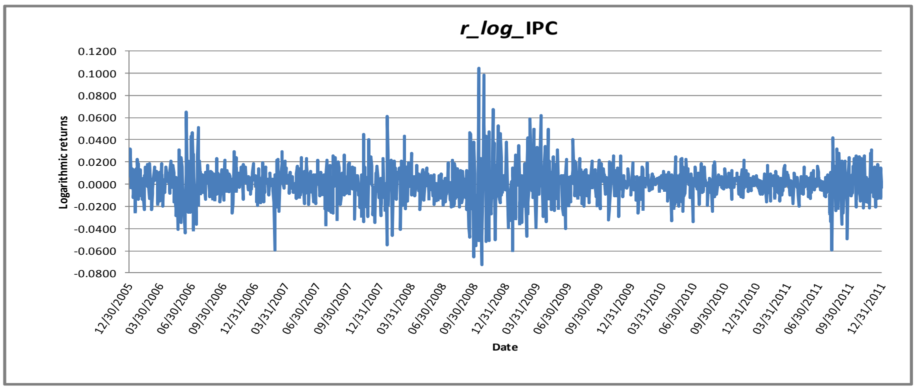


Source: Mexican Stock Exchange.

Figure 1. *IPC daily closing price*

Figure 2 shows the IPC logarithmic returns at time *t* according to the following formula:

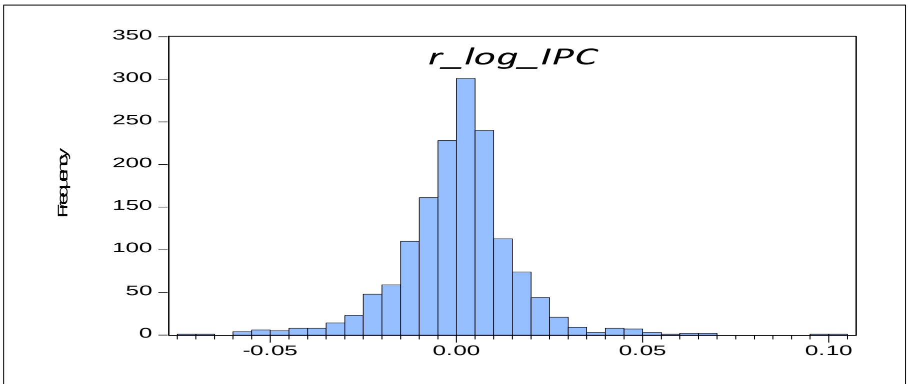
$$r\_log\ \_\mathrm{IPC_t} = \log\left(\frac{\mathrm{IPC}_t}{\mathrm{IPC}_{t-1}}\right)$$



Source: Mexican Stock Exchange.

Figure 2. *IPC Logarithmic returns*

Figure 3 shows the frequency histogram for the IPC logarithmic returns. Descriptive statistics show a mean of 0.000487, a median of 0.001267, a maximum of 0.104407, a minimum of -0.072661, a standard deviation of 0.015513, a skewness of 0.121546 and a kurtosis of 7.803962. The Jarque-Bera test yielded a result of 1,451.856000 with a probability of 0.000000, confirming that the return series are not normally distributed.



Plotting data: Eviews.

Figure 3. *Frequency histogram for the IPC logarithmic returns*

It is noted that the distribution of the IPC daily returns presents kurtosis excess compared to a normal distribution. The value of 7.803962 for kurtosis in the distribution of the IPC daily prices returns during the observed period confirms the presence of fat tails in the distribution. The leptokurtic distribution for the r_log_IPC series shows that there are high returns more frequently than expected.

We apply a stationarity test to r_log_IPC series and the results are presented in Table 1.

Table 1

Augmented Dickey-Fuller Test to *r_log*_IPC series

Null hypothesis: *r_log*_IPC has a unit root

		t Statistic	Probability
Augmented Dickey-Fuller test statistic		-35.54889	0.00000
Test critical values:	1% level	-3.43449	
	5% level	-2.86326	
	10% level	-2.56773	

Source: Eviews output.

From the results presented in Table 1, we can reject the hypothesis r_log_ipc series is not stationary; this allows us to state that for none of the intervals for which the result is evaluated, the series presents unit root, therefore, it is stationary.

We obtain a correlogram in order to detect problems related to autocorrelation and partial autocorrelation. Through the analysis of the correlogram, we performed the model that best fits the data series which is: ARIMA (34, 1, 30), as shown in Table 2.

Table 2

ARIMA Model

Dependent variable: *r_log*_IPC

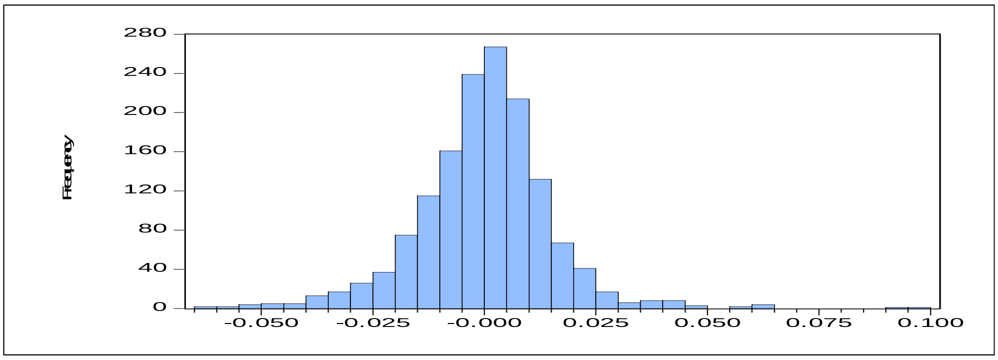
Variable	Coefficient	Standard error	t Statistic	Probability
C	0.000481	0.000389	1.235915	0.216700
AR(1)	0.072536	0.032713	2.217365	0.026800
AR(2)	-0.049913	0.025199	-1.980721	0.047800
AR(12)	0.067514	0.027937	2.416608	0.015800
AR(13)	0.068509	0.027537	2.487908	0.013000
AR(21)	0.256020	0.055284	4.631033	0.000000
AR(30)	-0.578213	0.061380	-9.420191	0.000000
AR(31)	0.060444	0.030666	1.971039	0.048900
AR(34)	-0.078108	0.027504	-2.839825	0.004600
MA(7)	-0.088359	0.024083	-3.668963	0.000300
MA(21)	-0.302324	0.053610	-5.639261	0.000000
MA(30)	0.560660	0.059149	9.478809	0.000000

Log likelihood	4,084.932000
F statistic	8.558195
Probability	0.000000
Akaike info criterion	-5.533874
Schwarz criterion	-5.490714

Source: Eviews output.

The coefficients for the proposed model are significantly different from zero at 95% of confidence level and the correlogram for the residuals shows that the autocorrelation and partial correlation values for 36 lags are between the bands, therefore, for a 95% confidence level, it is assumed that the residual series represents white noise.

In applying the Jarque-Bera testing to the residuals in order to prove normality for the developed ARIMA model of Table 2, we lead to the conclusion that we reject normality on the residuals, since the result is 1,033.885000 with a probability of 0.0. The frequency histogram for the residuals is presented in Figure 4. The skewness for the residuals is 0.194224 and the kurtosis is 7.087289.



Plotting data: Eviews.

Figure 4. *Frequency histogram for the residuals*

With the assistance of the ARCH-LM test we verify the existence of ARCH effects in the residuals; the results are presented in Table 3.

Table 3

ARCH test for the residuals

F-statistic	19.609230	Probability	0.000000
Observed R-squared	19.377270	Probability	0.000000

Source: Eviews output.

The probabilities for F-statistic and observed R-squared are less than 0.05; we chose one lag in order to incorporate the ARCH effect, according to the results, the null hypothesis of non existence of ARCH effects is rejected. To fit the appropriate GARCH model, we used the quasi-maximum likelihood Bollerslev and Wooldridge (1992) method, and the Marquardt optimization algorithm; the results are shown in Table 4.

Table 4

GARCH(1,1) model

Dependent variable: *r_log* _IPC

Method: ML - ARCH (Marquardt) - Distribución Normal

GARCH = C(11) + C(12)*RESID(-1)^2 + C(13)*GARCH(-1)

Variable	Coefficient	Standard error	z-Statistic	Probability
C	0.000703	0.000282	2.491160	0.012700
AR(1)	0.061713	0.026623	2.318059	0.020400
AR(13)	0.058377	0.023648	2.468557	0.013600
AR(21)	0.386558	0.088948	4.345904	0.000000
AR(30)	-0.408527	0.080175	-5.095451	0.000000
AR(31)	0.053435	0.022347	2.391172	0.016800
AR(34)	-0.050781	0.021745	-2.335316	0.019500
MA(7)	-0.063663	0.019547	-3.256954	0.001100
MA(21)	-0.425349	0.087911	-4.838411	0.000000
MA(30)	0.406395	0.080295	5.061294	0.000000

Variance equation

C	0.000003	0.000001	2.255087	0.024100
RESID(-1)^2	0.103287	0.021300	4.849245	0.000000
GARCH(-1)	0.886654	0.022481	39.439630	0.000000

Log likelihood	4,300.61
Akaike info criterion	-5.82555
Schwarz criterion	-5.778793

Source: Eviews output.

It was found that the model with the best fit was GARCH(1,1) according to the results presented in Table 4. From the correlogram of residuals, it is noted that the autocorrelation values and the partial autocorrelation values for 36 lags were in the band of confidence of 90%. Additionally, we applied the ARCH-LM test to the residuals we obtained with the regression and results are shown in Table 5.

Table 5

ARCH test for the residuals

F-statistic	0.000501	Probability	0.982100
Observed R-squared	0.000502	Probability	0.982100

Source: Eviews output.

According to the results of Table 5, the probability for F-statistic and for observed R-squared is greater than 0.05; therefore we accept the null hypothesis and it is stated that there are no ARCH effects affecting the estimation.

From the results, we selected the ARIMA model according to the parameters we show in Table 4:

$r\_log\_\text{IPC}_t =$ 0.000703 + 0.061713 $r\_log\_\text{IPC}_{t-1}$ + 0.058377 $r\_log\_\text{IPC}_{t-13}$ + 0.386558 $r\_log\_\text{IPC}_{t-21}$ - 0.408527 $r\_log\_\text{IPC}_{t-30}$+ 0.053435 $r\_log\_\text{IPC}_{t-31}$ - 0.050781 $r\_log\_\text{IPC}_{t-34}$ - 0.063663 ε_{t-7} - 0.425349 ε_{t-21} + 0.406395 ε_{t-30} + ε_t

We selected the GARCH(1,1) model to represent the variance as shown in Table 4.

σ_t^2 =0.000003 + 0.103287 u_{t-1}^2 + 0.886654 σ_{t-1}^2

The sum of ARCH and GARCH coefficients is 0.989941, indicating a high persistence of volatility shocks as mentioned by Engle (2001).

4.2. Symmetric GARCH model for the EMBI

In order to forecast the EMBI, we used 1,499 daily observations. We worked with the EMBI database in the same way we managed the IPC database. Descriptive statistics for the log first difference (*d_log*_EMBI) showed a mean of 0.000264, a median of 0.0, a maximum of 0.215975, a minimum of -0.192034, a standard deviation of 0.043041, a skewness of 0.139464 and a kurtosis of 5.216449. The Jarque-Bera test yielded a result of 311.487000 with a probability of 0.0, confirming that the log first difference series are not normally distributed.

We selected the following ARIMA model:

$d\_log\_\text{EMBI}_t =$ - 0.000974 - 0.741352 d_log $\_\text{EMBI}_{t-25}$ + 0.791185 $\varepsilon_{t-25} + \varepsilon_t$

We selected the following GARCH(1,1) model to represent the variance:

σ_t^2 =0.000065 + 0.101050 u_{t-1}^2+ 0.867361 σ_{t-1}^2

The sum of ARCH and GARCH coefficients is 0.968411, indicating a high persistence of volatility shocks as mentioned by Engle (2001).

4.3. Symmetric GARCH model for the weighted-average government funding rate

We used 1,509 daily observations to work with the weighted-average government funding rate. We handled the first difference between the current and previous data. Descriptive statistics for the first difference (*d*_tfondeo) showed a mean of -0.000024, a median of 0.0, a maximum of 0.005800, a minimum of -0.007500, a standard deviation of 0.000665, a skewness of -3.734920 and a kurtosis of 56.903310. The Jarque-Bera test yielded a result of 186,072.5 with a probability of 0.0, confirming that the first difference series are not normally distributed.

We selected the following ARIMA model:

$d\_tfondeo_t = -0.000007 + 0.025967\ d\_tfondeo_{t-8} +$
$+ 0.170460\ d\_tfondeo_{t-20} - 0.059123\ \varepsilon_{t-1} + 0.269492\ \varepsilon_{t-18} -$
$- 0.130003\ \varepsilon_{t-19} + \varepsilon_t$

We selected the following ARCH(1) model to represent the variance:

$\sigma_t^2 = 2.886024\ u_{t-1}^2$

The ARCH coefficient is greater than one, this indicates that the variance is not stationary as mentioned by Perez (2007); therefore, a more exhaustive study for this variable is needed.

4.4. Symmetric GARCH model for the fix exchange rate

In order to forecast the fix exchange rate, we used 1,509 daily observations. Descriptive statistics for the log first difference (*d_log*_FIX) showed a mean of 0.000180, a median of -0.000458, a maximum of 0.073328, a minimum of -0.055975, a standard deviation of 0.007578, a skewness of 0.873978 and a kurtosis of 17.578870. The Jarque-Bera test yielded a result of 13,546.790000 with a probability of 0.0, confirming that the log first difference series are not normally distributed.

We selected the following ARIMA model:

$d\_log\_\mathrm{FIX}_t = -0.000118 - 0.495221\ d\_log\_\mathrm{FIX}_{t-27} -$
$- 0.234008\ d\_log\_\mathrm{FIX}_{t-28} + 0.476132\ \varepsilon_{t-27} + 0.268737\ \varepsilon_{t-28} +$
$+ 0.056247\ \varepsilon_{t-31} + \varepsilon_t$

We selected the following GARCH(1,1) model to represent the variance:

$\sigma_t^2 = 0.127192\ u_{t-1}^2 + 0.869428\ \sigma_{t-1}^2$

The sum of ARCH and GARCH coefficients is 0.996620, indicating a high persistence of volatility shocks as mentioned by Engle (2001).

4.5. Symmetric GARCH model for the Mexican oil reference

In order to forecast the Mexican oil reference, we used 1,425 daily observations. Descriptive statistics for the log first difference (*d_log*_mezcla) showed a mean of 0.000572, a median of 0.001951, a maximum of 0.137918, a minimum of -0.118373, a standard deviation of 0.024456, a skewness of -0.198038 and a kurtosis of 7.079463. The Jarque-Bera test yielded a result of 997.434500 with a probability of 0.0, confirming that the log first difference series are not normally distributed.

$d\_log\_mezcla_t = 0.001238 - 0.578694\ d\_log\_mezcla_{t-23}$ -

$- 0.052692\ d\_log\_mezcla_{t-33}\ + 0.619774\ \varepsilon_{t-23} + 0.056456\ \varepsilon_{t-35} + \varepsilon_t$

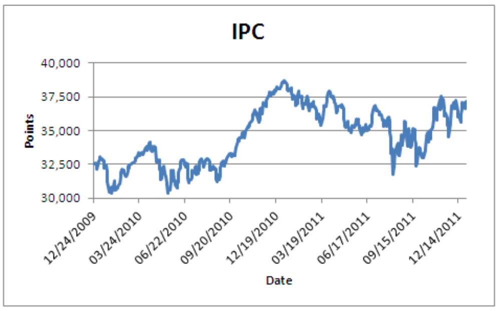
We selected the following GARCH(1,1) model to represent the variance:

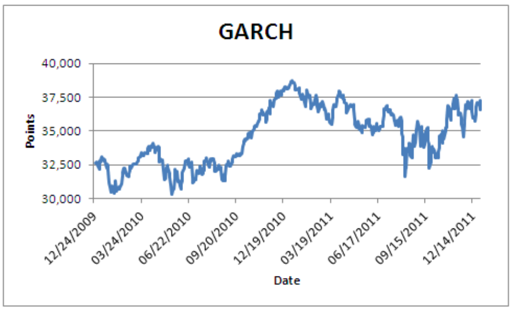
$\sigma_t^2 = 0.000005 + 0.0480380\ u_{t-1}^2 + 0.939831\ \sigma_{t-1}^2$

The sum of ARCH and GARCH coefficients is 0.987869, indicating a high persistence of volatility shocks as mentioned by Engle (2001).

5. In-sample and out-of-sample forecasting

According to the selected GARCH models, we proceeded to perform an in-sample forecasting from the 1,000 data hereinafter for each of the analyzed variables, the results are shown below.



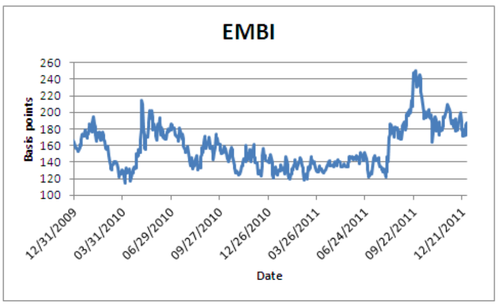


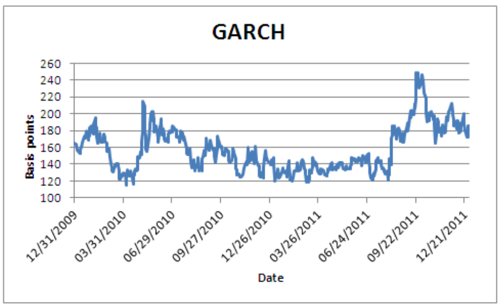
Source: Eviews output.

Figure 5. *Actual closing prices for the IPC*

Figure 6. *GARCH forecasting*

As shown in figures 5 and 6, the in-sample forecast follows the same trend as the actual IPC data.



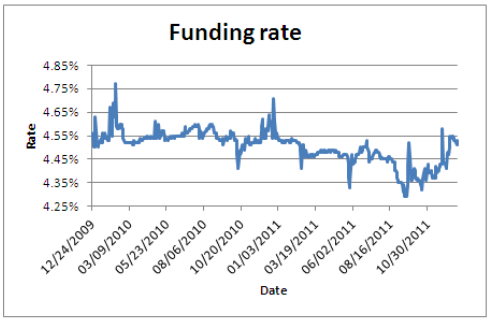


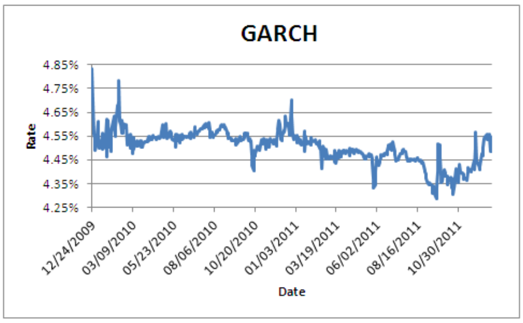
Source: Eviews output.

Figure 7. *EMBI real data*

Figure 8. *GARCH forecasting*

Figure 7 and Figure 8 show the fact that the EMBI data and the in-sample forecast look alike.



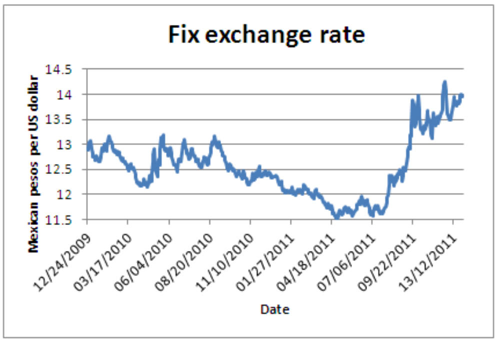


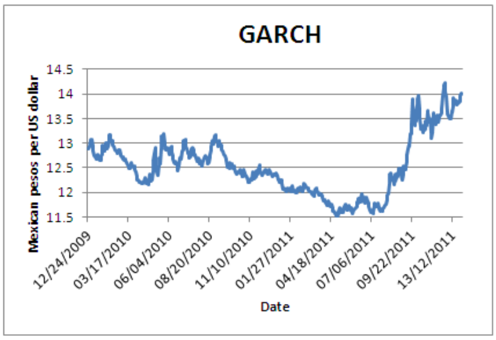
Source: Eviews output.

Figure 9. *Weighted-average government funding rate real data*

Figure 10. *GARCH forecasting*

Figures 9 and 10 show similar trajectories for the in-sample forecast and the real database for the weighted-average government funding real data.



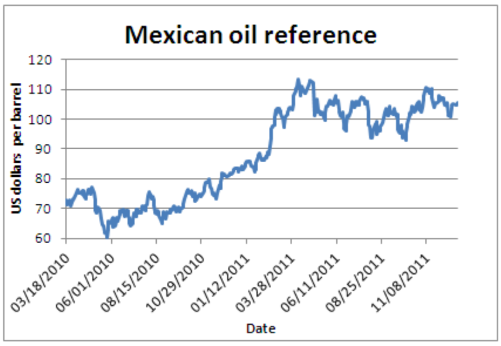


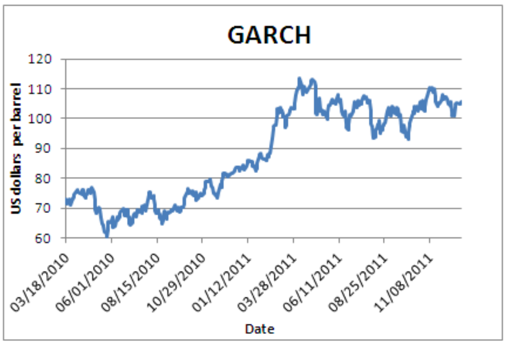
Source: Eviews output.

Figure 11. *Fix exchange rate*

Figure 12. *GARCH forecasting*

Figures 11 and 12 show quite similar paths, one of them represents the real fix exchange rate observations and the other shows the in-sample forecast.





Source: Eviews output.

Figure 13. *Mexican oil reference* **Figure 14.** *GARCH forecasting*

Figures 13 and 14 leave clear that the real information for the Mexican oil reference and the in-sample forecast follow similar trajectories.

By the other hand, we show a table for the variables we focused on where we included the most used indicators to assess the predictive capacity of each GARCH model we developed according to the methodology implemented by López-Herrera (2004) and Analia (2008).

Table 6

Evaluation of the in-sample forecast performance

INDICATOR	IPC	EMBI	Funding rate	Fix exchange rate	Mexican oil reference
Root mean squared error	368.189368	7.302710	0.000019	0.001750	1.507516
Mean absolute error	267.870842	5.470694	0.000179	0.063273	1.118887
Mean absolute percent error	0.780807	3.448249	0.397325	0.497716	1.286252
Theil inequality coefficient	0.005315	0.022939	0.003416	0.003560	0.008422
Bias proportion	0.001724	0.000641	0.000009	0.001441	0.000501
Variance proportion	0.000040	0.000004	0.005265	0.000901	0.000066
Covariance proportion	0.998236	0.999355	0.994726	0.997658	0.999433

Source: Eviews output.

Among the important results we emphasize that the root mean squared error, the mean absolute error and the mean absolute percent error for these variables represent less than 1% of the original level data. As long as the above indicators result close to zero, we obtain greater success in predicting with the referred models. The Theil inequality coefficient will always result between zero and one. The Theil inequality coefficient tends to zero for these five variables, indicating that there is an adequate fit for all forecasts. The Theil inequality coefficient is rescaled and decomposed into three proportions of inequality, the bias proportion, the variance proportion and the covariance proportion. The sum of the three proportions of inequality is equal to one.

The bias proportion is an indication of the systematic error. We expect the bias proportion is always close to zero. A large bias proportion indicates a systematic error with regard to the prediction. The research showed a minimum systematic error for the five variables.

The variance proportion should tend to zero to indicate the ability of the forecasts to replicate the variability in the variable to be forecast. The study showed the condition for each of the variables.

The covariance proportion measures the unsystematic error; this indicator should concentrate the highest proportion of the Theil inequality coefficient. The results for the five variables showed the highest concentration of the inequality in this indicator.

Next we evaluated if the forecasts can replicate the descriptive statistics for the series of interest, the results show that the statistic indicators correspond to the characteristic of the original series for the time horizon selected, so we conclude that the forecasts are adequate.

Table 7

Reproduction of the central moments of the IPC

	IPC	GARCH
Mean	34,585.000000	34,600.270000
Median	34,828.040000	34,821.530000
Standard Deviation	2,183.844000	2,186.208000
Skewness	-0.041335	-0.035855
Kurtosis	1.787480	1.791072

Table 8

Reproduction of the central moments of the EMBI

	EMBI	GARCH
Mean	157.108200	156.923200
Median	150.000000	149.452300
Standard Deviation	26.481260	26.467540
Skewness	0.852728	0.860356
Kurtosis	3.429933	3.459937

Source: Eviews output.

Table 9

Reproduction of the central moments of the weighted-average government funding rate

	FUNDING RATE	GARCH
Mean	4.503800%	4.503900%
Median	4.520000%	4.517100%
Standard Deviation	0.069000%	0.071200%
Skewness	-0.502101	-0.276739
Kurtosis	3.759659	4.262270

Table 10

Reproduction of the central moments of the fix exchange rate

	FIX EXCHANGE RATE	GARCH
Mean	12.533950	12.530570
Median	12.477900	12.476410
Standard Deviation	0.603213	0.600532
Skewness	0.510473	0.502088
Kurtosis	2.781690	2.779479

Source: Eviews output.

Table 11

Reproduction of the central moments of the Mexican oil reference

	MEXICAN OIL REFERENCE	GARCH
Mean	88.132350	88.166100
Median	86.970000	86.720420
Standard Deviation	15.632130	15.644400
Skewness	-0.074285	-0.065289
Kurtosis	1.409585	1.408882

Source: Eviews output.

From the information hereby presented, we conclude with a 95% of confidence the forecasts obtained from estimated GARCH models adequately fit.

With estimated GARCH models we performed out-of-sample forecasting, creating five estimations from the last one that makes up the original sample, these forecasts were compared to the actual data in order to evaluate the accuracy of the estimations. The selected criterion considers that while the average of the forecast data and actual data lays around 100, the out-of-sample forecast is more accurate.

The evaluation of the forecast for these studied variables is presented in the form of table for each of them.

Table 12

Evaluation of the out-of-sample forecast for the IPC

Date	IPC forecast	IPC actual	Evaluation of the out-of-sample forecast
01/02/2012	37,041.093326	37,335.030000	99.212705
01/03/2012	37,063.837889	37,384.340000	99.142684
01/04/2012	37,076.381494	37,387.630000	99.167509
01/05/2012	37,063.005552	37,017.950000	100.121713
01/06/2012	37,139.790489	36,804.050000	100.912238
Average			**99.711370**

Source: Based on data provided by Mexican Stock Exchange.

Table 13

Evaluation of the out-of-sample forecast for the EMBI

Date	EMBI forecast	EMBI actual	Evaluation of the out-of-sample forecast
01/03/2012	187.128440	183.000000	102.255978
01/04/2012	185.175830	183.000000	101.188978
01/05/2012	185.340877	182.000000	101.835647
01/06/2012	184.933851	187.000000	98.895108
01/09/2012	184.430168	186.000000	99.156004
Average			**100.666343**

Source: Based on data provided by Bloomberg.

Table 14

Evaluation of the out-of-sample forecast for the weighted-average government funding rate

Date	FUNDING RATE forecast	FUNDING RATE actual	Evaluation of the out-of-sample forecast
01/03/2012	4.528914%	4.520000%	100.197205
01/04/2012	4.519290%	4.520000%	99.984297
01/05/2012	4.524981%	4.500000%	100.555137
01/06/2012	4.535460%	4.500000%	100.787992
01/09/2012	4.526414%	4.490000%	100.810998
Average			**100.467126**

Source: Based on data provided by Banco de México.

Table 15

Evaluation of the out-of-sample forecast for the fix exchange rate

Date	FIX EXCHANGE RATE forecast	FIX EXCHANGE RATE actual	Evaluation of the out-of-sample forecast
01/03/2012	13.963165	13.688200	102.008775
01/04/2012	13.972604	13.714400	101.882719
01/05/2012	13.974390	13.740900	101.699230
01/06/2012	13.997323	13.722800	102.000492
01/09/2012	13.994901	13.743700	101.827757
Average			**101.883794**

Source: Based on data provided by Banco de México.

Table 16

Evaluation of the out-of-sample forecast for the Mexican oil reference

Date	MEXICAN OIL REFERENCE forecast	MEXICAN OIL REFERENCE actual	Evaluation of the out-of-sample forecast
01/03/2012	106.115172	108.120000	98.145738
01/04/2012	106.135333	109.300000	97.104604
01/05/2012	106.128376	108.480000	97.832205
01/06/2012	106.121772	108.200000	98.079272
01/09/2012	106.288560	107.620000	98.762832
Promedio			**97.984930**

Source: Based on data provided by Bloomberg.

The results clearly show the ability of GARCH models to forecast and capture the behavior of the five studied variables, enriching the knowledge of the features of the series in order to perform the accurate analysis and estimation of the trend that they will follow in the future as an essential tool for investors and analysts.

Conclusions

The empirical evidence suggested the symmetric GARCH models estimated for each variable in the research represent appropriately the behavior and trend of the series and allow to obtain accurate forecasts, we also confirmed the convergence that is presented with regard to the conditional variance forecast for the studied variables, with the exception of the weighted-average government funding rate, for which we did not find consistent evidence on the presence of mean reversion.

We confirmed that GARCH processes are mean reverting when the ARCH and GARCH coefficients sum up to a number less than one; as it is closer to one, the volatility will persist.

It was found that estimated GARCH models allow forecasting data that present similar trends to those for actual data with regard to the studied variables.

The forecast testing that we performed showed an error lower than 1%, finding that it is suitable for practical purposes. It is important to note that when we analyzed the level variables, the error can be seen high, but when we normalized the variables, the proportion is quite low.

Among the main results it is important to point out that the root mean squared error, the mean absolute error and the mean absolute percent error for these variables represent less than 1% of the original level data. The lower the values of these indicators, the higher the success of the models to forecast.

The Theil inequality coefficient tends to zero for these five variables, indicating that there is an adequate fit for all forecasts. We observed a minimum systematic error and a more concentrated non-systematic error that corresponds to the optimal levels of the Theil inequality coefficient.

It was evaluated the capacity of the models to forecast according to the reproduction of the descriptive statistic of the studied series, as important results we found that the statistical indicators correspond to the features of the original time series for the time horizon selected, findings that allowed us to conclude that the forecasts adequately fit.

The results showed the ability of GARCH models to forecast and capture the behavior and trend of the studied financial variables, providing an additional tool for investors and financial analysts in order to evaluate objectively the performance of different assets among the wide range of investment choices that exist in the financial markets.

Note

(1) The term symmetric refers to the fact that good news and bad news affect in the same way the prices of the financial assets.

References

Analia, C. (2008). *Volatilidad Comparada a los Mercados de Valores de Países Emergentes Latinos*, Universidad Nacional de Cuyo, Mendoza, pp. 1-66

Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 31, pp. 307-327

Bollerslev, T., Wooldridge, J.M. (1992). "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances", *Econometric Reviews*, 1, pp. 143-173

Dávila, J., Núñez, J.A., Ruiz, A. (2009). "Volatilidad del precio de la mezcla mexicana de exportación", *Economía, Teoría y Práctica, Universidad Autónoma Metropolitana,* No. 25, pp. 37-52

Engle, R.F. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation", *Econometrica*, 50, pp. 987-1008

Engle, R.F., Manganelli, S. (1999). *CAVIaR: Conditional Autoregressive Value at Risk by Regression Quantiles*, University of California, San Diego, July, 1999, pp.1-51

Engle, R.F., Ng, V.K. (1993). "Measuring and Testing the Impact of News on Volatility", *The Journal of Finance*, Vol. 48, No. 5, December, pp. 1749-1778

Engle, R.F. (2001). "GARCH 101: The Use of Arch/Garch Models in Applied Econometrics", *Journal of Economic Perspectives*, 15(4), pp. 157-168

Ferson, W.E., Harvey, C.R. (1991). "The Variation of Economic Risk Premiums", *Journal of Political Economy*, 99, pp. 385-415

Franses, P.H. (1998). *Time Series Models for Business and Economic Forecasting*, Cambridge University Press, Nueva York

Glosten, L.R., Jagannathan, R., Runkle, D.E. (1993). "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *The Journal of Finance*, Vol. XLVIII, No. 5, December, pp. 1779-1801

López-Herrera, F. (2004). "Modelado de la Volatilidad y Pronóstico del Índice de Precios y Cotizaciones de la Bolsa Mexicana de Valores", *Revista Contaduría y Administración*, No. 213, Mayo-Agosto, pp. 43-72

Mandelbrot, B.B. (1963). "The Variation of Certain Speculative Prices", *Journal of Business*, 36, pp. 394-419

Merton, R.C. (1980). "On Estimating the Expected Return on the Market. An Exploratory Investigation", *Journal of Financial Economics*, 8, pp. 323-361

Nelson, D.B. (1991). *Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica*, Vol. 59, No. 2, March, 1991, pp. 347-370

Ng, V., Engle, R.F., Rothschild, M. (1992). "A Multi-Dynamic-Factor Model for Stock Returns", *Journal of Econometrics*, 52, pp. 245-266

Pérez, C. (2007). *Econometría Básica, Técnicas y Herramientas*, Pearson Eduación, S.A., Madrid, pp. 177-205

Poon, S. y Granger, C. (2003). "Forecasting Volatility in Financial Markets: A Review", *Journal of Economic Literature*, Vol. 41, No. 2, pp. 478-539

Prayag, C., van Rensburg, P. (2004). "Accuracy of Brokers´ Consensus Earnings Forecasts: The South African Case", *Investment Analysts Journal*, No. 59, pp. 21-30

Rabemananjara, R., Zakoian, J.M. (1993). "Threshold Arch Models and Asymmetries in Volatility", *Journal of Applied Econometrics*, Vol. 8, Issue 1, pp. 31-49

Schwert, G.W., Seguin, P. (1990). "Heteroskedasticity in Stock Returns", *The Journal of Finance*, 45, pp. 1129-1155

Taylor, S.J. (1986). *Modeling Financial Time Series,* John Wiley

www.banxico.org.mx
www.bmv.com.mx

Appendix 1

The Mexican IPC index (IPC) is the main indicator which tracks the performance of leading companies listed on the Mexican Stock Exchange (BMV); it is made of a balanced weighted and representative selection of a group of shares that are listed on the Mexican stock market. The IPC is an indicator of the stock market fluctuations; an important feature is the fact that the IPC is representative and it reflects the behavior and the operating dynamics of the Mexican stock market and several shares that integrate it. The IPC is expressed in points. We consulted www.bmv.com.mx to obtain the information. We considered the closing prices.

The Emerging Markets Bond Index (EMBI) is the main indicator of country risk. It is calculated daily by the bank JP Morgan Chase, it measures the spread between the interest rate paid in bonds issued by emerging countries and denominated in US dollars and the return paid by the United States treasury bonds. The spread is expressed in basis points. JP Morgan Chase calculates the indicator for Russia, Ukraine, Brazil, Argentina, Indonesia, Bulgaria, Venezuela, Egypt, Colombia, Morocco, Nigeria, Mexico, Panama, Poland, Peru, Turkey, the Philippines, Ecuador and South Africa. We consulted Bloomberg to obtain the information with regard to the EMBI Mexico.

The weighted-average government funding rate is an indicator provided and calculated by the central bank of Mexico, Banco de México that considers the interest rate representative of the wholesale operations carried out by banks and brokerage firms. It is the average transaction-amount-weighted interest rate for one-day repo transactions with government securities settled through the system managed by INDEVAL, the Mexican securities clearing house. Transactions among institutions that belong to the same financial group and their customers are excluded. We consulted www.banxico.org.mx to obtain the information. We considered the weighted-average.

The fix exchange rate is determined by Banco de México on banking days, it is the result of an average of quotations of the exchange market of wholesale operations to be settled on the second banking day of its determination. Banco de México calculates three samples during the banking day; between 9 a.m. and 12 p.m. Banco de México informs the resulting average from 12 p.m. onward each banking day. The fix exchange rate is published by Banco de México in the Diario Oficial de la Federación on the next banking day of its determination. The quotation is expressed in Mexican pesos per US dollar. We consulted www.banxico.org.mx to obtain the information.

The Mexican oil reference, according to Dávila, Núñez y Ruiz (2009), is the export basket of crude oil which includes three types of oil: 1) The Olmeca, a light crude oil of 39 degrees API (American Petroleum Institute), the indicator shows the relationship of the weight of petroleum product in relation to water. If it is lighter than water and floats on water, the API degrees are over 10. The higher the API degrees, the petroleum product is heavier. 2) The Istmo of 32 degrees API and, 3) the Maya of 22 degrees API. Participation in weighting of each of these three types of oil is determined by Petróleos Mexicanos (PEMEX), however, the Maya has the greatest influence in the weighting. The information is released to the public every banking day approximately at 6 p.m. Prices are expressed in US dollars per barrel of crude oil. We consulted Bloomberg to obtain the information.