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A labor utility index to measure worker welfare and labor market performance

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Abstract. *This study develops an asymmetric labor utility index (LUI) that is determined by worker preferences for labor and leisure, based upon the target employment preference for individual workers. The welfare index is illustrated for monthly data frequency that is consistent with the current employment data, although any time increment could be used. Once the utility values are computed for a representative sample of workers in a defined region, the average of the data is used to create an aggregate labor utility index (ALUI) for the regional or national labor market. This provides an additional empirical insight into current economic performance beyond the standard U-1 to U-6 labor underutilization measures regularly computed by the United States Bureau of Labor Statistics (BLS). The U-6 provides some measure of underemployment, but it does not define a welfare measure that allows comparing the labor market performance from month to month. Moreover, none of the BLS measures account for the disutility of over-employment. We generate an index that accounts for unemployment, underemployment, and overemployment when determining the current overall aggregate labor market welfare in the region.*

Keywords: Aggregation, Labor Utility Index, Overemployment, Underemployment, Unemployment.

JEL Classification: C43, C80, J01, J40, J60.

1. Introduction

The United States Bureau of Labor Statistics (BLS) and other international organizations collect data on labor underutilization (www.bls.gov). The BLS computes the monthly variables U-1 through U-6, which provide direct measures of aggregate unemployment and underemployment(1). However, these variables are insufficient in two ways. First, they do not measure labor overutilization, which is a global problem, since over 20 percent of the world's workers are working more than 48 hours per week (Lee et al., 2007, p. 53). Second, these underutilization measures do not produce an individual or aggregate labor utility measure that can be tracked from over time, and that allows for international utility comparisons.

Labor underutilization correctly remains the primary focus of labor market measurement, but overwork creates both massive welfare losses and productivity losses. Previous literature suggests that mismatched work hours cause welfare losses (Golden and Gebreselassie, 2007). Using the U.S. BLS Current Population Survey (CPS) data, Golden and Gebreselassie (2007) find that underwork is a bigger problem than overwork, since only about 7% would like to work less than their current work hours, despite the resulting lower income, while 25% of their sample would prefer to work longer hours.

Overwork has consistently been a pervasive problem for Japanese workers since at least the 1960s (Weathers and North, 2009). Overwork problems in the U.S. have been popularized since Schor (1993), and have been studied globally by using a sample of 53 countries using ILO and World Bank databases (Lee et al. 2007). In the period 1990 and 2012, Peru and the Republic of Korea had the largest average working hours per person engaged, while Greece, Hungary, Japan, and the U.S. all have large average working hours (Lee et al., 2007; C.W. and A.J.K.D., 2013). Lee et al. (2007) defined excessive overtime, or long hours, as being an average workweek of more than 48 hours. In 2004-2005, about 50.9% of Peruvian workers exceeded at 48-workweek, and about 49.5% of Korean workers exceeded a 48-hour workweek (Lee et al., 2007).

Jacobs (1998) evaluates the claim that worker self-reporting of hours, such as in the U.S. CPS, are upwardly biased since workers tend to exaggerate the time they spend on the job. Jacobs (1998) concludes that the standard self-reported measure of working time is a reasonably reliable measure, and that it does not appear to overstate the workweek for individuals working long hours.

Overwork has led to at least four problems: productivity loss, workaholism, work-life balance issues, and karoshi, which is the most severe problem related to overwork (Kanai, 2008). In 2001, a Japanese Ministry of Health, Labor and Welfare (MHLW) medical commission found that long work hours in Japan led to both physical and mental illness (Weathers and North, 2009). The term karoshi includes severe mental and/or physical deterioration due to overwork. It has been used as socio-medical term in relation to workers' compensation, and includes cases of both death and permanent disability (Iwasaki et al., 2006). Hamermesh and Slemrod (2008) define workaholism "as an addiction to work that is acquired as a consequence of working early in a career, and that manifests itself as an increase in one's subsequent labor supply." Hamermesh and

Slemrod (2008) suggest that individuals may become workaholics because they develop an addiction to consumption goods that were increasingly acquired by increasing their market work. This implies that these workers have a preference for consistently working an overtime schedule.

Using data from Australia, Germany, the U.S., and Korea, Hamermesh and Lee (2007) find that additional market work generates time stress. Their study concludes that additional earnings increase time stress, controlling for hours of market work and house work. Tausig and Fenwick (2001) provide evidence that perceived employee control over work hours improves work-life balance, accounting for work and family characteristics. White et al. (2003) conclude that practices that increase work demand, such as longer actual hours worked and group work, increase negative work-to-home spillover, while practices that provide worker flexibility concerning work demands, such as control over hours worked, decrease negative work-to-home spillover. Lee et al. (2007) and C.W. and A.J.K.D. (2013) use cross-sectional data from OECD countries to show that average labor productivity falls as a country's average hours worked per person increases.

On the other end of the spectrum, Japan's economic downturn in the late 1990s and the first decade of the 2000s has led to a bipolarized trend where part of the labor force is working longer hours, and a growing number of non-regular employment where workers are part-time or temporary (Kanai, 2008). Whereas overwork may lead workers to maintain higher income and consumption levels, underemployment may lead to stress and underconsumption where the material standard of living is less than a worker's desired level.

1.1. Purpose and Scope

The purpose of this analysis is to fill in the void left by the two deficiencies in the current measures of the labor market activity: the lack of a direct welfare measure, and the failure to integrate the disutility of overemployment at the individual and aggregate level. We define the properties that a *labor utility index* (*LUI*) function must satisfy in order for it to serve as an appropriate measure of individual and aggregate labor market welfare, and to be pragmatically implemented within the current employment data collection system. Then, we develop a *LUI* function that satisfies these properties. Our *LUI* can be applied to each individual worker, where the welfare level is determined by that person's preferences for labor and leisure. The parameters in the *LUI* are automatically determined as a by-product of the target employment preferences that are stated by individual workers, so that the resulting statistic does not contain any arbitrary assumptions.

This *LUI* captures the welfare losses from both underemployment and overemployment. Moreover, it is asymmetrical, allowing for underemployment to cause greater decreases in worker welfare than overemployment. The welfare index is illustrated for monthly data frequency, which is consistent with the current employment data, although any time frequency could be used. Once the *LUI* values are computed for a representative sample of workers in a defined region, the average of the data can be used to create an *aggregate labor utility index* (*ALUI*) for any regional or national labor market. The *ALUI* allows for both time-series and cross-sectional comparisons of the aggregate welfare in the labor market.

2. Required properties of a labor welfare measure

The jobs, unemployment, and labor market data are collected and presented monthly by the U.S. BLS. Since the number of days within a month is non-standard and varies between the minimum of 28 in (a non-leap year) February, 30 days in April, June, September, and November, and 31 days in the remaining months, any measure based on a month will yield different labor hours for months of different lengths. The monthly labor welfare measurements are further complicated since a full-time job is defined as working 40 hours per week, which further is broken down as being 8 hours per workday. But, the number of weeks in a month is also non-standard due to the variance of the number of days in a month. Moreover, the definition of full-time hours during a week also varies for salaried employees depending on the number of holidays and vacation days that occur within a given week.

Property 1: The *LUI* must provide a consistent definition for *full-time monthly hours* and *percentage of time devoted to work* across all monthly data collection periods, and should be defined for entire range of potential hours worked.

Measuring worker's labor welfare requires reconciling the hour, day, month, and year measurement periods. Although there is not a unique reconciliation method, we will achieve this by employing a direct conversion based on a 360-day working year. The definitions used in the calculations are defined in Table 1.

Table 1. *Time definitions*

Time Unit	Quantity
hours in a full day	24
days in a working month	30
working months in working year	12
days in a working year	360
days in a week	7
weeks in a working year	51.4285714
days in a full-time workweek	5
hours in an full-time workday	8
hours in a full-time working month	171.4285714
average hours worked per full day	0.238095

There are 24 hours in a full day, and 7 days in a week. Define one working month as consisting of 30 days, where there are 12 working months in a 360-day working year. Let H_t denote the actual number of hours worked in month t. The labor hours per month are H_t = 0 if the individual is unemployed for the entire period. The maximum potential workload is H_t^{Max} = 720 hours per working month, in which case an individual would be working 24 hours per day for the entire 30 days (if it were physically possible).

Under this definition, there are 51 3/7 working weeks in a working year. There are 8 hours in a full-time workday. We define a full-time workweek as 5 days, so that each week has 5 workdays and 2 leisure days. Thus, full-time workers will work an amount given by H_t^{Full} = 171.428571 hours per month. That is found by calculating the following: [(8 hours/workday) * (5 workdays/week) / (7 days/week) * (360 days/working year) / (12 months / working year)] = 171 3/7.

Under this specification, a full-time worker will devote an average of 23.8095% of the time in any period to labor, and devote the remaining 76.1905% of the time to leisure. The time devoted to labor by a full-time worker is calculated as follows:

[(8 hours/workday) * (5 workdays/week) / (7 days/week) / (24 hours/day)] = 0.238095. This means that the average full-time U.S. worker spends about 24% of their time working, and about 76% of their time on leisure. Since an actual calendar year has 365 days, the actual percentage of time devoted to labor per year is actually slightly smaller than 23.8095%.

Property 2: The welfare level generated by *LUI* function must only depend on the labor hours actually worked (*H*), and the optimally desired hours (*H**).

Since the *LUI* must be calculated from the data collected by labor survey in a given country or region, such as the U.S. BLS, the *LUI* cannot be framed as a model that depends upon the wage rate, working conditions, type of industry, or any other variable other than the actual and desired hours. The overall welfare level for any individual depends upon a wide variety of variables that affect consumption and labor, but these are rendered irrelevant by the single issue that is the subject of the labor survey. When an individual is surveyed, the consolidation of the wage rate, desired consumption and saving, and all the other variables determines the desired optimal level of monthly work hours for any given worker. The desired labor hours thus encapsulates all of these factors, and simplifies the preferences within a single quantity. Given the actual monthly work hours, the level of welfare resulting from the monthly labor welfare can be determined based on the *LUI*.

Property 3: Define a continuous positive real-valued *LUI* function as *LUI*(*H*): $H \rightarrow LUI$, $H \in \mathbb{R}^{+}$, $LUI \in \mathbb{R}^{+}$. In each month *t*, the *LUI* function must provide a unique interior *bliss point*, or welfare-maximizing combination of monthly hours worked (*H**) and labor utility index level (*LUI**), as specified as follows:

$\exists$ (H^*, LUI^*) s.t. $\{(H, LUI): LUI^*(H^*) > LUI(H),\ \forall\ H \neq H^*,$

$0 < H < 720, LUI > 0\}$

Marginally attached workers and individuals in the labor force must determine their desired work hours and consumption spending based upon their initial endowments, job opportunities, and expected attainable future income stream. Rational individuals know that higher levels of consumption spending require more labor hours in order to generate the necessary purchasing power. Let *H** denote a given worker's optimally desired hours in month *t*. Each worker will have a desired bliss point that defines the optimal balance combination of labor and leisure. Workers will achieve less than optimal welfare when they underemployed or overemployed. Thus, the *LUI* function must be designed so that it has only one bliss point, which is the optimum level of labor hours that will uniquely provide the maximum level of welfare for any given individual in the data set.

Property 4: Given a continuously differentiable *LUI* function that satisfies *Property 3*, the individual's labor utility should continually decrease as the difference between the *actual hours worked*, and the *optimal number of hours worked*, increases in the same direction. Thus, the first-order condition on the derivative is:

$$\frac{d\,LUI\,(H)}{d\,H} > 0 \ \forall \ H < H^*, \text{ and } \frac{d\,LUI\,(H)}{d\,H} < 0 \ \forall \ H < H^*, 0 < H < 720.$$

Under *property 4*, the greater the level of underemployment, the greater the loss in utility. Workers who are underemployed encounter a decrease in their welfare as their number of monthly hours declines farther below their desired hours. For example, individuals who prefer to work full-time at 171 monthly hours will have a lower level of utility if their work hours are restricted to 150 hours per month, as compared to the utility level when they work 160 hour per month. Similarly, the greater the level of overemployment, the greater the loss in welfare. Suppose that a worker desires to work only full-time, but is working overtime. In that case, the level of utility will be greater when the total monthly hours is only 180 hours (still working 9 overtime hours) than when the overtime is larger, such as when the monthly labor hours are 200 (resulting in 29 overtime hours).

Property 5: Given *LUI* function that satisfies *Property 4*, the *LUI* should produce a rate of loss in utility that will continually increase as the difference between the *actual hours worked*, and the *optimal number of hours worked*, increases in the same direction. Therefore, second-order condition on derivative is:

$$\frac{d^2\,LUI\,(H)}{d\,H^2} < 0 \ \forall \ H, 0 < H < 720.$$

Under *property 5*, the *LUI* function will be strictly concave from the origin. The *LUI* function will be relatively flat in the neighborhood close to the desired optimal number of hours worked, and will continually become relatively steeper as the level of underemployment or overemployment increases. Consider an underemployed individual that desires to work full-time at 171 hours. When *property 5* holds, the welfare level will decrease more when the number of monthly hours falls from 160 hours to 155 hours than when the number of hours falls from 170 hours to 165 hours. Similarly, an overemployed worker will suffer a greater welfare loss when the monthly hours increase from 185 to 190 than when the hours increases from 175 to 180.

This means that individuals whose actual working hours differ only slightly from desired number of hours will suffer only a very small drop in welfare. This property is crucial for capturing the true underlying properties worker utility. Underemployed workers who only work hours that are slightly below full-time can make budgetary changes with minor alterations in consumption so that the effect on utility and well-being is very small. However, individuals that suffer large levels underemployment encounter large budget shortfalls, and are forced into drastic reductions in consumption.

Additional levels of overemployment impose larger utility losses as workers begin to spend large fractions of their time at work. Work-life balance conditions diminish much faster when individuals work large amounts of overtime, rather than just a few hours over the optimal preference. If the required overtime continues to increase, then the employees could experience karoshi. The minimum karoshi borderline that leads severe mental and/or physical deterioration due to overwork is defined as working at least 80 hours of overtime per month (Iwasaki et al., 2006). When full-time is defined as 171 hours as in figure 1, the karoshi borderline converts to about 251 total work hours per month. Thus, individuals who are working 251 hours or more per month are suffering extreme deterioration in health and welfare. The level of utility therefore declines much faster as for monthly labor increases, given that the initial level of overemployment is already large.

Property 6: Based on the fact that individuals experience karoshi (severe physical and mental deterioration) when working over 80 hours of overtime per month, the *LUI* should obtain a maximum at (H^*, LUI^*) $\forall$ H, $0 < H < 720$, but should primarily be analyzed for economic scenarios where $H^* < 251$ monthly hours.

Property 7: The *LUI* should by asymmetric around the desired optimal level of monthly work hours (H^*), such that any given quantity of underemployment generates lower welfare level than the identical quantity of overemployment. Thus, for any positive real number $\varepsilon > 0$,

$LUI(H^* - \varepsilon) < LUI(H^* + \varepsilon)$ where $\{ H: 0 < H < 720; 0 < H^* < 251\}$.

The primary barometer of macroeconomic performance and the current state of a region's economy is the level of underemployment. The foregone output production due to underemployment creates a loss for the economy, whereas overemployment does not result in any loss in potential output. For individuals, the lack of resources for consumption and saving that result from underemployment are more severe than the foregone leisure that results from an equivalent amount of overemployment. Overemployed individuals who only work a small amount of overtime can rearrange their schedules so that the extra consumption largely offsets the welfare losses in leisure time. Moreover, paid labor hours are generally more flexible downward than upward.

Property 8: Given any utility maximizing desired labor hours, $0 < H^* < 720$, The *LUI* should have a maximum value of $LUI^* = 1$, so that maximum utility level for any worker in the survey is 100%.

Property 9: Given any utility maximizing desired labor hours, $0 < H^* < 720$, The *LUI* should have a minimum value of $LUI^{min} = 0$, so that minimum utility level for any worker in the survey is 0%. Moreover, this minimum value should be obtained under two cases: unemployment, and when a worker is working the theoretical maximum (but physically impossible) amount of 720 monthly hours. This requires that $LUI(0) = LUI(720) = 0$.

Property 9 ensures that any individual who desires to work for any number of hours, and who is unemployed, will receive no utility from the attempted participation in the labor market. Also, any individual who works at every moment (if it were possible), and thus has no time to devote to leisure, will experience no utility. Properties 8 and 9 allow for an aggregate labor market index to be constructed. In order to compare the utility across equally weighted persons, each unemployed individual must have 0 utility, and each individual who is working their personally preferred optimal amount of hours must have a 100% satisfaction level.

3. Index derivation for individual workers

This section derives an *LUI* function that satisfies the seven properties in the previous section. Although this function does not uniquely satisfy these conditions, it is the simplest such function, due to two advantages. First, it has only one parameter, which is internally defined based on an individual's desired work hours. Second, it is relatively easy to compute, and thus it can be programmed by the data collectors so that the *LUI* number for each individual in the survey is automatically generated as soon as the actual and desired labor hours are entered into the database.

Let H_t^* denote a given worker's optimally desired hours in month *t*, and let λ represent the optimal percentage of given month devoted to labor as determined by a worker's preference. The worker's welfare maximizing percentage of time devoted to labor is

$$\lambda_t = \frac{H_t^*}{H_t^{Max}} \quad 0 < \lambda < 1 \tag{1}$$

The value for λ cannot equal 0, since would mean that the worker would drop out of the labor force. The value for λ cannot equal 1, since would mean that the worker is working every moment of the day, which is physically impossible. The simplest *LUI* which captures a person's preferences while satisfying all 9 properties can be expressed as follows.

$$LUI\,(H_t) = \left[\frac{H_t}{H_t^{Max}}\right]^{\lambda}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{1-\lambda} + \left[\frac{H_t}{H_t^{Max}}\right]^{\lambda}\left[\left(\frac{H_t^{Max} - H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{1-\lambda} \tag{2}$$

$$+ \left[1 - (\lambda)^{\lambda}(1-\lambda)^{1-\lambda}\right] \qquad 0 < \lambda < 1$$

There will always be a unique maximum value for the *LUI* index, and it will occur at $LUI^* = LUI(H^*) = 1$ for $0 < H^* < 720$. This scales the index so that maximum welfare level for every worker is 1, regardless of the individual's optimal labor preference. Thus, the index satisfies *property 3* and *property* 8. The *LUI* also satisfies *property 9*, since *LUI*(0) = *LUI*(720) = 0. Therefore, any individual with no labor hours, or no leisure hours, experiences 0 utility.

The index is asymmetric for all values of λ except for $\lambda = 0.5$. The value $\lambda = 0.5$ corresponds to a desired optimal monthly labor hour value of $H^* = 360$. This would require a person to work a 12-hour workday every day with no vacation days in a month.

The minimum border for karoshi to occur is 251 hours per month, which is far less than 360 hours. Thus, the *LUI* in equation (2) satisfies the asymmetry condition in *property 7* over the values $0 < H^* < 360$, which includes the relevant hourly range of $0 < H^* < 251$, where any value of $H > 251$ leads to extreme health issues due to overwork. This insures that the *LUI* function provides a greater penalty for underemployment than for overemployment for all desired monthly labor hours in the range $0 < H^* < 251$.

There are three general preferences that can occur for workers in the labor force. These cases include: (1) workers who desire to work full-time; (2) workers desire to work part-time; and (3) those who desire to work overtime.

3.1. Case 1: Full-Time Work Preference

Figure 1 and table 2 consider case (1) where individuals optimally desire to work full-time, and maximize their welfare by working the full-time amount 8 hours per day during each day of a 5-day workweek. This is represented by

$$\lambda_t = \frac{H_t^*}{H_t^{Max}} = \frac{171.4285714}{720} = 0.238095 \qquad (3)$$

As shown in figure 1, workers who prefer to work full-time for 8 hours during each workday will desire to work 171 hours per month. If a worker actually works 171 hours during the month, then their welfare level is $LUI = 1$. If the individual is unemployed for the month, then their welfare level is $LUI = 0$.

Figure 1. *Full-time Labor Preference*

Worker that prefers to work 8 hours per workday: $H^* = 171.428571$; $\lambda = 0.238095$

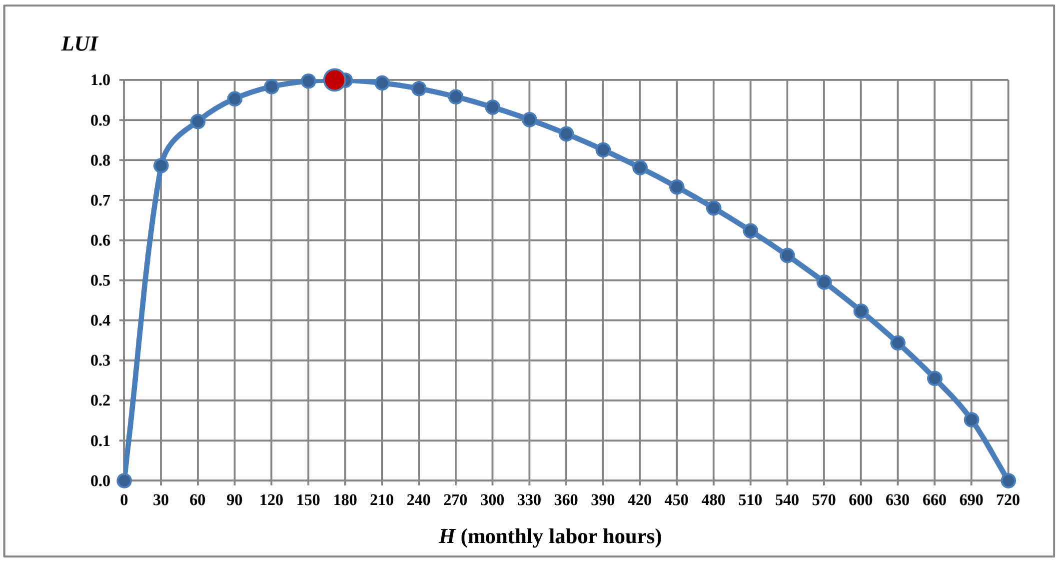

Table 2. *Full-time Labor Preference*

Worker that prefers to work 8 hours per workday: H^* = 171.428571; λ = 0.238095

H	*LUI*
0	0.0000
131.4286	0.9904
151.4286	0.9977
171.4286	1.0000
191.4286	0.9980
211.4286	0.9923
720	0.0000

The index embodies a designed asymmetry that assigns a greater penalty for welfare losses due to underemployment than for welfare losses due to overemployment. If the worker is underemployed and only works for 131 hours, which 40 hours less than the desired level of employment, then the welfare level is *LUI* = 0.9904. Alternatively, if the worker is overemployed and works for 211 hours, which 40 hours more than the desired level of employment, then the welfare level is *LUI* = 0.9923. This illustrates the fact that each hour of underemployment relative to the optimum preference will diminish worker utility more than each hour of overemployment, as specified in *property 7*.

In addition, utility decreases by the larger amount of 0.0057 when the labor hours increase from 191 to 211 hours as contrasted with the smaller amount of 0.0020 when the hours increase from 171 to 191 hours. Furthermore, the utility decreases by the larger amount of 0.0096 when the labor hours decrease from 151 to 131 hours than smaller amount of 0.0023 when the hours increase from 171 to 151 hours. This demonstrates that the *LUI* function is strictly concave from the origin, so that *property 4* and *property 5* are satisfied.

3.2. Case 2: Part-Time Work Preference

Case (2) includes all workers who desire to work part-time. Figure 2 and table 3 show the case of workers who desire to work 4 hours per workday, which translates into about 85.7 hours per month. When an individual actually works 85.7 hours during the month, then their welfare level is *LUI* = 1. When these workers are unemployed for the month, then their welfare level is *LUI* = 0.

Figure 2. *Part-time Labor Preference*

Worker that prefers to work 4 hours per workday: $H^* = 85.1428571$; $\lambda = 0.119048$

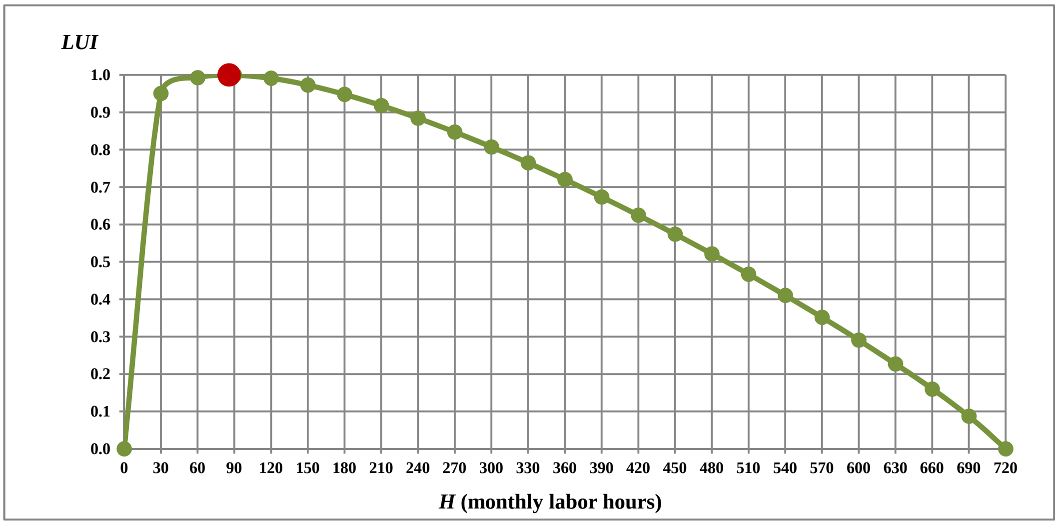

Table 3. *Part-time Labor Preference*

Worker that prefers to work 4 hours per workday: $H^* = 85.1428571$; $\lambda = 0.119048$

H	*LUI*
0	0.0000
45.7143	0.9793
65.7143	0.9957
85.7143	1.0000
105.7143	0.9967
125.7143	0.9883
720	0.0000

The index again demonstrates the asymmetry that results in larger welfare losses for underemployment than during situations of overemployment. If the worker suffers an episode of underemployment and only works for 46 hours, which 40 hours less than the desired level of employment, then the welfare level is $LUI = 0.9856$. Alternatively, if the worker is overemployed and works for 126 hours, which 40 hours more than the desired level of employment, then the welfare level is $LUI = 0.9919$.

3.3. Case 3: Overtime Work Preference

The third category includes all workers who desire to work overtime. Figure 3 and table 4 show the case of workers who desire to work 12 hours per workday, resulting in about 257 hours per month. Actually working 257 hours during the month leads to the welfare level $LUI = 1$. When these workers are unemployed, the welfare level is $LUI = 0$.

Figure 3. *Overtime Labor Preference*

Worker that prefers to work 12 hours per workday: ***H**** = **257.1428571**; ***λ*** = **0.357143**

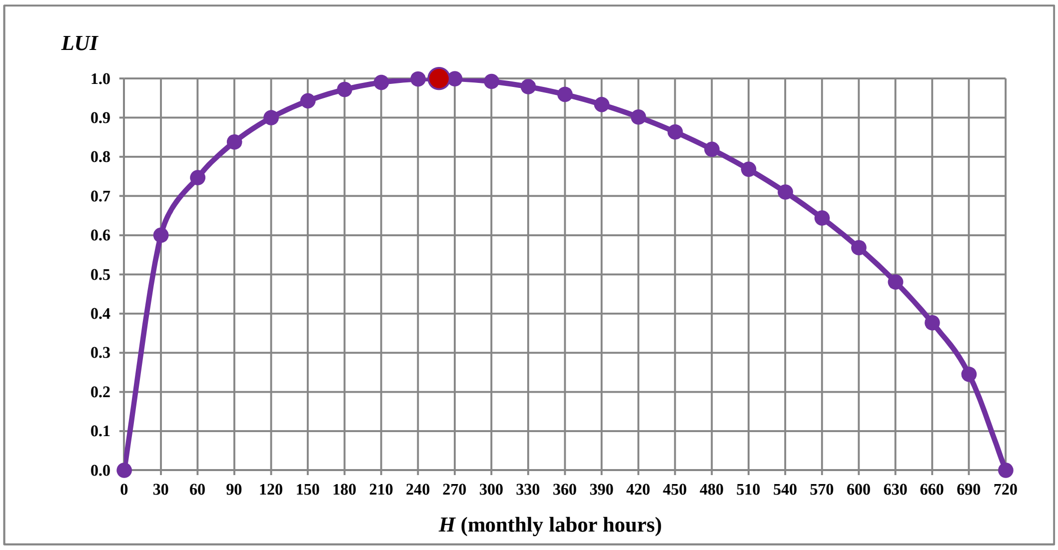

Table 4. *Overtime Labor Preference*

Worker that prefers to work 12 hours per workday: ***H**** = **257.1428571**; ***λ*** = **0.357143**

H	*LUI*
0	0.0000
217.1429	0.9929
237.1429	0.9983
257.1429	1.0000
277.1429	0.9984
297.1429	0.9936
720	0.0000

Situations of underemployment where the individual only works for 217 hours, which is 40 hours less than the desired level of hours, correspond to a welfare level of *LUI* = 0.9929. Overemployment where an individual works for 297 hours, which 40 hours more than the desired level of employment, corresponds to a welfare level of *LUI* = 0.9936. Thus, this case again demonstrates the asymmetry of the employment gap whereby a given number of hours of underemployment is penalized more than the same number of hours of overemployment.

In summary, the *LUI* function will always have a maximum value of *LUI* = 1.0 whenever the individual works the desired number of hours. When workers prefer to spend less than half of their time working, then $\lambda < 0.5$ and the desired number of hours is $H^* < 360$. For all of these cases, the *LUI* will be asymmetric and will assign larger penalties for underemployment than for overemployment. This includes all of the cases above, including the overemployment case 3, where a consistent 12-hour workday for each 5-day workweek resulted in the individual working 297 hours per month. The only case where there the *LUI* would be symmetric occurs when workers prefer to spend half of their time working, so that $\lambda = 0.5$ and the desired number of hours is $H^* = 360$. Although this last case is of theoretical interest, it does not have a practical application, since the optimal level of hours would be less than $H^* < 251$, due to human physical and mental health restraints.

As the desired number of hours decreases, both H^* and λ will fall. When the desired quantity of labor hours approaches 0, then $H^* \to 0$, $\lambda \to 0$. The *LUI* is undefined for $\lambda = 0$; this is consistent with the U.S. BLS data definitions, since any worker who does not desire to work is classified as not in the labor force.

The response of an individual's *LUI* to a change in number of hours worked can also be expressed. Consider a worker with fixed labor preference parameters H^* and λ, where $0 < \lambda < 1$. Whenever the actual level of hours worked changes, the resulting change in the welfare level will be determined by the first derivative of equation (2), which is given in equation (4):

$$\frac{d\,LUI\,(H_t)}{d\,H_t} = \frac{\lambda H_t^{\lambda-1}}{\left(H_t^{Max}\right)^{\lambda}}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{1-\lambda} - \frac{(1-\lambda)\,H_t^{\lambda}}{\left(H_t^{Max}\right)^{\lambda+1}}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{-\lambda}$$
$$+ \frac{\lambda H_t^{\lambda-1}}{\left(H_t^{Max}\right)^{\lambda}}\left[\left(\frac{H_t^{Max} - H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{1-\lambda} - \frac{H_t^{\lambda}}{\left(H_t^{Max}\right)^{\lambda+1}}\left[\left(\frac{H_t^{Max} - H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{-\lambda} \qquad (4)$$

Equation (4) shows that equation (2) satisfies the first derivative condition in *property 4*, and this fact is illustrated in figures 2 – 4. Thus, the *LUI* function is has a positive first derivative when $H < H^*$, and a negative first derivative when $H > H^*$.

The second derivate of equation (2), which is the first derivative of equation (4), is given by equation (5) as follows:

$$\frac{d^2\,LUI\,(H_t)}{\left(d\,H_t\right)^2} = \frac{\lambda(\lambda-1)H_t^{\lambda-2}}{\left(H_t^{Max}\right)^{\lambda}}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{1-\lambda} - \frac{\lambda H_t^{\lambda-1}}{\left(H_t^{Max}\right)^{\lambda+1}}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{-\lambda}$$
$$- \frac{\lambda(1-\lambda)H_t^{\lambda-1}}{H_t^{Max}}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{-\lambda} - \frac{\lambda(1-\lambda)H_t^{\lambda}}{\left(H_t^{Max}\right)^{\lambda+2}}\left[\frac{H_t^{Max} - H_t}{H_t^{Max}}\right]^{-(\lambda+1)} \qquad (5)$$

$$+\ \frac{\lambda(\lambda-1)H_t^{\lambda-2}}{\left(H_t^{Max}\right)^{\lambda}}\left[\left(\frac{H_t^{Max}-H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{1-\lambda}-\frac{\lambda H_t^{\lambda-1}}{\left(H_t^{Max}\right)^{\lambda+1}}\left[\left(\frac{H_t^{Max}-H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{-\lambda}$$

$$-\ \frac{\lambda H_t^{\lambda-1}}{\left(H_t^{Max}\right)^{\lambda+1}}\left[\left(\frac{H_t^{Max}-H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{-\lambda}-\frac{\lambda H_t^{\lambda}}{\left(H_t^{Max}\right)^{\lambda+2}}\left[\left(\frac{H_t^{Max}-H_t}{H_t^{Max}}\right)\left(\frac{1}{1-\lambda}\right)\right]^{-(\lambda+1)}$$

Property 6 requires that the *LUI* must be strictly concave, such that its second derivative is always negative. Note that $0 < \lambda < 1$, so that $(1 - \lambda) < 0$. Thus, all eight terms in equation (5) are negative, and this ensures that *property 6* is satisfied. This strictly concave curvature of the *LUI* function is also illustrated in figures 1 – 3.

3. The Aggregate Index

The *LUI* can be computed by the data collection agency by obtaining the answers to two questions for all workers in the labor force. The first question is "How many hours did you desire to work?" The second question is "How many hours did you work?" Based on the answers to these two questions, the researcher can determine H^*, λ, and LUI_{it} for each individual *i* who is a in the labor force in period *t*.

Then, the monthly Aggregate Labor Utility Index (*ALUI*) can be computed by averaging the individual LUI_{it} values. Let *n* be the number of individuals in the labor force in a given sample, where the individuals are numbered $i = 1, \ldots, n$. The aggregate labor utility index value for month *t* is given by equation (6).

$$ALUI_t = 100\ \frac{\sum_{i=1}^{n} LUI_{it}}{n} \qquad i = 1, \ldots, n \qquad 0 < ALUI_t \leq 100 \tag{6}$$

The larger the value of the $ALUI_t$, the better the economic performance of the labor market in month *t*. This aggregate index can be compared across regions, demographic groups, countries, and over time. Since the *LUI* in equation (2) satisfies *property 8*, the maximum value is $ALUI_t = 100$. This would occur if every person in the labor force was working their optimally desired number of hours, so that $LUI_{it} = 1$ for each worker, $i = 1, \ldots, n$. Since *property 9* is satisfied, the minimum value is $ALUI_t = 0$. This would occur if every individual were unemployed, since this would result in $LUI_{it} = 0$ for each worker, $i = 1, \ldots, n$.

The *natural unemployment rate*, where only normal frictional job turnover is present, is about 5% for the United States labor force. If 5% of the workforce is unemployed, and the remaining individuals are each working their optimally desired number of hours, then the aggregate index will have the value $ALUI_t = 95$. Thus, the largest upperbound value that the aggregate index that could consistently be achieved in the U.S. over the long-run is 95.

Consider the following examples. Suppose that there are an equal number of case (1), case (2), and case (3) workers. Assume that half of the case (1) workers are working full-time at H^* = 171.4 monthly hours, and the other half are 40 hours underemployed, working only 131.4 hours. Assume that half of the case (2) individuals are 40 hours underemployed, and are working 45.7 monthly hours; and, the other half are working 125.7 hours, and thus are overemployed by 40 hours relative to their preference. Finally, assume that half of the case (3) individuals are working 217.1 hours (40 underemployment hours), and the other half are working 297.1 hours (40 overemployment hours) per month. In this example, the aggregate index would be given by $ALUI_t = 99.0737$.

All else constant, if the underemployed workers in the case (1) group were unemployed, rather than just underemployed, then the index would be $ALUI_t = 82.5669$. This demonstrates that unemployment will have a significant impact on the index. All else constant, if these same case (1) workers who were originally underemployed were working 211.4 monthly hours (which is 40 overemployment hours), then the index value would be $ALUI_t = 99.1049$.

4. Conclusion

The employment data collection agencies, such as the U.S. BLS, can directly measure the current welfare level of a given regional or national labor market that incorporates the dual allocation problems of underemployment and overemployment. This requires having each worker answer two questions within the monthly survey regarding the specifics of (1) the number of hours worked during the month, and (2) the desired number of hours of monthly work. Data on the number of hours per worked per month is already collected internationally by various agencies, including the U.S. BLS CPS; and, Jacobs (1994) concluded that worker's self-reported data on workings is reasonably reliable. The desired number of monthly hours worked data does not provide any *a priori* reasons to believe that it would be biased, although follow-up studies would be recommended, once the question is implemented. After collecting the monthly data from the answers to these two questions, the methods developed in this paper can then be employed to obtain the *LUI* for each individual in the survey. These *LUI* statistics can be used to compute the aggregate *ALUI* measures that can track the labor market over time, thus providing information on economic performance for workers as an aggregate and for subgroups based on regional demographic characteristics.

Note

(1) The BLS defines U1-U6 as follows: "U-1: Persons unemployed 15 weeks or longer, as a percent of the civilian labor force; U-2: Job losers and persons who completed temporary jobs, as a percent of the civilian labor force; U-3: Total unemployed, as a percent of the civilian labor force (official unemployment rate); U-4: Total unemployed plus discouraged workers, as a percent of the civilian labor force plus discouraged workers; U-5: Total unemployed, plus discouraged workers, plus all other persons marginally attached to the labor force, as a percent of the civilian labor force plus all persons marginally attached to the labor force; U-6:Total

unemployed, plus all persons marginally attached to the labor force, plus total employed part time for economic reasons, as a percent of the civilian labor force plus all persons marginally attached to the labor force."
Source: http://www.bls.gov/news.release/empsit.t15.htm. Retrieved on 2-22-2015.

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