Theoretical and Applied Economics
Volume XXV (2018), No. 2(615), Summer, pp. 27-50

Corruption, governments' debts, trade, and global growth

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Abstract. *The purpose of the study is to examine interdependence between national growth, global growth, income and wealth distributions between workers and officials and between countries, government's debts, and corruption. We are especially concerned with how corruption in any economy can affect global trade pattern and global economic growth. The model is constructed within a dynamic general equilibrium framework on the basis of the Solow growth model, the Oniki-Uzawa-two-country global growth model, Diamond's growth model with government's debt, and Zhang's model with corruption. The dynamic interdependence between endogenous labor supply, government's debt, and corruption for a-country world economy is described by nonlinear differential equations. We simulate the model and conduct comparative dynamic analyses conducted for a 3-country global economy. We get many insights from comparative dynamic analyses. For instance, if they become more corrupt, the officials in the world become richer, consume more, and get more corrupt incomes. All the workers have less wealth, consume less, and work almost the same hours. The output levels are slightly affected and each country employs more capital as global wealth is increased. Each government has slightly less debt and spends more.*

Keywords: corruption, trade pattern, government's debts, tradable and non-tradable, economic growth.

JEL Classification: F11, O41.

1. Introduction

Corruption has been well recognized as an important factor for analyzing economic growth (e.g., Lui, 1996). There are some theoretical studies on corruption and economic growth (Becker, 1968; Rose-Ackerman, 1999; Shi and Temzelides, 2004; Dzhumashev, 2014; D'Agostino et al., 2016). There are only some theoretical studies about how corruption affect global economic growth and issues related to income and wealth distribution. There are also many studies applying different models to empirically examine corruption and economic growth (e.g., Mauro, 1995, 1998; Ehrlich and Lui, 1999; Del Monte and Papagni, 2001; Rivera-Batiz, 2001; Glaeser and Saks, 2006; Swaleheen and Stansel, 2007; Teles, 2007; Gyimah-Brempong, 2002; Chea, 2015). There are different aspects of corruption and different researchers emphasize different aspects of corruption and economic growth. The situation is described by Chea (2015: 187) as follows: "From the theoretical background and empirical evidence from various studies spread in various countries in the world using different methods over the years, there are mostly negative findings of economic growth. Nevertheless, there are also positive findings found on the effects of corruption on economic growth." An early well-mentioned possible positive effects of corruption on economic development was mentioned by (Leff, 1964). Huntington is well known for emphasis on "efficient corruption". Through bribery some firms might get and complete projects more quickly by passing over bureaucratic complexity (Mo, 2001). Possibly negative impact is obvious. Myrdal (1968) provided an example. For instance, corruption makes the economic system less effective if officials delay permissions of projects in order to get bribes. Dzhumashev (2014: 203) summarizes the review on the literature on research of corruption: "the existing literature lacks a more general approach for interpreting the role of governance, the size government, and the level of development in the relationship between corruption and growth." The purpose of this study is to introduce corruption into a dynamic equilibrium trade theory.

Basing on the growth model for a two-sector two-group economy by Zhang (2017), we are concerned with a multi-country global economy where each national economy is composed of one industrial sector and one public sector, and workers and officials. The main differences of this paper from Zhang's previous study is that this study is concerned with international economies. Due to power, officials get incomes not only from wages paid by the government but also takes bribes from different sources. Industrial sector is basically the same as in the Solow one-sector growth model and trade is based on the Oniki-Solow model. The introduction of public goods is influenced by Barro (1990) and Turnovsky (2000, 2004). Rather than the Ramsey utility, this study applies Zhang's utility function (Zhang, 1993, 2005, 2009).

This study also emphasizes dynamic interdependence between fiscal policies and other variables. Fiscal policy has been introduced formal economic modelling in different ways (Barro, 1990; King and Rebelo, 1990; Stokey and Rebelo, 1995; Leith and Wren, 2000; Rankin and Roffia, 2003; Zhang, 2005; Ho et al., 2007). Another important issue is about international transmission of changes in fiscal policy (Mussa, 1979; Branson and Rotemberg, 1980; and Schmid, 1982). Nevertheless, these studies are not conducted with a genuine international analytical framework. As Ganelli (2007) observed, many studies in the literature of fiscal policies are not based on rigorous economic foundations. This study makes a contribution to the literature by a trade model with fiscal policies with microeconomic bases. The basic framework of our model is based on Zhang (2012) which is influenced by contemporary neoclassical growth trade theory characterized by the Oniki-Uzawa model (Oniki and Uzawa, 1965; Ikeda and Ono, 1992; Deardorff, 1973; Ruffin, 1979; Findlay, 1984; Frenkel and Razin, 1987; Sorger, 2003).

Government debts are obviously important for understanding long-term national economic growth. National debts are complicated as they have close relations with government's social and economic activities, taxation structures, income and wealth distributions, national output, economic structures, population structure, international trade, and economic growth. Most of theoretical growth models with national debts are built within analytical frameworks with simplified taxation. Moreover, debts are explicitly related to taxation. This study attempts to make a contribution to the literature on growth with debts by considering that taxes are dependent on national debts in a competitive global economy. It should be noted that this paper synthesizes two papers recently proposed by Zhang (2012, 2017). Zhang (2012) proposes a global growth model with tradable good, national public good, and fiscal policies. Zhang (2017) deals with a national economy with corruption. Different from the growth models with the Ramsey approach, we use the alternative utility function proposed by Zhang (1993) to determine saving and consumption. The rest of the paper is organized as follows. Section 2 defines the international trade model with global economic growth, wealth and income distributions between households and between countries, governments' debts, and corruption. Section 3 provides a computational procedure to follow movement of global economy and simulates the global economy. Section 4 conducts comparative dynamic analysis to see how changes in some parameters affect dynamic paths of the global economy. Section 5 concludes the study. The appendix checks the main results in Section 3.

2. The model

The world economy produces a single industrial good. Let the price of the industrial good be unity. The global economy is composed of J countries, indexed by $j = 1, ..., J$. Country $j's$ fixed population is denoted by N_j, $j = 1, ..., J$. Each country consists of one private and one public sectors. Let prices be measured in terms of the commodity. The commodity's price is unity. The private sectors supply the homogenous commodity which is used either for consumption and investment. We describe most aspects of production sectors as in neo-classical one-sector growth model (Burmeister and Dobell 1970, Zhang, 2005, 2009). Markets are perfectly competitive. There is free trade between countries. There is no transaction cost between countries and within each national economy. Households own all assets. Households distribute their disposable incomes to consumption and saving. Industrial sectors use capital and labor. Factor markets work well. Factors are inelastically supplied and are fully employed. We use subscript index, i and p, to stand for respectively the production sector and public sector. No international migration is allowed. The fixed labor force in each country is distributed between the two sectors. Each country's population is classified into officials and workers. Each group's population is fixed and homogenous. The industrial sector employs all national workers. Officials are fully employed by the public sector. The public sector is finally supported by the government which taxes households and the industrial sector and issues debts. We introduce the following variables

q - subscripts for national economies, $q = 1, ..., Q$;

j - subscripts for workers $(j = 1)$ and officials $(j = 2)$, respectively;

$\bar{N}_{qj}$ and $\bar{N}_q$ - fixed population of group (q, j) and the total population of country q, $\bar{N}_q = \bar{N}_{q1} + \bar{N}_{q2}$;

$r(t)$ and $w_{qj}(t)$ - rate of interest and group $(q, j)'s$ wage rate per unit of time;

$T_{qj}(t)$, $\bar{T}_{qj}(t)$ - work and leisure time of group $(q, j)'s$ household;

$c_{qj}(t)$ and $s_{qj}(t)$ - consumption and saving of group $(q, j)'s$ household;

$N_{qj}(t)$ and h_{qj} - labor supply and human capital of group (q, j);

$K_j(t)$ and $K(t)$ - total capital employed by economy q and global wealth;

$a_{qj}(t)$ - value of wealth owned by group $(q, j)'s$ household;

$\tau_q(t)$ - government's tax rate on industrial sector's output;

$\tau_{aq}(t)$, $\tau_{wq}(t)$ and $\tau_{cq}(t)$ - government's tax rates on household $(q, j)'$ wealth income, wage income, and consumption;

$\bar{\tau}_{x2}(t)$ - $\bar{\tau}_{x2}(t) \equiv 1 - \bar{\tau}_{x2}(t)$, $x = a, w, c$;

ϕ_q and $\bar{\phi}_q(t)$ - fixed corruption rate on output level of industrial sector q and $\bar{\phi}_q(t) \equiv 1 - \phi_q - \tau_q(t)$;

ϕ_{aq} and $\bar{\phi}_{aq}(t)$ - corruption rate on workers' income from wealth and $\bar{\phi}_{aq}(t) \equiv 1 - \phi_{aq} - \tau_{aq}(t)$;

ϕ_{wq} and $\bar{\phi}_{wq}(t)$ - corruption rate on workers' wage income and $\bar{\phi}_{wq} \equiv 1 - \phi_{wq} - \tau_{wq}(t)$;

ϕ_{cq} and $\bar{\phi}_{cq}(t)$ - corruption rate on workers' consumption level and $\bar{\phi}_{wq} \equiv 1 - \phi_{wq} - \tau_{wq}(t)$;

δ_{kq} - depreciation rates of physical capital.

We have $N_j(t)$ as follows

$$N_{q1}(t) = h_{q1}\bar{N}_{q1}T_{q1}(t), \;\; N_{qj} = h_{qj}\bar{N}_{qj}T_{q2}, \;\; q = 1, ..., Q.$$

The industrial sector

We specify the industrial sector's production function as follows

$$F_q(t) = A_q G_q^{\theta_q} K_q^{\alpha_q}(t) N_{q1}^{\beta_q}(t), \;\; \alpha_q + \beta_q = 1, \;\; A_q, \theta_q, \alpha_q, \beta_q > 0, \tag{1}$$

where A_q, θ_q, α_q and β_q are parameters. We take the approach different from Chen and Guo (2014) but similar to de Vaal and Ebben (2011) in that national public goods $G_q(t)$ has positive impact on productivity. We ignore possible corruption on firms' capital and labor force and assume positive corruption rate on product of the industrial sector. The marginal conditions imply

$$r(t) + \delta_{qk} = \frac{\alpha_q \bar{\phi}_q(t) F_q(t)}{K_q(t)}, \;\; w_{q1}(t) = \frac{\beta_q \bar{\phi}_q(t) F_q(t)}{N_{q1}(t)}. \tag{2}$$

Workers' disposable income budget constraint

The household wealth $a_{qj}(t)$ is composed of the value of physical wealth $\bar{k}_{qj}(t)$ and debts $d_{qj}(t)$

$$a_{qj}(t) = \bar{k}_{qj}(t) + d_{qj}(t).$$

If there is no corruption on the worker, the worker's current income is $r(t)a_{q1}(t) + w_{q1}(t)h_{q1}T_{q1}(t)$. The total value of wealth that the representative household is $a_{q1}(t)$. This value can used to purchase goods and to save. Here, it is assumed that selling and buying wealth can be done without any transaction cost and time delay. The disposable income is

$$a_{q1}(t) + r(t)a_{q1}(t) + h_{q1}w_{q1}(t)T_{q1}(t).$$

The representative household pays taxes to the government and corruption fees to the officials. The representative household's disposable income $\hat{y}_{ql}$ is thus given by

$$\hat{y}_{q1}(t) \equiv (1 + r(t))\bar{\phi}_{qa}(t)a_{q1}(t) + h_{q1}\bar{\phi}_{qw}(t)w_{q1}(t)T_{q1}(t). \tag{3}$$

The worker's disposable income is distributed between saving $s_{q1}(t)$ and consumption $c_{q1}(t)$. The budget constraint is

$$(1 + \tau_{cq}(t))c_{q1}(t) + s_{q1}(t) = \hat{y}_{q1}(t). \tag{4}$$

This equation states that the consumers' disposable income is fully spent on consumption and savings. The time constraint for the worker is

$$T_{q1}(t) + \overline{T}_{q1}(t) = T_0 . \qquad (5)$$

Insert (5) and (3) in (4)

$$(1 + \tau_{cq}(t))c_{q1}(t) + s_{q1}(t) + \overline{w}_{q1}(t)\overline{T}_{q1}(t) = \overline{y}_{q1}(t) \equiv \overline{\phi}_{aq}(t)R(t)a_{q1}(t) + \overline{w}_{q1}(t)T_0 , \qquad (6)$$

where

$$R(t) \equiv 1 + r(t), \ \ \overline{w}_{q1}(t) \equiv \overline{\phi}_{wq}(t)h_{q1}\, w_{q1}(t).$$

The worker's utility function and optimal decision

The representative worker chooses three variables, $c_{q1}(t)$, $s_{q1}(t)$ and $\overline{T}_{q1}(t)$ subject to the budget constraint. The utility level $U_{q1}(t)$ is dependent on the three variables as follows

$$U_{q1}(t) = u_{q1}(G_q(t), t)\overline{T}_{q1}^{\,\widetilde{\sigma}_{q1}}(t)c_{q1}^{\widetilde{\xi}_{q1}}(t)s_{q1}^{\widetilde{\lambda}_{q1}}(t), \ \ \widetilde{\sigma}_{q1}, \widetilde{\xi}_{q1}, \widetilde{\lambda}_{q1} > 0 ,$$

where u_{q1} is a time-dependent variable, $\widetilde{\sigma}_{qj}$, $\widetilde{\xi}_{qj}$ and $\widetilde{\lambda}_{qj}$ are called respectively the worker's propensity to stay at home, the propensities to consume good and to hold wealth. Maximize U_{qj} subject to (6)

$$\overline{w}_{q1}(t)\overline{T}_{q1}(t) = \sigma_{q1}\,\overline{y}_{q1}(t), \ \ c_{q1}(t) = \xi_{q1}(t)\overline{y}_{q1}(t), \ \ s_{q1}(t) = \lambda_{q1}\,\overline{y}_{q1}(t), \qquad (7)$$

where

$$\rho_{q1} \equiv \frac{1}{\widetilde{\sigma}_{q1} + \widetilde{\xi}_{q1} + \widetilde{\lambda}_{q1}}, \ \ \sigma_{q1} \equiv \rho_{q1}\,\widetilde{\sigma}_{q1}, \ \ \xi_{q1}(t) \equiv \frac{\rho_{q1}\,\widetilde{\xi}_{q1}}{1 + \tau_{qc}(t)}, \ \ \lambda_{q1} \equiv \rho_{q1}\,\widetilde{\lambda}_{q1} .$$

Officials' disposable income and budget constraint

Let $\overline{k}_{q2}(t)$ stand for the capital wealth owned by the representative official. We assume that the representative official's wage income per unit of qualified work time is paid in proportion to the worker's wage rate as follows

$$\overline{w}_{q2}(t) = u_q\, h_{q2}\, w_{q1}(t), \qquad (8)$$

where $u_q > 0$ is fixed by the government. The official wage rate is proportional to the worker's market wage rate. The representative official gets the following income via corruption

$$w_{cq}(t) \equiv \frac{\phi_q(t)F_q(t) + \phi_{aq}\, R(t)a_{q1}(t)\overline{N}_{q1} + \phi_{wq}\, h_{q1}\, w_{q1}(t)T_{q1}(t)\overline{N}_{q1}}{\overline{N}_{q2}}. \qquad (9)$$

The representative official's disposable income $\hat{y}_{q2}$ is

$$\hat{y}_{q2}(t) \equiv \bar{\tau}_{a2}(t)R(t)a_{q2}(t) + \bar{\tau}_{w2}(t)\bar{w}_{q2}(t)T_{q2} + w_{cq}(t). \tag{10}$$

The official allocates the disposable income between saving $s_{q2}(t)$ and consumption $c_{q2}(t)$. The budget constraint is

$$(1 + \tau_{cq})c_{q2}(t) + s_{q2}(t) = \hat{y}_{q2}(t). \tag{11}$$

The official's utility functions and optimal decisions

The representative official decides two variables, $c_{q2}(t)$, and $s_{q2}(t)$ subject to budget constraint. The utility level $U_{qj}(t)$ is related to the two variables as follows

$$U_{qj}(t) = u_{qj}(G_q(t), t)c_{qj}^{\tilde{\xi}_{qj}}(t)s_{qj}^{\tilde{\lambda}_{qj}}(t), \ \ \tilde{\xi}_{qj}, \tilde{\lambda}_{qj} > 0,$$

where u_{qj} is a time-dependent variable, $\tilde{\xi}_{qj}$ and $\tilde{\lambda}_{qj}$ are called respectively the official's propensities to consume good and to hold wealth. Maximize U_{q2} subject to (11)

$$c_{q2}(t) = \xi_{q2}(t)\bar{y}_{q2}(t), \ \ s_{q2}(t) = \lambda_{q2}\,\bar{y}_{q2}(t), \tag{12}$$

where

$$\rho_{q2} \equiv \frac{1}{\tilde{\xi}_{q2} + \tilde{\lambda}_{q2}}, \ \ \xi_{q2}(t) \equiv \frac{\rho_{q2}\,\tilde{\xi}_{q2}}{1 + \tau_{cq}(t)}, \ \ \lambda_{q2} \equiv \rho_{q2}\,\tilde{\lambda}_{q2}.$$

It should be remarked that our approach does not make the assumption of identical preference across the countries. Traditional neoclassical growth trade theory often requires that countries have the same preference. As pointed by Bianconi and Turnovsky (1997: 61), "We shall assume that the rate of time discount of the foreign resident equals that of the domestic agent. As is well known, under the assumption of perfect foresight and perfect capital markets, this assumption is necessary in order for a well-defined steady state to exist." The assumption in the traditional approach is obviously too strict.

The wealth accumulation

According to the definitions of $s_{qj}(t)$ and $a_{qj}(t)$, the change in the household's wealth is given by

$$\dot{a}_{qj}(t) = s_{qj}(t) - a_{qj}(t). \tag{13}$$

This equation simply tells that wealth change is equal to saving minus dissaving.

The government incomes, expenditures, and public service supply

The public sector is financially supported by the government. Government q's tax income is

$$I_q(t) = \tau_q F_q(t) + \tau_{wq}(t) h_{q1} w_{q1}(t) T_{q1}(t) \bar{N}_{q1} + \tau_{wq}(t) \bar{w}_{q2}(t) T_{q2} \bar{N}_{q2} + \sum_{j=1}^{2} \left[\tau_{cq}(t) c_{qj}(t) + \tau_{aq}(t) R(t) a_{qj}(t) \right] \bar{N}_{qj} . \tag{14}$$

We assume that the tax rates in each national economy are nonlinearly related to its debt as follows

$$\tau_q(t) = \hat{\tau}_q \exp[\tilde{\tau}_q D_q(t)], \quad \tau_{qx}(t) = \hat{\tau}_{qx} \exp[\tilde{\tau}_{qx} D_q(t)], \quad x = w, c, a. \tag{15}$$

This specified form implies that if a government's is high, it tends to have higher tax rates producers and consumers. It should be noted that in the Diamond model and other growth models with debts, it is often assumed that tax rates are constant. We assume (15) so that debts will not be arbitrarily high. We assume that the service that the public provides is dependent only on the officials' labor input N_{q2}, described by the following public sector production function

$$G_q = A_{pq} N_{q2}^{\gamma_q}, \quad A_{pq}, \gamma_q > 0. \tag{16}$$

Government $q's$ expenditure is

$$Y_q(t) = \bar{w}_{q2}(t) T_{q2} \bar{N}_{q2} . \tag{17}$$

For simplicity of analysis we don't introduce possible corruption of the officials on governance and government expenditures. The interaction between corruption and governance may affect the efficiency of public spending (e.g., Dzhumashev, 2014).

The dynamics of governments' debts

The governments' debts can be owned by domestic as well as foreign households. Country j's government debt follows the following dynamics

$$\dot{D}_q(t) = r(t) D_q(t) + Y_q(t) - I_q(t).$$

Wealth being owned by households

The sum of the wealth owned by the two groups is the national wealth

$$a_q(t) = a_{q1}(t) \bar{N}_{q1} + a_{q2}(t) \bar{N}_{q2} . \tag{18}$$

The global wealth balances

$$\sum_{q=1}^{Q} a_q(t) = K(t) + \sum_{q=1}^{Q} D_q(t). \tag{19}$$

$$\sum_{q=1}^{Q} \left[d_{q1}(t) N_{q1} + d_{q2}(t) N_{q2} \right] = \sum_{q=1}^{Q} D_q(t). \tag{20}$$

Full employment of global capital implies

$$\sum_{q=1}^{Q} K_q(t) = K(t). \tag{21}$$

The total demand for the product is equal to the total supply. The change in the total capital is equal to the global output minus the global consumption and total depreciations

$$\dot{K}(t) = \sum_{q=1}^{Q} F_q(t) - \sum_{q=1}^{Q} \left(\sum_{j=1}^{2} c_{qj}(t) \overline{N}_{qj} \right) - \sum_{q=1}^{Q} \delta_{qk} K_q(t). \tag{22}$$

According to the concepts, we have the national debt $\overline{D}_j(t)$ as follows

$$\overline{D}_j(t) = D_j(t) + K_j(t) - a_j(t). \tag{23}$$

We built a multi-country model with trade, economic growth, physical distribution, governments' debts, and national debts, and corruption in the world economy in which the domestic markets of each country are perfectly competitive, international product, capital markets are freely mobile, and governments' policies on public good supplies are endogenous. The model integrates some of main ideas in economic growth theory and trade theory in a comprehensive framework.

3. The dynamics and equilibrium

The economic system is composed of nonlinear dynamic relations between many variables. It is impossible to get general analytical solutions of such complicated nonlinear different equations. Nevertheless, it is not a difficult task to follow motion of the system if we specify functional forms, parameter values, and initial conditions. In order to simulate the model, this section provides a computational procedure. First, we introduce a variable $z_1(t)$ by

$$z_1(t) \equiv \frac{r(t) + \delta_{k1}}{w_{11}(t)}.$$

We now show that the dynamics can be expressed by differential equations with $z_1(t)$, $\{a_{q1}(t)\} \equiv (q_{21}(t), ..., q_{Q1}(t))$, $(a_{q2}) \equiv (q_{12}(t), ..., q_{Q2}(t))$, and $(D_j) \equiv (D_1(t), ..., D_Q(t))$ as variables.

Lemma

The motion of $3Q$ variables, $z_1(t)$, $\{a_{q1}(t)\}$, $(a_{q2}(t))$, and $(D_j(t))$ is given by the following $3Q$ differential equations

$$\dot{z}_1(t) = \Lambda_{11}(z_1(t), \{a_{q1}(t)\}, (a_{q2}(t)), (D_q(t))),$$

$$\dot{a}_{qj}(t) = \Lambda_{qj}\left(z_1(t), \{a_{q1}(t)\}, (a_{q2}(t)), (D_q(t))\right), \ \ q = 2, ..., Q, \ \ j = 1, 2,$$
$$\dot{D}_q(t) = \Psi_q\left(z_1(t), \{a_{q1}(t)\}, (a_{q2}(t)), (D_q(t))\right), \ \ q = 1, ..., Q. \tag{24}$$

where $\Lambda_{qj}(t)$ and $\Psi_{qj}(t)$ are functions of $z_1(t)$, $\{a_{q1}(t)\}$, $(a_{q2}(t))$, and $(D_j(t))$ defined in the appendix. The values of the other variables are given as functions of $z_1(t)$, $\{a_{q1}(t)\}$, $(a_{q2}(t))$, and $(D_j(t))$ at any point in time by the following procedure:

$r(t)$ by (A2) $\rightarrow$ $z_j(t)$ by (A3) $\rightarrow$ $w_{q1}(t)$ by (A2) $\rightarrow$ $\bar{w}_{q1}(t)$

by definition $\rightarrow$ $\bar{w}_{q2}(t)$ by (8) $\rightarrow$ $\tau_q(t)$ and $\tau_{q1}(t)$ $\rightarrow$ $N_{q2}(t)$

by definition $\rightarrow$ $G_q(t)$ by (16) $\rightarrow$ $a_{11}(t)$ by (A15) $\rightarrow$ $K_q(t)$

by (A12) $\rightarrow w_{cq}(t)$ by (A8) $\rightarrow$ $\hat{y}_{q2}(t)$ by (A9) $\rightarrow$ $c_{q2}(t)$ and $s_{q2}(t)$ by (12) $\rightarrow$ $F_q(t)$

by (A7) $\rightarrow$ $N_{q1}(t)$ by (A6) $\rightarrow$ $Y_q(t)$ by (17) $\rightarrow$ $a_q(t)$ by (18) $\rightarrow$ $\bar{y}_{qj}(t)$ by (6) $\rightarrow$ $c_{q1}(t)$, $\bar{T}_{q1}(t)$, and $s_{q1}(t)$ by (7) $\rightarrow$ $T_{q1}(t) = T_0 - \bar{T}_{q1}(t)$ $\rightarrow$ $I_q(t)$ by (14) $\rightarrow$ $K(t)$ by (21).

The lemma guarantees us that we can use computer to follow the motion of global economic system at any point in time. We consider that the world is composed of three national economies, i.e., $Q = 3$. We specify the following relations between the tax rates and national debts

$$\tau_q(t) = 0.01e^{0.0003D_q(t)}, \ \ \tau_{qa}(t) = 0.001e^{0.0003D_q(t)}, \ \ q = 1, 2, 3, \ \ x = a, w, c. \tag{25}$$

The specified functions imply that if debts are not very high, tax rates are slightly affected by debts. But if debts are high, governments will increase tax rates. We specify the parameter values as follows

$$T_0 = 24, \ \bar{N}_{11} = 300, \ \bar{N}_{12} = 30, \ T_{12} = 8, \ h_{11} = 2, \ h_{12} = 3, \ \bar{N}_{21} = 200, \ \bar{N}_{22} = 20,$$
$$T_{22} = 8, \ h_{21} = 3, \ h_{22} = 4, \ \bar{N}_{31} = 100, \ \bar{N}_{32} = 10, \ T_{12} = 8, \ h_{11} = 5, \ h_{12} = 6, \ \alpha_1 = 0.3,$$
$$\alpha_2 = 0.32, \ \alpha_3 = 0.34, \ A_1 = 1, \ A_2 = 1.1, \ A_3 = 1.3, \ \gamma_1 = 0.5, \ \gamma_2 = 0.6, \ \gamma_7 = 0.7,$$
$$A_{p1} = 1, \ A_{p2} = 1.2, \ A_{p3} = 1.4, \ u_1 = 1.5, \ h_2 = 1.3, \ h_3 = 1.1, \ \theta_1 = 0.3, \ \theta_2 = 0.35,$$
$$\theta_3 = 0.4, \ \varphi_1 = \varphi_{x1} = 0.01, \ \varphi_2 = \varphi_{x2} = 0.007, \ \varphi_3 = \varphi_{x3} = 0.005, \ x = a, w, \ \tilde{\lambda}_{12} = 0.6,$$
$$\tilde{\xi}_{12} = 0.1, \ \tilde{\lambda}_{22} = 0.65, \ \tilde{\xi}_{22} = 0.1, \ \tilde{\lambda}_{32} = 0.7, \ \tilde{\xi}_{32} = 0.1, \ \tilde{\lambda}_{11} = 0.6, \ \tilde{\xi}_{11} = 0.1,$$
$$\tilde{\sigma}_{11} = 0.15, \ \tilde{\lambda}_{21} = 0.6, \ \tilde{\xi}_{21} = 0.12, \ \tilde{\sigma}_{21} = 0.17, \ \tilde{\lambda}_{31} = 0.55, \ \tilde{\xi}_{31} = 0.12, \ \tilde{\sigma}_{31} = 0.2,$$
$$\delta_{k1} = 0.06, \ \delta_{k2} = \delta_{k3} = 0.05. \tag{26}$$

Country 1, 2 and 3's worker populations are respectively 300, 200 and 100; Country 1, 2 and 3's official populations are respectively 30, 20 and 10. Country 1 has the lowest human capital among the corresponding groups in the world; Country 3 has the highest human capital. The initial conditions are as follows

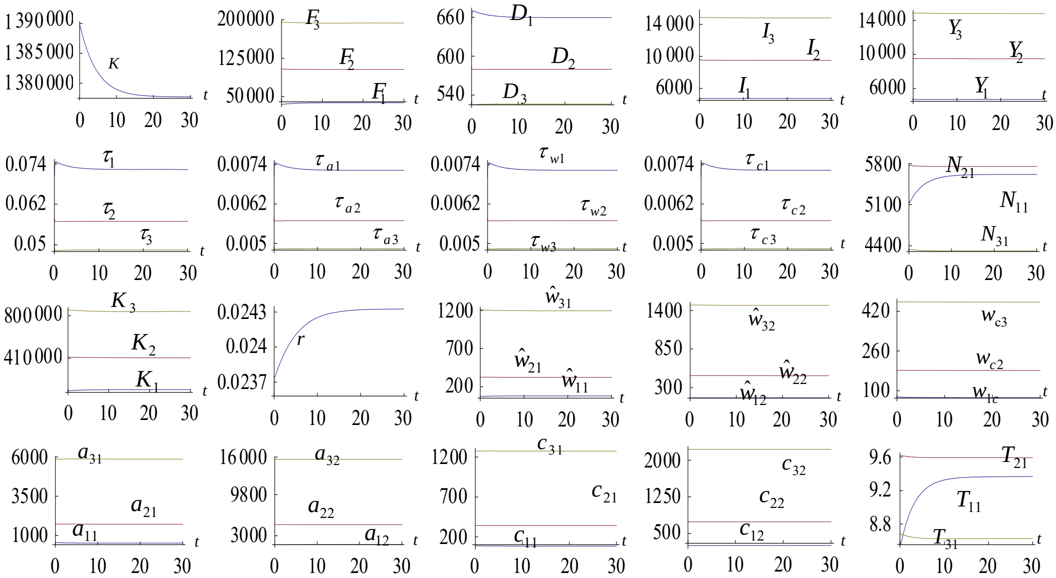
$$z_1(0) = 0.038,\ \ a_{21}(0) = 1572,\ \ a_{31}(0) = 5145,\ \ a_{12}(0) = 2284,\ \ a_{22}(0) = 7080,$$
$$a_{32}(0) = 23535,\ \ D_1(0) = 480,\ \ D_2(0) = 404,\ \ D_3(0) = 337.$$

The motion of the system is given in Figure 1. In the figure the disposable wage incomes are defined as follows

$$\hat{w}_{q1}(t) \equiv \bar{\varphi}_{wq}(t) h_{q1}\, \bar{w}_q(t) T_{q1}(t),\ \ \hat{w}_{q2}(t) \equiv \bar{\tau}_{wq}(t) \bar{w}_{q2}(t) T_{q2}.$$

As our initial conditions are not far from the long-term equilibrium point, the variables change not very much during the simulation period. The simulation does not show long-term convergence in the national incomes, per capita income, wage rates and wealth in the long term. It should be noted that issues related to income and wealth convergence between nations are often addressed in the literature of economic growth and development. Nevertheless, the theoretical literature has failed to provide important insights on global income and wealth distribution and global growth. A main reason is the lacking of a proper theoretical framework. Most of theoretical discussions based on the insights from analyzing models of closed economies (e.g., Barro and Sala-i-Marti, 1995; Barro, 2013).

Figure 1. *The motion of the global economy*



Following the lemma and (23), we calculate the equilibrium values of the variables as follows

$K = 1377730$, $r = 0.024$,

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 38211 \\ 103091 \\ 193434 \end{pmatrix}, \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} 659 \\ 580 \\ 526 \end{pmatrix}, \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 4734 \\ 9499 \\ 14810 \end{pmatrix}, \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 4718 \\ 9485 \\ 14797 \end{pmatrix}, \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} 0.072 \\ 0.057 \\ 0.049 \end{pmatrix},$$

$$\begin{pmatrix} \tau_{a1} \\ \tau_{a2} \\ \tau_{a3} \end{pmatrix} = \begin{pmatrix} \tau_{w1} \\ \tau_{w2} \\ \tau_{w3} \end{pmatrix} = \begin{pmatrix} \tau_{c1} \\ \tau_{c2} \\ \tau_{c3} \end{pmatrix} = \begin{pmatrix} 0.0047 \\ 0.006 \\ 0.005 \end{pmatrix}, \begin{pmatrix} N_{11} \\ N_{21} \\ N_{31} \end{pmatrix} = \begin{pmatrix} 5619 \\ 5756 \\ 4312 \end{pmatrix}, \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 124751 \\ 415459 \\ 837518 \end{pmatrix},$$

$$\begin{pmatrix} \hat{w}_{11} \\ \hat{w}_{21} \\ \hat{w}_{31} \end{pmatrix} = \begin{pmatrix} 80.4 \\ 324 \\ 1197 \end{pmatrix}, \begin{pmatrix} \hat{w}_{12} \\ \hat{w}_{22} \\ \hat{w}_{32} \end{pmatrix} = \begin{pmatrix} 156 \\ 472 \\ 1473 \end{pmatrix}, \begin{pmatrix} w_{c1} \\ w_{c2} \\ w_{c3} \end{pmatrix} = \begin{pmatrix} 72.4 \\ 182.2 \\ 457.6 \end{pmatrix}, \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} = \begin{pmatrix} 503 \\ 1717 \\ 5867 \end{pmatrix},$$

$$\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} = \begin{pmatrix} 1526 \\ 4830 \\ 15629 \end{pmatrix}, \begin{pmatrix} c_{11} \\ c_{21} \\ c_{31} \end{pmatrix} = \begin{pmatrix} 83.2 \\ 314 \\ 1274 \end{pmatrix}, \begin{pmatrix} c_{12} \\ c_{22} \\ c_{32} \end{pmatrix} = \begin{pmatrix} 252.5 \\ 738.8 \\ 2222 \end{pmatrix}, \begin{pmatrix} T_{11} \\ T_{21} \\ T_{31} \end{pmatrix} = \begin{pmatrix} 9.4 \\ 9.6 \\ 8.6 \end{pmatrix}. \tag{27}$$

It is straightforward to calculate the nine eigenvalues as follows

$$\{-78.3, -47.5, -47.5 \pm 0.5, -8.97, -0.34, -0.31, -0.14, -0.12, -0.11\}.$$

The negative parts of all the eigenvalues implies a stable equilibrium point. This result is important as it guarantees the efficiency of comparative dynamic analysis in the long run as the system will not move along a stable path over time. We conduct comparative dynamic analysis to follow transitory processes and long-term equilibrium when different exogenous events happen in the global economy.

4. Comparative dynamic analysis

We now conduct comparative dynamic analysis to examine questions such as how a change in corruption affects national economies and global economy in transitory processes and long-term equilibrium. For instance, if a country's officials increase their corrupt charges on firms, it is reasonable to expect that different aspects of national and international economies will be affected. We can easily answer this kind of questions as our model is a dynamic general equilibrium model and we provide a computational procedure to follow the movement of the dynamic system with computer. We first introduce a variable $\overline{\Delta} x(t)$ to stand for the change rate of the variable $x(t)$ in percentage due to changes in parameter values.

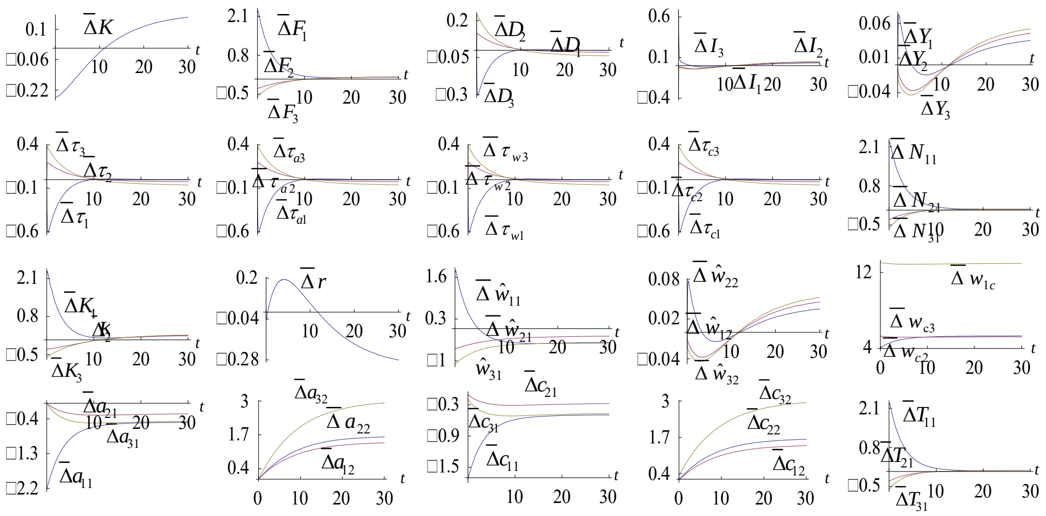
4.1. All the corruption rates on the wage incomes rise

There are different combinations of corruption in the global economy. For instance, it is possible to have a case that some countries more corrupt in labor markets while some other countries become more transparent and more fairly competitive. We are now concerned with a case that all the labor markets become more corrupt. We study what happen to the global economic system if all the countries' officials become more corrupt in the following way

$$\varphi_{w1} : 0.01 \Rightarrow 0.015, \quad \varphi_{w2} : 0.007 \Rightarrow 0.01, \quad \varphi_{w3} : 0.005 \Rightarrow 0.01. \qquad (28)$$

The simulation result under (28) is plotted in Figure 2. As the world becomes more corrupt, all the officials in the global economy become richer and consume more. All the officials' corrupt incomes are increased. All the workers have less wealth and consume. The workers in country 1 work longer hours initially and almost the same hours in the long term. The workers in the other two countries country 1 work longer hours initially and almost the same hours in the long term. In the long term no country's labor force is changed much. In the long term workers' wage incomes fall and officials' wage income rise in the long term. Country 1 employs more capital and produce more initially, while the other two economies employ less capital and produce less initially. In the long term the output levels are slightly affected and each country employs slightly a little more capital. Initially country 1's government has less debts and other two countries' have more debts. In the long term each government has slightly less debt and spends slightly more. The tax rates are changed in the same direction as their country's government debt. It should be noted that with regard to why global physical capital is increased in the long term, one of the forces which enable global capital to rise is that the enforced corruption changes income and wealth distribution. As we assume that the officials have higher propensities to save than the workers within each country, societies tend to save more as wealth is accumulated to the groups who have higher propensities to save. As officials get more shares from the national income, each national economy tends to save more. As the economy is characterized of neoclassical economic growth, higher propensities to save leads to higher equilibrium level of capital.

Figure 2. *All the corruption rates on the wage incomes rise*



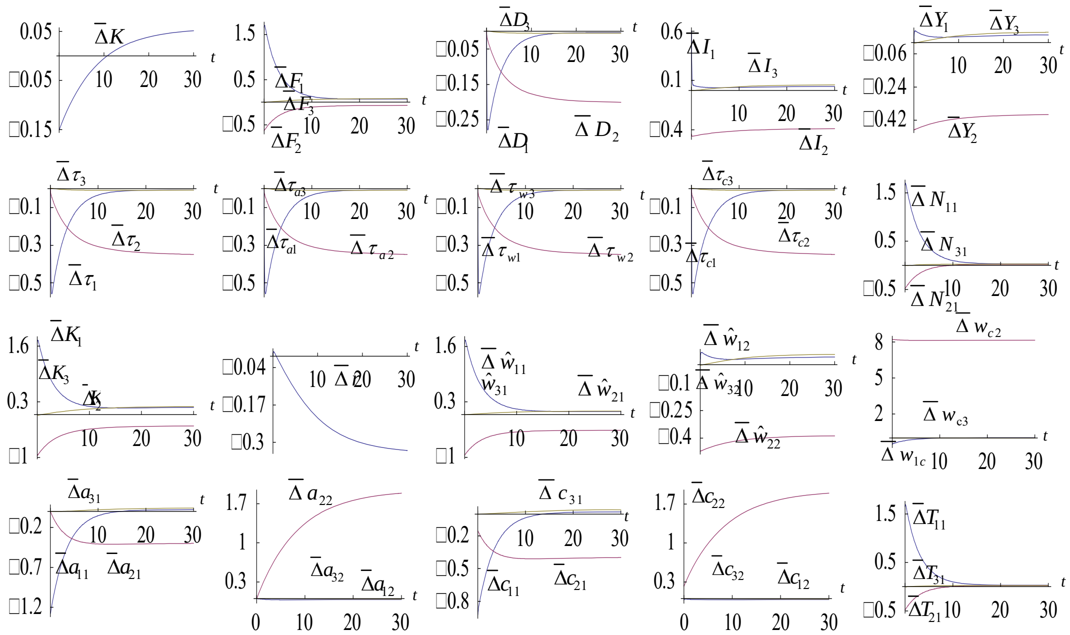
4.2. Country 2's corruption rate on the production sector rises

We now show how the following rise in country 2's corruption rate affects economic development of nations and the global economic system

$$\varphi_2 : 0.007 \;\Rightarrow\; 0.01 . \qquad (29)$$

The impact is plotted in Figure 3. Country 2 has lower output level and employs less capital. More corruption in country 2 makes the other two economies to produce more and employ more capital. The more corrupt economy uses less capital and produces less output. The global capital stock falls initially and rises in the long term. The workers in country 2 have less wealth and consume less and the officials in the same country have more wealth and consume more. The workers in the other two economies have more wealth and consume more in the long term. The corrupt income in country 2 rises and corrupt incomes in the other two economies change slightly. All the countries' government debts fall and all the tax rates fall. The rate of interest falls. Country 2's government has lower income and spends less, while the other two countries' governments have higher incomes and spend more.

Figure 3. *Country 2's corruption rate on the production sector rises*



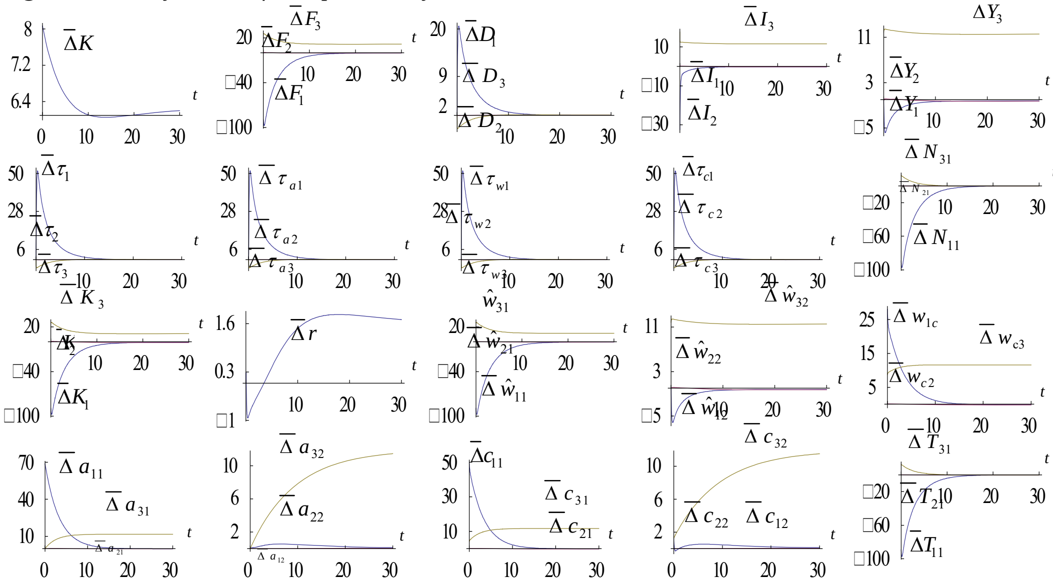
4.3. Country 3's total factor productivity is enhanced

We now show how the following rise in country 2's corruption rate affects economic development of nations and the global economic system

$$A_3 : 1.3 \Rightarrow 1.4. \tag{30}$$

The impact is plotted in Figure 4. Country 3 produces more, while the other two countries produce less. The global capital stock rises. The rite of interest falls initially and rises in the long term. Country 3 employs more capital. The other two economies employ less capital stocks initially and almost the same levels of capital in the long term. The officials' corrupt income in country 3 rises. The officials' corrupt incomes in the other two economies change slightly in the long term. Both the officials and workers have more wealth and consume more. The wealth and consumption levels of officials and workers in the other two countries are reduced.

Figure 4. *Country 3's total factor productivity is enhanced*



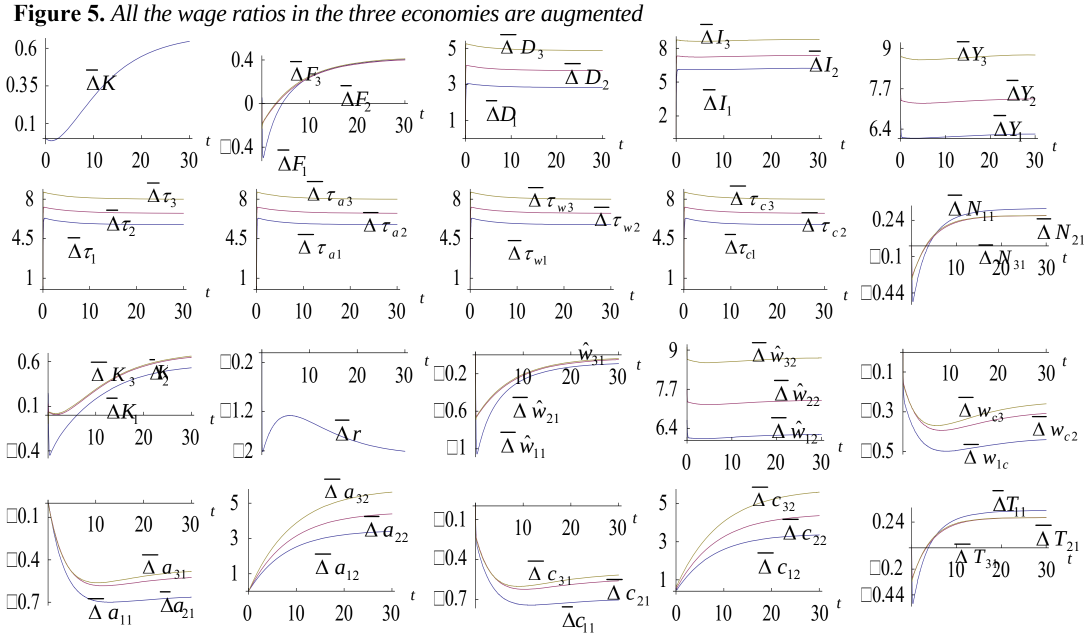
4.4. All the wage ratios in the three economies are augmented

We now examine what will happen to the global economy if all the countries increase the wage ratio between the officials and workers as follows

$$u_1 : 1.5 \Rightarrow 1.6, \;\; u_2 : 1.3 \Rightarrow 1.4, \;\; u_3 : 1.1 \Rightarrow 1.2. \tag{31}$$

The impact is plotted in Figure 5. The global capital stock is increased. The output levels of the three production sectors fall slightly initially and rise in the long term. The three economies' governments have higher incomes and spend more. The government debts are enhanced. All the tax rates are increased. The workers in the three economies work less hours initially and work more hours in the long term. The total supplies fall initially and rise in the long term. The workers have less wealth and consume less. The officials own more wealth and consume more. The wage incomes of the workers in the three economies are reduced. The wage incomes of the officials in the three economies are augmented. The corrupt incomes in the three economies are all reduced. The rate of interest falls.

Figure 5. *All the wage ratios in the three economies are augmented*



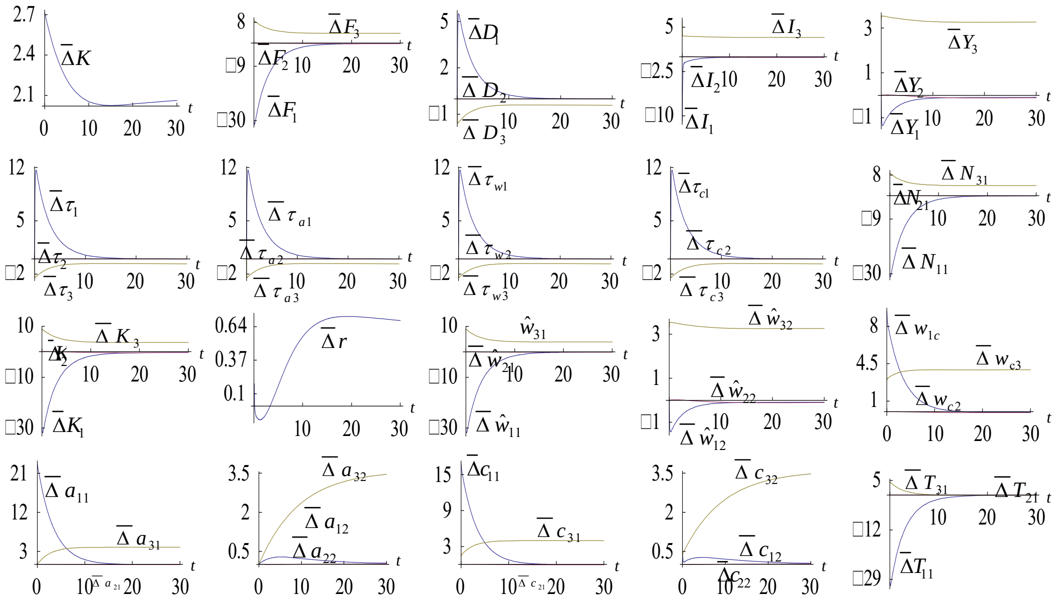
4.5. The officials' and workers' levels of human capital are enhanced in country 3

We now examine what will happen to the global economy if the officials' and workers' levels of human capital are enhanced as follows

$$h_{31}: 5 \Rightarrow 5.2, \quad h_2: 6 \Rightarrow 6.2. \qquad (32)$$

The impact is plotted in Figure 6. The global capital stock is augmented. The output level of the production sector, capital employed and government's income and expenditure are all augmented in country 3. Country 3's government debt falls. These variables are affected slightly in the long term. In country 3 the wage incomes of the two groups and the corrupt incomes are increased. In country 3 the wealth and consumption levels of the two groups are augmented.

Figure 6. *The levels of human capital are enhanced in country 3*



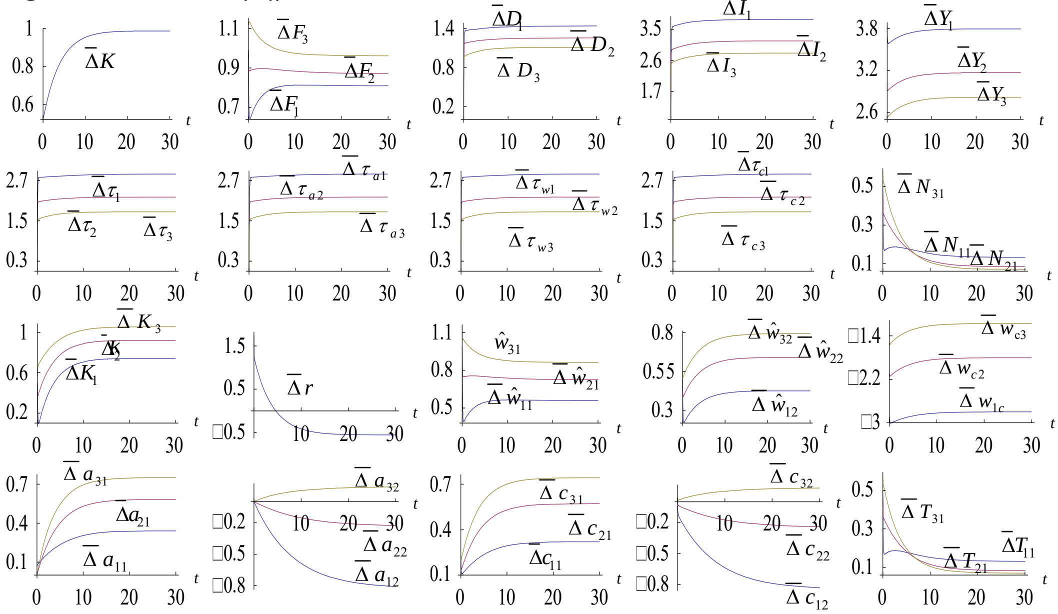
4.6. The numbers of officials are increased in the three countries

We now examine what will happen to the global economy if the numbers of officials are increased as follows

$$\bar{N}_{12}:30 \Rightarrow 31, \ \bar{N}_{22}:20 \Rightarrow 20.5, \ \bar{N}_{12}:10 \Rightarrow 10.2. \tag{33}$$

The effects are plotted in Figure 7. As each country increase its total population, the world has more total physical capital. Each country employs more capital and produces more. Each government has more debts and increases all the tax rates. Each government also has more income and spends more. The workers work more hours and each country's total labor supply is increased. The rate of interest rises initially and falls in the long term. The workers and officials all have higher wage incomes. The corrupt incomes fall in all the countries. All the workers have more wealth and consume more. Country 3's official has more wealth and consume more. The official in either of the other two countries has less wealth and consume less.

Figure 7. *The numbers of officials are increased in the three countries*



5. Concluding remarks

This study constructed a global economic growth model with any number of national economies and free trade. Our purpose is to show interdependence between national growth, global growth, income and wealth distributions between workers and officials and between countries, and government's debts with corruption. We are especially concerned with how corruption in any economy can affect global trade pattern and global economic growth. This study applied an alternative approach to behavior of household proposed by Zhang to a neoclassical growth trade model with corruption and governments' debts. The model is constructed within a dynamic general equilibrium framework. It was based on the Solow growth model, the Uzawa-Oniki two-country global growth model, Diamond's growth model with government's debt, and Zhang's model with corruption. National economies supply one globally homogeneous tradable good (which can be consumed and invested as in the Solow model) and domestically consumed public services. Populations are fixed and classed as workers and officials. The public sector uses single input, officials, to supply services. Governments obtain incomes through taxing production, consumption, incomes from wealth, and wage incomes. We specially assume that a country's tax rates are positively related to its government debt. Productivities and welfare are affected by public services. Production sectors employ uses workers and capital. Officials are corrupt as they take bribes from the private sector and households. The corruption is measured by the corruption rates on the output level of the private sector, the wealth interest returns of the workers, and wage incomes of the workers. The dynamic interdependence between endogenous labor supply, government's debt, and corruption, for a Q -country world economy is described by $3Q$ nonlinear differential equations. We simulated the model and demonstrated dynamic properties of the model. We identified a stable equilibrium point and plotted the motion of the dynamic system. The comparative dynamic analyses conducted for a 3-country global economy show transitory processes to the new equilibrium point when some exogenous conditions are changed. We get many insights from comparative dynamic analyses. For instance, if all the officials in the world become more corrupt, they become richer and consume more. Their corrupt incomes are increased. All the workers have less wealth and consume less and they work almost the same hours in the long term. In the long term workers' wage incomes fall and officials' wage income rise. In the long term the output levels are slightly affected and each country employs slightly a little more capital. In the long term each government has slightly less debt and spends slightly more. Irrespective of its complicated interdependence, the model simplifies reality. The model should be generalized and extended. We may more general production functions and utility functions. It is also interesting to examine various combinations of parameter changes. For instance, one may ask what happen to the world economy if one country becomes more corrupt and others become cleaner. In our approach, the channels of corruption are too simplified. Specially, there are great differences between officials with regard to ethic behavior. Corruption should be endogenous. Solow's model, the Oniki-Uzawa's trade model, and Diamond's growth model with debts are most well-known models in the literature of growth theory. Possible limitations, extensions and generalizations of our model are obvious in the light of the sophistication of the literature.

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Appendix: Proving the lemma

From (2) we obtain

$$z_q \equiv \frac{r + \delta_q}{w_{q1}} = \frac{N_{q1}}{\bar{\beta}_q K_q}, \tag{A1}$$

where

$$\bar{\beta}_q \equiv \frac{\beta_q}{\alpha_q}, \ \ q = 1, ..., Q.$$

From (A1) and (3), we obtain

$$r = \tilde{\alpha}_q \bar{\phi}_q z_{q1}^{\beta_q} - \delta_{kq}, \ \ w_{q1} = \frac{r + \delta_q}{z_q}, \tag{A2}$$

where

$$\tilde{\alpha}_j \equiv \alpha_q \bar{\beta}_q^{\beta_q} A_q G_q^{\theta_q}.$$

From (A2) we also have

$$z_q = \left(\frac{r + \delta_{kq}}{\tilde{\alpha}_q \bar{\phi}_q} \right)^{1/\beta_q}. \tag{A3}$$

From (6) and (7) we have

$$\bar{T}_{q1} = \frac{\sigma_{q1} \bar{\phi}_{aq} R a_{q1}}{\bar{w}_{q1}} + \sigma_{q1} T_0, \ \ c_{q1} = \xi_{q1} \bar{\phi}_{aq} R a_{q1} + \xi_{q1} T_0 \bar{w}_{q1}. \tag{A4}$$

Equations (A4) and (5) imply

$$T_{q1} = (1 - \sigma_{q1}) T_0 - \frac{\sigma_{q1} \bar{\phi}_{aq} R a_{q1}}{\bar{w}_{q1}}. \tag{A5}$$

From (A5) and the definition of N_{q1}, we have

$$N_{q1} = n_{0q} - n_q a_{q1}, \ \ q = 1, ..., Q, \tag{A6}$$

where

$$n_{0q} \equiv (1 - \sigma_{q1}) h_{q1} \bar{N}_{q1} T_0, \ \ n_q \equiv \frac{h_{q1} \bar{N}_{q1} \sigma_{q1} \bar{\phi}_{aq} R}{\bar{w}_{q1}}.$$

From (1), (A6) and (A1) we have

$$F_q = \tilde{n}_{0q} - \tilde{n}_q a_{q1}, \tag{A7}$$

where

$$\tilde{n}_{0q} \equiv \frac{A_q G_q^{\theta_q}}{\bar{\beta}_q^{\alpha_q} z_q^{\alpha_q}} n_{0q}, \quad \tilde{n}_q \equiv \frac{A_q G_q^{\theta_q} n_q}{\bar{\beta}_q^{\alpha_q} z_q^{\alpha_q}}.$$

Insert (A6) and (A7) in (9)

$$w_{cq} = \bar{n}_{0q} + \bar{n}_q a_{q1}, \tag{A8}$$

where

$$\bar{n}_{0q} \equiv \frac{\phi_q \tilde{n}_{0q} + \phi_{wq} w_{q1} n_{0q}}{\bar{N}_{q2}}, \quad \bar{n}_q \equiv \frac{\phi_{aq} R \bar{N}_{q1} - \phi_q \tilde{n}_q - \phi_{wq} w_{q1} n_q}{\bar{N}_{q2}}.$$

Insert (A8) in (10)

$$\hat{y}_{q2} = R a_{q2} + \bar{w}_{q2} T_{q2} + \bar{n}_{0q} + \bar{n}_q a_{q1}. \tag{A9}$$

Insert (A9) in (12)

$$c_{q2} = \xi_{q2} R a_{q2} + m_q + \xi_{q2} \bar{n}_q a_{q1}, \tag{A10}$$

where

$$m_q = \bar{w}_{q2} \xi_{q2} T_{q2} + \xi_{q2} \bar{n}_{0q}.$$

From (A1) and (A6) we have

$$K_q = \frac{n_{0q} - n_q a_{q1}}{\bar{\beta}_q z_q}, \quad q = 1, ..., Q. \tag{A11}$$

From (19) we have

$$\sum_{j=1}^{Q} a_q = \sum_{j=1}^{Q} K_q + \sum_{j=1}^{Q} D_q. \tag{A12}$$

Insert (A12) in (A13)

$$\sum_{q=1}^{Q} \hat{n}_q a_{q1} = \tilde{n}_0, \tag{A13}$$

where

$$\hat{n}_q \equiv \bar{N}_{q1} + \frac{n_q}{\bar{\beta}_q z_q}, \quad \tilde{n}_0 \equiv \sum_{q=1}^{Q} \left(\frac{n_{0q}}{\bar{\beta}_q z_q} - a_{q2} \bar{N}_{q2} \right) + \sum_{q=1}^{Q} D_q.$$

Solve (A14) with a_{11} as the variable

$$a_{11} = \Lambda\left(z_1, \{a_{q1}\}, (a_{q2}), (D_j)\right) = \left(\tilde{n}_0 - \sum_{q=2}^{Q} \hat{n}_q \, a_{q1}\right)\frac{1}{\hat{n}_1}. \tag{A15}$$

It is straightforward to check that all the variables can be expressed as functions of z_1, $\{a_{q1},\}$ (a_{q2}), and (D_j) at any point in time as follows: r by (A2) $\rightarrow$ z_j by (A3) $\rightarrow$ w_{q1} by (A2) $\rightarrow$ $\bar{w}_{q1}$ by definition $\rightarrow$ $\bar{w}_{q2}$ by (8) $\rightarrow$ τ_q and τ_{q1} $\rightarrow$ N_{q2} by definition $\rightarrow$ G_q by (16) $\rightarrow$ a_{11} by (A15) $\rightarrow$ K_q by (A12) $\rightarrow$ w_{cq} by (A8) $\rightarrow$ $\hat{y}_{q2}$ by (A9) $\rightarrow$ c_{q2} and s_{q2} by (12) $\rightarrow$ F_q by (A7) $\rightarrow$ N_{q1} by (A6) $\rightarrow$ Y_q by (17) $\rightarrow$ a_q by (18) $\rightarrow$ $\bar{y}_{qj}$ by (6) $\rightarrow$ c_{q1}, $\bar{T}_{q1}$, and s_{q1} by (7) $\rightarrow$ $T_{q1} = T_0 - \bar{T}_{q1}$ $\rightarrow$ I_q by (14) $\rightarrow$ K by (21). From this procedure, and (13) and (15), we have

$$\dot{a}_{11} = \Lambda_0\left(z_1, \{a_{q1}\}, (a_{q2}), (D_j)\right) \equiv s_{11} - a_{11}, \tag{A16}$$

$$\begin{aligned} \dot{a}_{qj} &= \Lambda_{qj}\left(z_1, \{a_{q1}\}, (a_{q2}), (D_q)\right) \equiv s_{qj} - a_{qj}, \quad q = 2, ..., Q, \quad j = 1, 2, \\ \dot{D}_q &= \Psi_q\left(z_1, \{a_{q1}\}, (a_{q2}), (D_q)\right) \equiv r\,D_q + Y_p - I_q, \quad q = 1, ..., Q. \end{aligned} \tag{A17}$$

Taking derivatives of (A15) with respect to t yields

$$\dot{a}_{11} = \frac{\partial \Lambda}{\partial z_1}\dot{z}_1 + \sum_{q=2}^{Q}\Lambda_{q1}\frac{\partial \Lambda}{\partial a_{q1}} + \sum_{q=1}^{Q}\Psi_j\frac{\partial \Lambda}{\partial D_j}, \tag{A18}$$

where we also use (A17). From (A18) and (A16), we have

$$\dot{z}_1 = \Lambda_{11}\left(z_1, \{a_{q1}\}, (a_{q2}), (D_q)\right) \equiv \left(\Lambda_0 - \sum_{q=1}^{Q}\Lambda_{q2}\frac{\partial \Lambda}{\partial a_{q2}} - \sum_{q=2}^{Q}\Lambda_{q1}\frac{\partial \Lambda}{\partial a_{q1}} - \sum_{q=1}^{Q}\Psi_q\frac{\partial \Lambda}{\partial D_q}\right)\left(\frac{\partial \Lambda}{\partial z_1}\right)^{-1}. \tag{A19}$$

We determine the motion of the system with (A17) and (A19) and the rest variables by the procedure provided before. In summary, we proved the lemma.