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Diversifying the risk through portfolio investment

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Abstract. *The stock exchange markets are characterized by high dynamics of the investment activity, mainly portfolio investments. The existing relation between yield and risk, on one side, and the portfolio diversification, on the other side, are two basic aspects allowing the investors to build up a portfolio founded on the yield and risk targets which they are aiming. In the frame of this article we have applied the Marcowitz model on a number of portfolios of equities issued by commercial companies listed at the Bucharest Stock Exchange on the main REGS market.*

Keywords: financial instrument, portfolio, risk diversification, yield, Markowitz model.

JEL Classification: G11, G32.
REL Classification: 11B.

1. Introduction

In terms of capital market, we talk about the interaction modality between legal persons or individuals holding, at a certain time, an excess of capital – entering this market as investors – and those entities which are facing a certain need of capital and therefore, are showing their intention to draw the available amounts through emissions of equities or other specific financial instruments (Anghelache, 2009).

The intermediation between the two categories of players referred above is achieved by specialized entities, called financial investment services companies.

Both the theoretical works and the actual activity the emphasize goes maximizing the profit function under the conditions of minimizing the risks connected to the analyzed transactions. When considering minimizing the risks specific to the transactions on financial instruments the portfolio investments are to be compulsory considered.

2. Literature review

The economists running their activity both during the second half of the century XX and nowadays paid a high consideration to the analysis of the typical activities of the capital market and, implicitly, to the management of the financial instruments portfolio.

In this respect, we have to underline the contributions brought by Harry Markowitz *(Portfolio selection. Efficient Diversification of Investments,* 1959), William Sharpe *(A Simplified Model of Portfolio Analysis,* 1963), John Lintner *(The valuation of risk assets and the selection of risk portfolios and capital budgets,* 1965), Jan Mossin *(Equilibrium in a Capital Asset Market,* 1966), Stephan Ross *(The Arbitrage Theory of Capital Asset Pricing – APT,* 1976). These works are submitting – from theoretical point of view – a series of economic-mathematical models for analyzing the yield and risk of the financial instruments which may be considered as the very basis for all the analyses being performed on the subject of the financial instruments portfolios.

The theory of the management of the financial instruments portfolio is originating back to the first half of the XX century when the first models for the analysis of risk and profitableness for the investments in financial instruments have been drawn up. Thus, it is by the beginning of the previous century that a number of models for the analysis of the financial investments made themselves conspicuous: the "fair game" model (Louis Bachelier, *The speculation theory,* 1900); the "martingale" and "sub martingale" models; "random walk" model. The

"fair game" model has been put in discussion by Louis Bachelier (1900) by his dissertation work "The speculation theory", being subsequently submitted by Paul Cootner in his work *The Random Character of Stock Market Prices*, issued in 1964.

These attempts to systemize the notions referring to the investments on the capital markets are however considered as modest and therefore, replaced by the modern theory of the portfolio.

In the case of the portfolio modern theory, the investments are statistically modeled taking into account the level of the expected profit and the degree of volatility of the financial instrument which, in fact is considered as the specific risk bearer for each instrument. This theory is meant to let each investor identify the accepted level of risk and thereafter to identify that specific portfolio of the highest yield for the respective level.

The starting points of these theories go back to the beginning of the second half of the XX century and are to be found out in the works issued by the American Professor Harry Markowitz. Giving up the classical approach of the financial investment analysis (based on the technical and fundamental analysis), he focused to the analysis of the overall performance of a portfolio of financial instruments (which analysis is grounded on the ratio yield/risk of the components of a portfolio). The activity achieved by Professor Markowitz materialized in the publication of his work titled *Portfolio selection. Efficient Diversification of Investments* (1959) for which he get awarded with the Nobel Prize for Economics in the year 1990.

On the other hand, William Sharpe is the one who elaborated the CAPM model, through his work *Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk* published by the *Journal of Finance* in September 1964. The CAPM model meant a real qualitative jump in the frame of the financial theory, from the normative models to a model of equilibrium between the demand and the offer for risky assets meant to generate unique prices for the financial assets.

Further on, in 1976, Stephan Ross submitted – by his work *The Arbitrage Theory of Capital Asset Pricing – APT* – an alternative to the Markowitz and Sharpe models of analysis for the profitableness of a financial instruments portfolio. The supporting basis of this theory is given by the hypothesis that all the financial instruments should by identified in an identical way on the different capital markets.

The above mentioned models for the analysis of the financial instruments portfolios are representing the starting point for a large number of studies being performed during the last years and published in specialized international reviews.

It is obvious that, during the respective period, number of specialists in the field of the financial investments have analyzed the models created during the second half of the previous century and accommodated them to the specific requirements of the economy at the beginning of the III millennium.

There is in this context that we have to mention also the works published by Baule Rainer *(Optimal portfolio selection for the small investor considering risk and transaction costs*, 2010), Hagstromer Bjorn, Binner Jane M. *(Stock portfolio selection with full-scale optimization and differential evolution,* 2009), Kini Omesh, Mian Shehzad, Rebello Michael, Venkateswaran Anan *(On the Structure of Analyst Research Portfolios and Forecast Accuracy,* 2009), Lisa Koonce, Marlys Gascho Lipe and Mary Lea Mcanally *(Judging the risk of financial instruments: Problems and potential remedies,* 2005), Marco Taboga *(A simple model of robust portfolio selection*, 2005), Ralph Steuer and Yue Qi *(Developments in multi-attribute portfolio selection,* 2006), Considine G. *(Projecting portfolio risk and return,* 2007), Bhalla V.K. *(Investment management,* 2008), Ameur Hachmi Ben, Prigent Jean Luc *(Behavior towards Risk in Structured Portfolio Management,* 2010), etc.

We are meeting the same problems of how to optimize the financial instruments portfolios under risk conditions when considering the work *On portfolio selection under extreme risk measure* published in 2006 by Stephan Clemencon and Skander Slim.

Meanwhile, in the frame of his article *Projecting portfolio risk and return* published in 2007, Geoff Considine is applying to an informatics instrument (QPP – Quantext Portfolio Planer) meant to forecast the evolution of a portfolio formed by 20 equities issued by American companies and quoted on the NASDAQ market. QPP is combining the recent evolution of the instruments defined by the user with long term relations concerning the connection between the profit and the risk for forecasting the evolution of the 20 instruments over a 33 years period.

3. Reasons concerning the portfolio and the portfolio management

The concept of portfolio is a complex one so that, over the time, the specialists in the field, both in the country and abroad, paid enough efforts in order to identify those elements helping the definition of this notion. The portfolio is representing a „combination of financial instruments held by an individual or a company with the purpose to multiply the invested capital under the conditions of minimizing the risk through diversification" (Roman, 2003).

It is recommended that the activity of building up and managing the portfolio takes into account to select equities out of the under-valued category, to choose

the best moment of entering the market and, when considering to sell and get profit, to wait for the moment when the price reaches a concordance with the financial outcomes, taking into consideration the market fluctuations, so that the opportunities they are offering are not getting lost (Anghel and Dinu, 2013).

When building up a portfolio, it is required to consider an as diversified range as possible so that the undertaken risk is as small as possible. The number of the financial instruments which might make part of a portfolio is depending on the investor but the specialists in the field are alleging that this must be minimum 7 in order to allow compensating a loss which might be recorded in the case that one or more out of these instruments are facing an unexpected drop. Meantime, these specialists are recommending that the instruments should be diversified on activity sectors (Anghel, 2013).

The management of a financial instruments portfolio is an important activity because the factors leading to the best performances for the respective investment are basically the adequate structure and the good administration of the portfolio.

Although advisable that the portfolio management is run by brokers or by placement consultants, even those with no qualification in the field can administrate their own portfolios with good results provided they are following and applying a series of principles and techniques.

Basically, the decision of any investor as to place the capital he is holding in financial instruments must take into account the adopted investment strategy, the modality to identify and select the financial instruments, the number of selected instruments, the achievement of the optimum combination of the built up portfolio and, mainly, its management in time depending the degree of adversity of the investor as against the risk and the degree of profitableness expected by this one (Manole and Anghel, 2013).

By the time the decision of building up a portfolio is taken it is aiming to get an as high as possible capitalized value of the future benefits. Under such circumstances, the risk of getting a variable capitalization rate of the benefit may occur. The risk of the portfolio may be diminished through diversification.

If to define the portfolio management, it comprises the totality of methods and models through which an optimum combination of the assets is achieved, based on the correlation yield-risk. Under these conditions, the decisions being taken in the activity of the portfolio management are based on the portfolio yield and the corresponding risk. This is a complex activity which requires to be continuously carried out because of the fact that modifications arousing on the determinant variables for the yield and risk of the portfolio component instruments would imply modifications to be made within the portfolio structure (Dragotă, 2009).

The valuation of the portfolio is made on the basis of the profitableness-risk criterion. The profitableness means the level of the gain provided by an investment. Between the profitableness of an investment and the connected risk there is a directly proportional relation: to the extent the risk is smaller, the investment profitableness which might be achieved is smaller but to the extent the undertake risks are bigger the investment profitableness is increasing accordingly (Anghelache and Anghel, 2014).

Consequently, it may be said that the investor must take into consideration the type of financial instruments which provides an optimum ratio between the profitableness and the risk, a ratio adjusted depending on his adversity to the risk as well as on his expectations as regards the portfolio and, meanwhile, the available financial possibilities.

4. The Markowitz model

The entire portfolio theory is meant to create the required framework for identifying an optimum portfolio, namely that specific combination of financial assets „which provides the best possible profitableness for a certain level of risk or bears the lowest level of the possible risk for a certain profitableness rate (Roman, 2003).

The concept of optimum portfolio has been used for the first time by Harry Markowitz, back in 1959. He argued that it was possible to meet various portfolios being associated with different levels of risk and profit. Under the circumstances, each investor has to decide the maximum level of the risk he would accept but considering also the value of the expected profit out of the forecasted investment. Depending on the decision previously made, the investor can select the optimum solution for his financial assets portfolio diversification.

In his work, *Portfolio selection. Efficient Diversification of Investments*, Harry Markowitz proved that the decision as to choose the portfolio can be achieved through studying *„the portfolio expected rate of gain and the dispersion or the average squared deviation, as risk measure"* (Markowitz, 1959).

The practical application of the Markowitz model allows us to set up the level of the individual dispersions of the financial instruments yields, both for a simplified portfolio of instruments (formed of two financial assets only) and for a portfolio formed up „n" financial instruments (Anghel, 2013).

Even if considering two or more assets from different markets, the construction of an efficient portfolio implies the gradual running through the following stages (Anghel, 2013):

- Identification of the risk-gain profile for each alternative combination of the assent in the frame of the portfolio;
- Setting up the combination of risky assets with minimum variance depending on the adversity degree of each investor;
- Setting up the complete portfolio by combining the portfolio of minimum variance with risk free assets which the investor intends to take in his portfolio;

The Markowitz model for diversifying the financial instruments portfolio may lead to the identification of some optimum portfolios of risky assets, respectively of portfolios providing a maximum of the estimated yield for a certain level of the risk which the capital investors are willing to undertake depending on their behaviour against the risk.

The simplest model of portfolio which can be analyzed with the support of the Markowitz model is formed of two financial instruments.

The mathematical expectation of the yield rate for the portfolio (E_p) is set up by using the relation:

$E_p = X_1E_1 + X_2E_2,$

where:
X_1, X_2 = the weights of participation to the portfolio;
E_1, E_2 = the yields of the two assets.

The second element which has to be analyzed in order to characterize the portfolio efficiency is given by the portfolio dispersion, as measure of the risk connected to the investment. In this respect, the following mathematical relation is used (Anghelache and Anghel, 2014):
$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X1X_2\sigma_{12}$

The portfolio dispersion is influenced by the following elements (Roman, 2003):
- The dispersion of each asset included in the portfolio;
- The proportions in which the two financial assets are combined;
- The covariance between the two considered assets.

The covariance between the two assets (σ_{12}) (Anghelache and Anghel, 2014)

$$\sigma_{12} = \frac{1}{T-1} \cdot \sum_{t=1}^{T} (R_{1_{(t)}} - \overline{R}1)(R_{2_{(t)}} - \overline{R}2)$$

where:
t = 1, ..., T (number of observations in time on the yield rates;
$R_{1(t)}$ = the yield of the asset "1" at the moment "t";
$\overline{R}1$ = the average yield of the asset "1";

$R_{2(t)}$ = the yield of the asset "2" at the moment "t";
$\overline{R}2$ = the average yield of the asset "2".

Based on the previously presented methodology, it is possible to determine the performance and risk for any type of financial instruments portfolio, regardless the number of titles included in its structure.

$$E_p = \sum_{i=1}^{n} X_i E_i$$

Also, in the case of portfolios made of „n" financial titles, the profitability of portfolio depends on the estimated profitability of each title included in the structure of the portfolio, and as well on the weight they hold in this structure.

$$\sigma_p^2 = \sum_i \sum_j X_i X_j \sigma_{ij}$$

The covariance between the asset "i" and the asset "j" (σ_{ij}) (Anghelache, 2013)

$$\sigma_{ij} = \frac{1}{T-1} \cdot \sum_{t=1}^{T} (R_{i_{(t)}} - \overline{R}i)(R_{j_{(t)}} - \overline{R}j)$$

Dispersion $\boldsymbol{\sigma_i^2}$

$$\sigma_i^2 = \frac{1}{T-1} \cdot \sum_{t=1}^{T} (R_{i_{(t)}} - \overline{R}i)^2$$

In the case of the portfolio risk it is to note that its level is influenced by (Stancu, 2007):

- The individual risks of each asset included in the portfolio;
- The weight of each asset in the portfolio structure;
- The covariance between the assets yields, considered two by two.

5. The analysis of the yield and risk for a portfolio formed up of two assets issued by companies transacted at the Bucharest Stock Exchange

We applied the Markowitz mode in order to analyze the yield and risk of a simple portfolio formed up of two equities. In this respect, we have built up a portfolio of assets issued by two commercial companies from our country transacted through the intermediary of the Bucharest Stock Exchange. The two selected companies are VRANCART SA and MECANICA CEAHLĂU.

VRANCART SA is running its activity in the field of "Manufacturing of paper and corrugated cardboard" (Cod CAEN Rev.2 – 1721), while MECANICA

CEAHLĂU is running its activity in the field of „Manufacturing of machines and equipment for agriculture and forestry exploitations" (Cod CAEN Rev.2 – 2830).

The main information referring to the assets issuers are synthesized as follows (Table 1):

Table 1. *Details on issuers*

Company	**VRANCART S.A.**	**MECANICA CEAHLĂU**
Location	Adjud, Vrancea County	Piatra Neamţ, Neamţ County
Symbol	VNC	MECF
Category	II	II
Market	Main, REGS	Main, REGS
Date start transacting	15.07.2005	15.02.2006
Number shares	863.717.920	239.908.460
Nominal value per share	0,1	0,1
Social capital	86.371.792	23.990.846

Based on the weekly evolution of the prices recorded by the two assets in the year 2013 (03.01.2013 – 30.12.2013), we can proceed to estimating the yield and the connected risk for the year 2014. In this respect, the average weekly yield and the corresponding risk degree recorded for 2013 have been calculated.

Table 2. *Weekly yields of the shares included in the portfolio, recorded in the year 2013*

Week	**Yield VRANCART**	**Yield MECANICA**	**Week**	**Yield VRANCART**	**Yield a MECANICA**
1	0.026900	0.020500	27	0.004360	0.055634
2	-0.184593	0.047840	28	0.017366	0.050033
3	0.228164	-0.013991	29	0.001422	-0.047014
4	0.004354	-0.000747	30	0.034091	-0.040000
5	0.013006	0.006726	31	0.002747	0.075694
6	0.062767	0.017075	32	-0.004110	0.049064
7	0.004027	-0.010949	33	-0.012380	-0.144615
8	-0.008021	0.005904	34	0.019499	0.029496
9	0.037736	0.008070	35	-0.006831	0.030049
10	0.000000	-0.017467	36	-0.016506	0.007463
11	0.000000	-0.005185	37	-0.001399	-0.041751
12	-0.020779	0.000745	38	-0.007003	-0.026001
13	-0.014589	-0.017113	39	0.049365	0.060606
14	0.013459	0.000757	40	-0.018817	-0.011565
15	-0.009296	-0.014372	41	-0.038356	-0.014453
16	-0.009383	0.036071	42	-0.028490	-0.039804
17	0.012179	0.037037	43	-0.020528	-0.054545
18	-0.004011	0.015714	44	-0.002994	0.042308
19	-0.067114	-0.029536	45	0.042042	0.073801
20	0.000000	-0.041304	46	-0.004323	0.001375
21	-0.010072	0.020408	47	-0.007236	-0.025395
22	-0.011628	0.014815	48	0.039359	0.031690
23	0.014706	0.021898	49	-0.008415	-0.063481
24	-0.014493	-0.003571	50	0.002829	0.020408
25	0.007353	-0.028674	51	-0.031030	-0.025714
26	0.004380	0.047970	52	-0.004367	0.021261

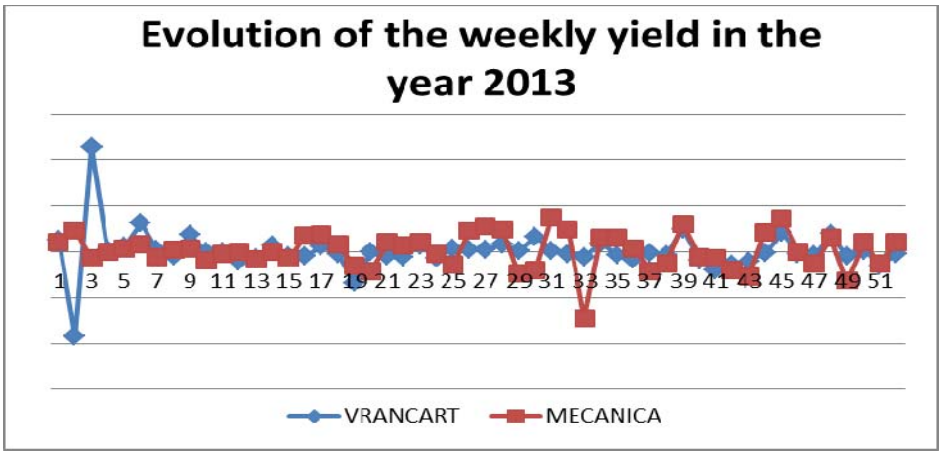

Table 3. *The yield and the average weekly volatility for the equities included in the portfolio in the year 2013*

No.	Company name	Symbol	Yield	Volatility
1	VRANCART SA	VNC	0.001449	0.046684
2	MECANICA CEAHLAU	MECF	0.002561	0.039465

The estimated yield of the considered portfolio is established as follows:

$$E_p = X_{VNC}E_{VNC} + X_{MECF}E_{MECF}$$

Simultaneously, we have calculated the risk connected with this investment, according to the relation:

$$\sigma_p^2 = X_{VNC}^2\sigma_{VNC}^2 + X_{MECF}^2\sigma_{MECF}^2 + 2X_{VNC}X_{MECF}\sigma_{VNC/MECF}$$

The investor has the possibility to make his options on the weights allocated to each type of shares within the portfolio he holds. The influence of these weights on the evolution of the portfolio is submitted in the following table:

Table 4. *The estimation of the weekly yield and risk for the portfolio formed up of the two equities for the year 2014 (for different weights of participation)*

P	X_{VNC}	X_{MECF}	E_P	σ^2_P	σ_P
P1	100%	0%	0.001449	0.002179	0.046684
P2	95%	5%	0.001505	0.001982	0.044515
P3	90%	10%	0.001560	0.001801	0.042442
P4	85%	15%	0.001616	0.001639	0.040480
P5	80%	20%	0.001671	0.001493	0.038645
P6	75%	25%	0.001727	0.001366	0.036957
P7	70%	30%	0.001783	0.001256	0.035437
P8	65%	35%	0.001838	0.001163	0.034107
P9	60%	40%	0.001894	0.001088	0.032989
P10	55%	45%	0.001949	0.001031	0.032107
P11	50%	50%	0.002005	0.000991	0.031480
P12	45%	55%	0.002061	0.000969	0.031124

P	X_{VNC}	X_{MECF}	E_P	σ^2_P	σ_P
P13	40%	60%	0.002116	0.000964	0.031047
P14	35%	65%	0.002172	0.000977	0.031252
P15	30%	70%	0.002227	0.001007	0.031733
P16	25%	75%	0.002283	0.001055	0.032479
P17	20%	80%	0.002338	0.001120	0.033471
P18	15%	85%	0.002394	0.001203	0.034688
P19	10%	90%	0.002450	0.001304	0.036108
P20	5%	95%	0.002505	0.001422	0.037708
P21	0%	100%	0.002561	0.001558	0.039465

By interpreting the previous results we can conclude the following:

- In the case of the equi-weighted portfolio formed up of the two equities, a weekly yield of de 0.2005% has been recorded in the conditions of an average squared deviation 3.1480% which means that the expected future yield will have the biggest probability to equal 0.2005% ± de 3.1480%, respectively to be comprised in the interval {-2.9475; 3.3485};
- No investor with adversity against the risk would accept anything else but those assets which record the higher yield for an unit of connected risk or, reversely, those assets recording the lowest risk per the forecasted yield unit.

6. The analysis of the yield and risk for a portfolio formed up of three assets issued by companies transacted at the Bucharest Stock Exchange

The portfolio previously analyzed, formed up of the two equities (VRANCART SA and MECANICA CEAHLĂU) can be further diversified by including in its structure a third equity "i" (SC ANTIBIOTICE SA). This company has been selected as a result of the economic and financial analysis achieved on its main indicators.

SC ANTIBIOTICE SA is a commercial company located in the district Iaşi, running its activity in the field of "Manufacturing of basic pharmaceutical products" (Cod CAEN Rev.2 – 2110). SC ANTIBIOTICE SA is holding a social capital amounting 6,133,804 lei, divided in 671,338,040 shares of a nominal value of 0.7000 lei/share.

The shares issued by the considered economic agent have been accepted for transactions in the frame of the Bucharest Stock Exchange on 16.04.1997, getting as symbol of identification the **ATB**. The shares issued by SC ANTIBIOTICE SA are presently transacted in the frame of the stock exchange BVB, on the main market REGS, being include in the category I of financial instruments.

Table 5. *The weekly yields of the ANTIBIOTICE S.A. shares recorded in the year 2013*

W	Yield ATB	S	Yield ATB	W	Yield ATB	W	Yield ATB
1	0.010900	14	-0.003928	27	0.011361	40	-0.002761
2	0.032326	15	-0.021468	28	-0.018723	41	-0.000639
3	0.014002	16	0.006940	29	-0.018868	42	-0.009802
4	0.030128	17	0.015562	30	-0.006050	43	0.012481
5	0.050207	18	0.002408	31	0.012174	44	0.020616
6	0.021815	19	-0.005023	32	0.006229	45	0.068930
7	0.000454	20	-0.026339	33	0.001281	46	0.069745
8	0.002043	21	-0.011722	34	0.003837	47	-0.023311
9	0.008382	22	0.067518	35	-0.021236	48	0.031885
10	0.017749	23	0.004274	36	0.011282	49	0.022949
11	0.016115	24	0.006383	37	-0.000429	50	-0.016605
12	-0.003476	25	0.004017	38	0.001932	51	0.009700
13	-0.000872	26	0.000842	39	0.008783	52	-0.003202

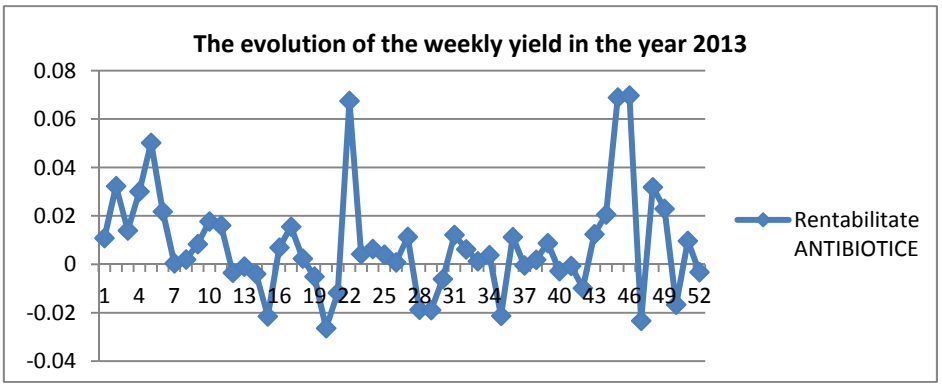

Table 6. *The yield and average weekly volatility of the equities included in the portfolio in the year 2013*

No.	Company name	Symbol	Yield	Volatility
1	VRANCART SA	VNC	0.001449	0.046684
2	MECANICA CEAHLAU	MECF	0.002561	0.039465
3	ANTIBIOTICE SA	ATP	0.007900	0.021442

Based on the methodology previously submitted, we have calculated the yield and the connected risk for the issued shares:

$$E_p = X_{VNC}E_{VNC} + X_{MECF}E_{MECF} + X_{ATB}E_{ATB}$$

Further on, the values of the correlation coefficients between the three financial instruments included in the portfolio structure will be set up:

Table 7. *The covariance matrix between the yields of the analysed equities*

σ	ATB	VNC	MECF
ATB	0.000460	0.000056	0.000221
VNC	0.000056	0.002179	0.000114
MECF	0.000221	0.000114	0.002179

$$\sigma_p^2 = X_{VNC}^2\sigma_{VNC}^2 + X_{MECF}^2\sigma_{MECF}^2 + X_{ATB}^2\sigma_{ATB}^2 + 2X_{VNC}X_{MECF}\sigma_{VNC/MECF} + 2X_{VNC}X_{ATB}\sigma_{VNC/ATB} + 2X_{MECF}X_{ATB}\sigma_{MECF/ATB}$$

Here again, the investor has the choice on the weights allocated to each type of equities out of the built up portfolio. The influence of this weight on the evolution of the portfolio yield and risk is submitted in the following table:

Table 8. *The estimation for the weekly yield and risk of the portfolio formed up of 3 equities for the year 2014 (with different weight of participation)*

P	weight VNC	weight MECF	weight ATB	E_P	σ^2_P	σ_P
P1	100%	0%	0%	0.001449	0.002179	0.046684
P2	90%	5%	5%	0.001827	0.001787	0.042270
P3	85%	5%	10%	0.002150	0.001605	0.040057
P4	85%	10%	5%	0.001883	0.001618	0.040220
P5	80%	10%	10%	0.002205	0.001447	0.038034
P6	80%	5%	15%	0.002472	0.001435	0.037881
P7	80%	15%	5%	0.001938	0.001466	0.038290
P8	70%	15%	15%	0.002583	0.001159	0.034043
P9	70%	10%	20%	0.002850	0.001142	0.033799
P10	70%	20%	10%	0.002316	0.001183	0.034400
P11	60%	20%	20%	0.002962	0.000924	0.030393
P12	60%	30%	10%	0.002428	0.000990	0.031469
P13	60%	10%	30%	0.003495	0.000889	0.029811
P14	50%	25%	25%	0.003340	0.000741	0.027222
P15	50%	20%	30%	0.003607	0.000715	0.026733
P16	50%	30%	20%	0.003073	0.000775	0.027844
P17	40%	30%	30%	0.003718	0.000611	0.024715
P18	40%	20%	40%	0.004252	0.000556	0.023583
P19	40%	40%	20%	0.003184	0.000697	0.026402
P20	30%	35%	35%	0.004096	0.000533	0.023090
P21	30%	30%	40%	0.004363	0.000497	0.022292
P22	30%	40%	30%	0.003829	0.000577	0.024025
P23	20%	40%	40%	0.004474	0.000508	0.022537
P24	20%	20%	60%	0.005542	0.000391	0.019766
P25	20%	60%	20%	0.003406	0.000751	0.027407
P26	10%	45%	45%	0.004852	0.000535	0.023134
P27	10%	40%	50%	0.005119	0.000489	0.022117
P28	10%	50%	40%	0.004585	0.000589	0.024272
P29	0%	50%	50%	0.005230	0.000615	0.024799
P30	0%	100%	0%	0.002561	0.001558	0.039465
P31	0%	0%	100%	0.007900	0.000460	0.021442
P32	33%	33%	34%	0.004009	0.000547	0.023390
P33	33%	34%	33%	0.003956	0.000555	0.023555
P34	34%	33%	33%	0.003945	0.000558	0.023620

Including in the portfolio structure the third equity (ATB) led to an increase of the portfolio yield, simultaneously with a diminishing of the risk connected to this investment.

Another variant for diversifying the financial instruments portfolio is given by the purchase of government bonds. These provide the big advantage that, contrary to the equities issued by economic entities, they are not bearing risks. In order to analyze this type of portfolio, we have included government bonds with a weekly yield of 0.1397%.

Table 9. *The yield of the equities included in the portfolio*

VNC	0.001449
MECF	0.002561
GB	0.001397

Also, in this case as well, we have simulated more variants of allocation for the equities weights of participation to the portfolio construction, the results being submitted in the following table:

Table 10. *The estimation for the weekly yield and risk of the portfolio formed up of two equities and one government bond for the year 2014 (with different weight of participation)*

P	Weight VNC	Weight MECF	Weight TS	E_P	σ^2_P	σ_P
P1	100%	0%	0%	0.001449	0.002179	0.046684
P2	90%	5%	5%	0.001502	0.001779	0.042184
P3	85%	5%	10%	0.001499	0.001588	0.039852
P4	85%	10%	5%	0.001558	0.001610	0.040119
P5	80%	10%	10%	0.001555	0.001429	0.037796
P6	80%	5%	15%	0.001497	0.001408	0.037521
P7	80%	15%	5%	0.001613	0.001457	0.038172
P8	70%	15%	15%	0.001608	0.001127	0.033568
P9	70%	10%	20%	0.001550	0.001099	0.033157
P10	70%	20%	10%	0.001666	0.001162	0.034088
P11	60%	20%	20%	0.001661	0.000874	0.029566
P12	60%	30%	10%	0.001777	0.000966	0.031075
P13	60%	10%	30%	0.001545	0.000814	0.028527
P14	50%	25%	25%	0.001714	0.000671	0.025896
P15	50%	20%	30%	0.001656	0.000630	0.025097
P16	50%	30%	20%	0.001772	0.000719	0.026816
P17	40%	30%	30%	0.001767	0.000516	0.022719
P18	40%	20%	40%	0.001651	0.000429	0.020716
P19	40%	40%	20%	0.001883	0.000634	0.025184
P20	30%	35%	35%	0.001820	0.000411	0.020268
P21	30%	30%	40%	0.001762	0.000357	0.018888
P22	30%	40%	30%	0.001878	0.000473	0.021739
P23	20%	40%	40%	0.001873	0.000355	0.018829
P24	20%	20%	60%	0.001640	0.000159	0.012592
P25	20%	60%	20%	0.002106	0.000675	0.025983
P26	10%	45%	45%	0.001926	0.000347	0.018639
P27	10%	40%	50%	0.001868	0.000280	0.016736
P28	10%	50%	40%	0.001984	0.000423	0.020555
P29	0%	50%	50%	0.001979	0.000389	0.019733
P30	0%	100%	0%	0.002561	0.001558	0.039465
P31	0%	0%	100%	0.001397	0.000000	0.000000
P32	33%	33%	34%	0.001798	0.000432	0.020777
P33	33%	34%	33%	0.001810	0.000443	0.021044
P34	34%	33%	33%	0.001799	0.000447	0.021143

In situations of a generalized economic and financial crisis it is advisable that the available capitals are invested in government bonds, these being financial instruments which do not involve risks and which might provide even higher yields as comparatively with the equities issued by the economic agents being transacted through the institutions of the capital market.

7. Conclusions

The portfolio diversification by including in its structure more equities is generating an increase of its yield simultaneously with the diminishing of the corresponding risk.

According to the Markowitz model, the choice of the portfolio is resuming in fact to the analysis of two specific indicators respectively the expected gain rate of the portfolio and its average squared deviation as measure of the risk

In the frame of this research, we have analyzed the model in the particular forms represented by the portfolios made up of two, respectively three equities. As result of the analysis we stated out that the portfolios yield is dependent on the estimated yields of each equity included in the portfolio structure, as well as on the weight they are holding in this structure. In the case of the portfolio risk, we have noticed that its level is influenced by: the individual risk of each equity included in the portfolio, the weight of each equity in the portfolio structure, the covariance between the equities yields, considered two by two. The Markowitz model allows the setting up of an optimum portfolio, respectively of that portfolio which provided the best possible profitableness for a certain level of the risk or which presents the lowest level of the possible risk for a certain rate of profitableness.

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