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Analysing movements in investor's risk aversion using the Heston volatility model

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Abstract. *In this paper we intend to identify and analyze, if it is the case, an "epidemiological" relationship between forecasts of professional investors and short-term developments in the EUR/RON exchange rate. Even that we don't call a typical epidemiological model as those ones used in biology fields of research, we investigated the hypothesis according to which after the Lehman Brothers crash and implicit the generation of the current financial crisis, the forecasts of professional investors pose a significant explanatory power on the futures short-run movements of EUR/RON. How does it work this mechanism? Firstly, the professional forecasters account for the current macro, financial and political states, then they elaborate forecasts. Secondly, based on that forecasts they get positions in the Romanian exchange market for hedging and/or speculation purposes. But their positions incorporate in addition different degrees of uncertainty. In parallel, a part of their anticipations are disseminated to the public via media channels. Since some important movements are viewed within macro, financial or political fields, the positions of profess-sional investors from FX derivative market are activated. The current study represents a first step in that direction of analysis for Romanian case. For the above formulated objectives, in this paper different measures of EUR/RON rate volatility have been estimated and compared with implied volatilities. In a second timeframe we called the co-integration and dynamic correlation based tools in order to investigate the relationship between implied volatility and daily returns of EUR/RON exchange rate.*

Keywords: implied volatility; smile volatility; dynamic correlation; error correction model.

JEL Codes: D81, G11.
REL Code: 11B.

1. Introduction

One of the most interesting and highly debated topics in economic theory is related to the stochastic properties of exchange rate, its links to the fundamental variables and the ability to forecast its short-run movements. Since 1983, Meese and Rogoff provide a seminal work about this debate whose results were published in two stages. They showed that macro fundamental variables have no explanatory power on the future changes of the exchange rate, which means the exchange rate pose the random walk property. In order to achieve these results they made use of the out-of-sample fits for several benchmark models of exchange rates. The result of Meese and Rogoff remained standing until 2005, when the fundamental work of the Engel and West brought a new light in regard with these topics. Engel and West showed that within a rational expectations Euler equation based model, the exchange rate, viewed as an asset price as Obsfeld and Rogoff (1996) underlined, follows a process which is at least near-random walk model under certain circumstances. These circumstances are given by the *I(1)* property of fundamentals and the discount factor located very closed to 1. In this context, it is not surprising that short-run exchange rate movements are almost impossible to predict under a very large discount factor and *I(1)* macro-variables. Rime, Sarno and Sojli (2007) studied the relationship existing between order flows and the daily movements of the exchange rates. Their estimates emphasized that order flows represents a good predictor for exchange rates return and also that order flows contains significant information on the current and the expected future states of macro variables. On the other hand, Xavier Gabaix suggests to Barro (2005) that a possible way to account for disaster probabilities is to use prices recorded by far *out-of-the money* options. Therefore, in the Gabaix's view, options characteristics bear important information on the macro states. Coming back to the fundamentals of Engel and West, Cochrane (1989) and Akerlof and Yelen (1985a, 1985b) noted that economic agents may adopt near-rational behaviours because it is too cheap to situate near-rationality (Cochrane) or because the agents don't have the proper tools to elaborate accurately evaluations. But later, Sargent (1993) and De Grawe (2010) assumed under different form that agents are learning during the time from their past decisions and have at least rational-expectation based intentions. De Grawe accentuated the role of heuristic rules. In that sense, Caroll (2001), using an epidemiological style model, underlined its high performance in forecasting inflation and unemployment. Later, Caroll (2003) showed that the economic agents try to be rational-expectations through the interpretation and use of the information provided by professional forecasters, which in turn are rational agents.

Summing-up all these considerations and based on recently elaborated analyses by Alupoaiei, Codîrlaşu and Săndică (2012), we investigate the case of existing "epidemiological" relationship between forecasts of professional investors and short-term developments in the EUR/RON exchange rate. Even if we don't call a typical epidemiological model as those ones used in biology fields of research, we investigated the hypothesis according to which after the Lehman Brothers crash and, implicit, the generation of the current financial crisis, the forecasts of professional investors pose a significant explanatory power on the futures short-run movements of EUR/RON. How does it work this mechanism? Firstly, the professional forecasters account for the current macro, financial and political states, then they elaborate forecasts. Secondly, based on that forecasts they get positions in the Romanian exchange market for hedging and/or speculation purposes. But their positions incorporate in addition different degrees of uncertainty. In parallel, a part of their anticipations are disseminated to the public via media channels. Since some important movements are viewed within macro, financial or political fields, the positions of professional investors from FX derivative market are activated. The current study represents a first step in that direction of analysis for Romanian case. For the above formulated objectives, in this paper there have been estimated different measures of EUR/RON rate volatility and compared with implied volatilities. In a second timeframe we called the co-integration and dynamic correlation based tools in order to investigate the relationship between implied volatility and daily returns of EUR/RON exchange rate.

2. Theoretical model

In this section we will briefly present the theoretical background of volatility smile concept of implied, stochastic and conditional volatility. Once we establish a base for the volatility features in Romanian FX market we move further to analyze the co-integration and dynamic correlation in order to investigate the relationship between implied volatility and daily returns of exchange rate.

2.1. Smile volatility

Currency options are in general used for strategies on foreign exchange rate movements. One type of strategy is risk reversal, which implies a long position on call and short position on put, the call having a higher strike than put. That strategy represents a measure of the relative value of options with strikes above or below the current at the money forward rate, actually expressing the skew that might exist in the volatility smile.

Forward looking financial indicators represents informative predictors of the behaviour of the asset price; therefore those options gained attention in recent years. Neftci (2008) explained how risk reversal represents a measure of the bias in a volatility smile, since a symmetric smile implies that zero cost risk reversal could be achieved.

The measure of an option's *moneyness* is called delta, such as *at the money* (ATM) options have a delta around 50 and the following relationship holds:

$$\sigma(25 - \text{deltaRR}) = \sigma(25 - \text{deltaput}) - \sigma(25 - \text{deltacall}) \quad (1)$$

Where $\sigma(25 - \text{deltaRR})$, $\sigma(25 - \text{deltacall})$, $\sigma(25 - \text{deltaput})$ represents the implied volatilites of a *risk reversal*, namely the 24 delta call and 25 delta *put*.

The curvature of the smile can be measured using *butterfly* strategy. One way to construct a *butterfly* is a long straddle which forms a V-shaped head of the *butterfly* and short position a strangle that creates the flattened wings.

$$\sigma(25 - \text{deltacall}) = \sigma(25 - \text{delta ATM}) + \sigma(25 - \text{deltaBFY}) + 0.5 \times \times \sigma(25 - \text{deltaRR}) \quad (2)$$

$$\sigma(25 - \text{delta put}) = \sigma(25 - \text{delta ATM}) + \sigma(25 - \text{deltaBFY}) - 0.5 \times \times \sigma(25 - \text{deltaRR}) \quad (3)$$

Nakisa (2010) plot the implied volatility for ATM options on the S&P 500 index against expiry term and determined two term structure curves: the first one for January 24th 2007, which had a low level of volatility, respectively the second one for November 20th 2008, which was just after the collapse of Lehman Brothers when volatility was across the entire term structure.

2.2. Implied volatility

In 1993, Heston proposes the first model with diffusion defined as state variable and follows a stochastic process. The main advantage of using this model is the calibration of the volatility "smile". Once the implied volatility is obtained, we have compared it with the stochastic and conditional volatility and with con on the same time frame.

2.2.1. Stochastic volatility

The stochastic volatility has been calculated using a Metropolis-Hastings algorithm. This uses multivariate normal proposals with mean the posterior mode in order to estimate the parameters. This algorithm performs a sequence of iterations summarized by the following steps:

1. Set starting values for $\beta = (\beta_0^{(t-1)}, \beta_1^{(t-1)}, \beta_2^{(t-1)}, \beta_3^{(t-1)})$;
2. Propose new values $\beta^* = (\beta_0^*, \beta_1^*, \beta_2^*, \beta_3^*)$ from normal distributions;
3. Calculate acceptance probability α=min$(1, \frac{f(y|\beta_0^*,\beta_1^*,\beta_2^*,\beta_3^*)f(\beta_0^*,\beta_1^*,\beta_2^*,\beta_3^*)}{f(y|\beta_0,\beta_1,\beta_2,\beta_3)f(\beta_0,\beta_1,\beta_2,\beta_3)})$;
4. Update $\beta^{(t)} = \beta^*$ with probability α or keep the same values with probability 1-α;
5. Repeat Steps 2,3 and 4 T times;
6. Take the average of the T draws $(\beta_j^{(1)}, \ldots, \beta_j^{(T)})$, $j = \overline{0,3}$.

Considering a simple stochastic volatility model:

$$y_t = \epsilon_t \sqrt{\exp\,(lnh_t)} \quad (4)$$
$$lnh_t = lnh_{t-1} + v_t$$
$$v_t \sim N(0, g)$$

where h_t is time-varying variance.

Jacquier et al. (2004) suggest applying a Metropolis Hastings algorithm at each point in time to sample from conditional distribution of h_t which is given by $f(h_t \backslash h_{-t}, y_t)$, where –t represents all other dates than t. The authors argue that because the transition equation of the model is a random walk, the knowledge of h_{t+1} and h_{t-1} captures all relevant information about h_t

$$f(h_t \backslash h_{-t}, y_t) = f(h_t \backslash h_{t-1}, h_{t+1}, y_t), \quad (5)$$

The density is a product of a normal density and log normal density and has the following form:

$$f(h_t \backslash h_{t-1}, h_{t+1}, y_t) = h_t^{-0.5} \exp\left(\frac{-y_t^2}{2h_t}\right) \times h_t^{-1} \exp\left(\frac{-(\ln h_t - \mu)^2}{2\sigma_h}\right), \quad (6)$$

where

$$\mu = \left(\frac{\ln h_{t+1} + \ln h_{t-1}}{2}\right), \sigma_h = \frac{g}{2} \quad (7)$$

The algorithm starts sampling h_0 and accepting the draw:

$$f(h_0 \backslash h_1,) = \mathrm{h}_0^{-1} \exp\left(\frac{-\left(\ln \mathrm{h}_0 - \mu_0\right)^2}{2\sigma_0}\right) \tag{8}$$

$$\mu_0 = \sigma_0 \left(\frac{\bar{\mu}}{\bar{\bar{\sigma}}} + \frac{\ln \mathrm{h}_1}{\mathrm{g}}\right), \sigma_0 = \frac{\bar{\bar{\sigma}}\mathrm{g}}{\bar{\sigma} + \mathrm{g}} \tag{9}$$

Jacquier et al. (2004) suggest sampling the final value of h_t using the following modified candidate generating density

$$\mathrm{q}(\phi^{G+1}) = \mathrm{h}_t^{-1} \exp\left(\frac{-(\ln \mathrm{h}_t - \mu)^2}{2\sigma_h}\right); \tag{10}$$

where:

$\mu = \ln \mathrm{h}_{t-1}, \sigma_h = \mathrm{g}$

2.2.2. Conditional volatility

If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedasticity (GARCH, Bollerslev(1986)) model. In that case, the GARCH (p, q) model (where p is the order of the GARCH terms $\sigma^2$and q is the order of the ARCH terms ϵ^2) is given by

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \\ &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \end{aligned} \tag{11}$$

The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model proposed by Glosten, Jagannathan and Runkle in 1993 captures the asymmetry in the ARCH process. The suggestion is to model $\epsilon_t = \sigma_t \times z_t$ where

$$\sigma_t^2 = K + \delta \sigma_{t-1}^2 + \alpha \epsilon_{t-1}^2 + \phi \epsilon_{t-1}^2 I_{t-1} \tag{12}$$

$$\begin{aligned} I_{t-1} &= 0 \;\; if \; \epsilon_{t-1} \geq 0 \\ I_{t-1} &= 1 \;\; if \; \epsilon_{t-1} < 0 \end{aligned} \tag{13}$$

2.3. Correlation and error correction model

The main problems in the modeling of multivariate processes of volatility are related to the large number of parameters which have to be simultaneously estimated and the positive definiteness of covariance matrix. In this spirit, Bollerslev, Engle and Wooldridge (1988) proposed the first GARCH representation of conditional covariance matrices, defining the so called VEC-model.

The general multivariate GARCH (p,q) model is given as:

$$VEC(\Sigma_t) = C + \sum_{i=1}^{q} A_i \times vec(\varepsilon_{t-i} \times \varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j \times vec(\Sigma_{t-j})^{(1)} \quad (14)$$

The fact that the model described by the above equation requires the estimation of a large number of parameters led to a development of the simplified diagonal VEC model by Bollerslev, Engle and Wooldridge (1988), where the A and B matrices are forced to be diagonal. The model can be written as follows:

$$\sigma_{ij,t} = c_{ij} + \sum_{h=1}^{p} a_{hij} \times \varepsilon_{t-h,i} \times \varepsilon_{t-h,j} + \sum_{h=1}^{q} b_{hij} \times \sigma_{t-h,ij} \quad {\scriptstyle 1\le i\le j\le k} \quad (15)$$

The diagonal VEC model is represented by the following equation:

$$\Sigma_t = C_0^* + \sum_{i=1}^{m} A_i^* \vartriangle\cdot (\varepsilon_{t-i} \times \varepsilon'_{t-i}) + \sum_{j=1}^{s} B_j^* \vartriangle \Sigma_{t-j} \quad (16)$$

where m and s are non-negative integers, and $\vartriangle$ denotes Hadamard product[(2)]. Σ_t must be a parameter matrix and Silberberg and Pafka in 2001 demonstrated that a sufficient condition to ensure the positive definiteness of the covariance matrix Σ_t is that the constant term C_0^* is positive definite and all other coefficient matrices are positive semi-definite. For a multivariate GARCH model to be plausible, $\Sigma_t$is required to be positive definite for all values of the disturbances. In order to solve this problem Engle and Kroner (1995) proposed a quadratic formulation for the parameters that ensured positive definiteness. This model is known as BEKK and has the following form:

$$\Sigma_t = C_0 \times C'_0 + \sum_{k=1}^{K}\sum_{i=1}^{q} A'_i \times \varepsilon_{t-i} \times \varepsilon'_{t-i} \times A_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} B'_{ki}\Sigma_{t-i}B_{ki} \quad (17)$$

The BEKK(1,1,1) model, $\Sigma_t = \Omega + A'\varepsilon_{t-i} \times \varepsilon'_{t-i}A + B'\Sigma_{t-i}B$, can be written as a VEC model:

$$\mathrm{VEC}(\Sigma_t) = \mathrm{VEC}(\Omega) + (A \otimes A)' vec(\varepsilon_{t-i} \times \varepsilon'_{t-i}) + (B \otimes B)' \mathrm{VEC}(\Sigma_{t-i}) \quad (18)$$

An error correction model is a dynamical system with the characteristics that the deviation of the current state from its long-run relationship will be fed into its short-run dynamics.

Given a VAR (p) of I(1) x's

$$x_t = \Phi_1 \times x_{t-1} + .. + \Phi_p \times x_{t-p} + \epsilon_t \quad (19)$$

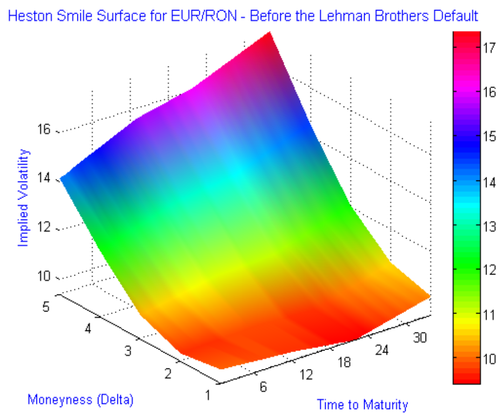
There always exists an error correction representation of the form

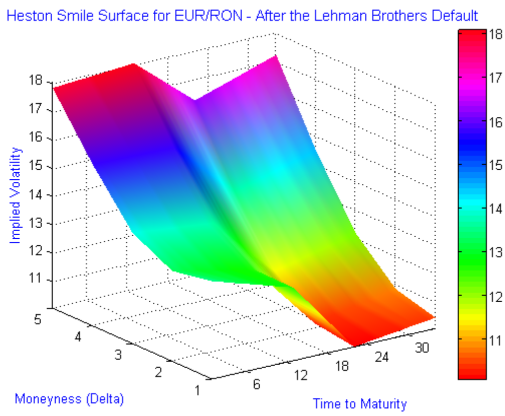
$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{p-1} \Phi_i^* \times \Delta x_{t-1} + \epsilon_t \quad (20)$$

where Π and the Φ_i^* are functions of the Φ. When $\Pi = 0$ then there is no cointegration(3).

3. Empirical results

The current analysis started from the empirical work of Alupoaiei, Codîrlaşu and Săndică (2012) that investigated how the expectations of professional investors from the local(4) Romanian are changing across an important event. The so called important event was set to the moment of Lehman Brothers default. In fact, the event of Lehman Brothers bankruptcy is considered by experts as the official start of the current financial crisis. They showed that, in the face of crucial event at global scale, the expectations of professional investors quoted as implied volatilities changes especially for the short horizons of time as we can observe from the following plot:





Source: Alupoaiei, Codîrlaşu and Săndică (forthcoming).

Figure 1. *Heston smile surface for EUR/RON rate calibrated on one-month data before the Lehman Brother default*

Figure 2. *Heston smile surface for EUR/RON rate calibrated on one-month data after the Lehman Brother default*

Given these results, we extended the philosophy of the above study in a dynamic manner. More exactly we called two classes of econometric techniques in order to obtain information on the relationship between investors expectations expressed as implied volatilities and the evolution of the EUR/RON rate.

On the first ground, we begin our analysis by getting as reference a common practice in trading activities. It is well known that, in derivative markets, investors actually trade volatility. In that sense, traders usually use the difference between forecasted volatility and implied volatility for hedging purposes. Taking into account these considerations, we firstly estimated the historical volatility of EUR/RON rate and expressed it in annual terms. In order to avoid limits implied by different methods of volatility fit[(5)], it has been restored two different approaches. The first method used to estimate the historical volatility is based on a conditional mixture model of the form AR (2)-GJR (1, 11). The selection and the usage of this model are made based on the study of Alupoaiei (2010). There are several studies that underlined the existence of fractional structures within the volatility of EUR/RON exchange rate with *d* parameter from FIGARCH model closed to 1. Starting from these facts, Alupoaiei empirically showed that when increasing the order of AR equation, the fits provided by the mixture model AR (2)-GJR (1, 1) approximate pretty well those ones provided by FIGARCH model. Alupoaiei's investigation was motivated by the complexity of using FIGARCH model in exercises of simulation or forecasting.

The second approach that we called here is based on a stochastic formulation on the volatility of underlined process. We used two different approaches for fitting historical volatility in order to assure robustness for the final results. For simplicity, we compare the fitted realized volatility in place of forecasted volatility with the implied volatility.

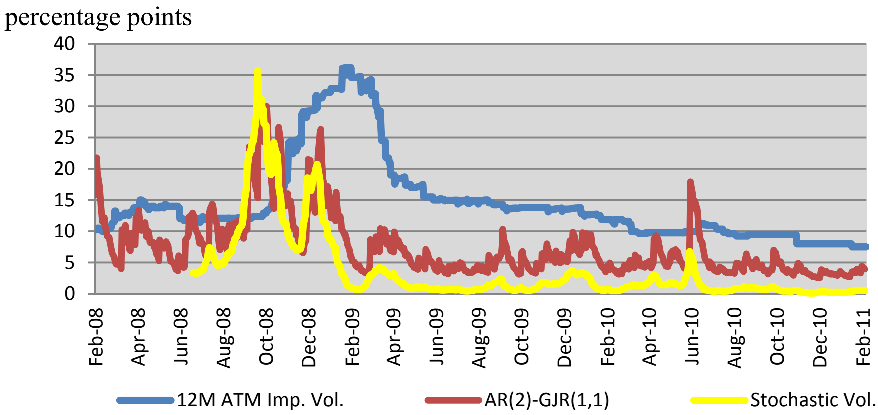


Figure 3. *Evolution of conditional, stochastic and implied volatilities*

In fact, the implied volatility represents the forecast of conditional volatility. Taking into account the shapes of estimated realized volatility shown in Figure 3, perhaps the predicted volatility would likely be a one step push before of the realized volatility. Looking at the above plot, we can observe that, almost every time, the implied volatility is situated above the realized one, with the exception of the period after Lehman Brothers crash. The trading practices in derivative markets states that if the implied volatility is above forecasted historical one, professional investors should enter on short position on EUR/RON, except the case when in the local exchange rate market is a lot of uncertainty regarding the future evolution of domestic currency. The main key insight of this analysis is the shock felt after the starting of current financial crisis posted permanent effects on implied volatility.

Therefore, in the second timeframe we come to analyse the link between implied volatility and the evolution of EUR/RON rate. We treat this problem econometric in two ways, namely through the usage of co-integration and the dynamic correlation between the two series. Using a VECM model, we determined how much the implied volatility and EUR/RON returns co-integrate in an econometric sense.

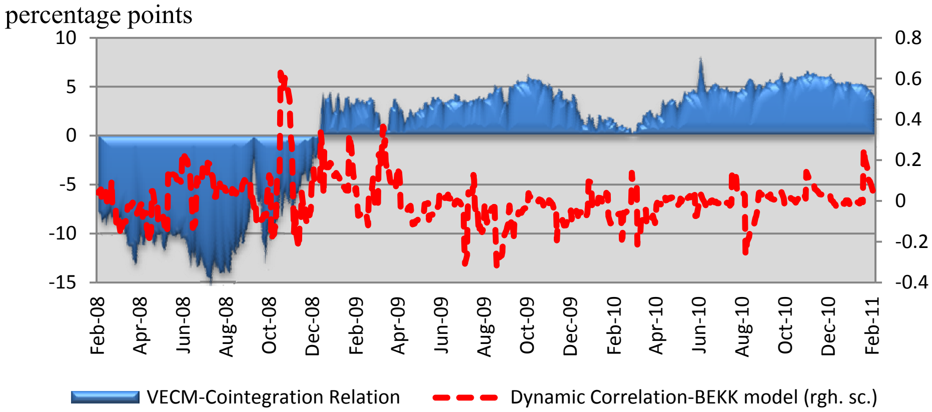


Figure 4. *Evolution of cointegration and correlation between 12M ATM implied volatility and EUR/RON evolution*

Estimated results showed that cointegrations between the two series became positive definitively after the Lehman episode, which means the EUR/RON returns had to increase in order to achieve the equilibrium. On the other hand, the dynamic correlation between 12M ATM implied volatility and EUR/RON evolution, estimated with BEKK model, fluctuated a lot within the interval + 0.2% and – 0.2%. Exception makes the period between October-December 2008, when the level of correlation ranged within 0.6% – 0.7%.

4. Conclusions

In this paper we intend to identify and analyze, if it is the case, an "epidemiological" relationship between forecasts of professional investors and short-term developments in the EUR / RON exchange rate. We used two different approaches for fitting historical volatility in order to assure robustness for the final results. The first method used to estimate the historical volatility is based on a conditional mixture model of the form AR (2)-GJR (1, 1). The second approach that we called here is based on a stochastic formulation on the volatility of underlined process. From the firs analysis we conclude that, almost every time, the implied volatility is situated above the realized one, with the exception of the period after Lehman Brothers crash. The trading practices in derivative markets states that if the implied volatility is above forecasted historical one, professional investors should enter on short position on EUR/RON, except the case when in the local exchange rate market is a lot of uncertainty regarding the future evolution of domestic currency. The main key insight of this analysis is the shock felt after the starting of current financial crisis posted permanent effects on implied volatility.

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Even the obtained results require further investigation in order to be able to talk about the structural relationship between implied volatility and the evolution of EUR/RON rate, this paper represent a first stage for this type of analysis focused on the Romanian exchange rate market and should be treated accordingly.

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Notes

(1) The VEC operator vectorizes a matrix by stacking its columns.
The Kronecker product of two matrices, A and B, where A is *m x n* and B is $p\ x\ q$ is defined as: $A \otimes B = \begin{pmatrix} A_{11}B & A_{12}B & \cdots & A_{1n}B \\ \vdots & & \ddots & \vdots \\ A_{m1}B & A_{m2}B & \cdots & A_{mn}B \end{pmatrix}$ which is an *mp x nq* matrix. There is an important relationship between the Kronecker product and the VEC operator: $vec(A\ X\ B) = (B^T \otimes A)vec(X)$

(2) For two matrices of the same dimensions, $X, Y \in R^{mxn}$ the Hadamard product $X \cdot Y$ is a matrix of the same $X \odot Y \in R^{mxn}$ with elements given by$(X \odot Y)_{i,j} = (A)_{i,j} \times (B)_{i,j}$. Note that the Hadamard product is a principal sub matrix of the Kronecker product.

(3) If a linear combination of I (1) series is stationary, i.e. I (0), the series are called cointegrated. If there are 2 processes x_t and y_t are both I(1) and $y_t - \alpha x_t = \epsilon_t$, with ϵ_t trend-stationary or simply I(0), then x_t and y_t are called cointegrated.

(4) Here the term local denotes that our analysis is concentrated only on the transactions rulled out on Romanian exchange market.

(5) For example, the obtianed conditional volatility with a model from the ARCH class it is sensitive to the conditional mean.

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