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GARCH based VaR estimation: An empirical evidence from BRICS stock markets

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Abstract. *This paper examines the adequacy of GARCH based VaR models in risk estimation for BRICS emerging stock markets. This study uses the daily data of stock indices in these markets for the period 25th September 1997 to 30th March 2018. Here we employ SGARCH, EGARCH and GJR-GARCH models to test volatility persistence and leverage effect of these markets. It is observed that the volatility persistence and leverage effect is present in all these markets. In GARCH estimation the error distribution - students t is found to be suitable for Brazil, Russia, India, and South Africa whereas GED for China. From the backtesting results of Kupiec and Christoffersen test, it is found that these models are appropriate for Brazil, Russia, India, and South Africa in risk estimation at 99% one day VaR.*

Keywords: emerging markets, GARCH models, volatility and leverage, VaR estimation, backtesting.

JEL Classification: C22, C52, G15.

1. Introduction

Globalization and financial sector reforms in emerging market economies (EMEs) led to greater integration of emerging stock markets with the developed economies. As a result, these EMEs have started experiencing an increase in both domestic and foreign investments. When the foreign institutional or portfolio investments are directed towards a EMEs, the inflow of foreign currency into the EMEs adds volume to their stock markets and long-term investment for the infrastructure projects. As this capital flows increase, they bring profit opportunities along with high risk in these markets.

In the last three decades, the financial liberalization and integration of markets have resulted in unrestricted access to the cross-border capital flows which is an important source of external finance for emerging countries. Among these emerging countries, the BRICS countries gained a lot of importance as these BRICS countries are the most rapidly growing economies in terms of their GDP and the size of the stock markets. The market capitalization is a commonly measured indicator used to measure the size of the markets and the development levels of the markets across various countries markets. In the last two decades, the BRICS stock markets have increased its share in the world market and played a significant role in the growth of the world economy. The stock markets of BRICS grew from US$1.19 trillion to US$13 trillion during the period of 1997 to 2017 (WDI, 2019) which accounts for 17% of world market capitalization. Despite the robust growth experienced by these five markets in terms of size, the presence of excess volatility in the market returns would adversely affect the investment decisions and increases the risks in the investments. Also, the increased disorders during the crisis necessitate study on volatility behavior and accurate market risk prediction in the BRICS stock markets.

The outbreak of financial distress and uncertainty in the 1990's induced intensive research from financial institutions, regulators, and researchers to design sophisticated tools for measuring and forecasting risk. Though there exists several statistical risk measures such as standard deviation, variance, and Beta, Value-at-Risk (VaR) is the better measure to estimate the risk (Mehmet and Bulent (2012)). Value-at-Risk is described as the probable maximum loss to occur over a targeted period at a given level of confidence. The concept of VaR as a method of risk management was first introduced by J.P. Morgan in 1994 with its Risk Metrics system. Since then the researchers like Philippe (1996), Darrell and Jun (1997), and Kevin (1998) contributed for improving the accuracy of VaR estimates. In financial risk management, the application of VaR models gained a lot of importance and several studies have been conducted to compare the relative performance of various methods. However, researchers observed that there is no single measure to obtain accurate risk estimates, for example, Manganelli and Engle (2001), Christoffersen et al. (2001), Wong et al. (2002), Angelidis et al. (2004), Harmantzis et al. (2006), Raghavan et al. (2017).

The objective of this study is twofold. First, among the various types of VaR models, we employed the symmetric (SGARCH) and asymmetric (EGARCH, GJRGARCH) GARCH models under three error distributions of BRICS stock markets. The reason for using GARCH based VaR is that the GARCH model gives accurate time-varying volatility forecasts which are important in the calculation of VaR measures.(1) Second, to compare the relative performance of GARCH models by evaluating the estimated VaR through backtesting measures under a risk management framework. The present study contributes to the existing literature on the following aspects. This study offers a comparison of various VaR models to the existing emerging market literature under GARCH framework. This study also encompasses the entire time period since the inception of VaR applications to the most recent period i.e. from 25th September 1997 to 30th March 2018.

The remainder of the paper is arranged as follows: In section 2 the relevant literature is reviewed. Section 3 describes the methodology. In section 4 we present data and empirical results of this study. Section 5 ends the paper with a summary and conclusions.

2. Literature review

The empirical studies available in the estimation of VaR for developing or emerging markets is limited compared to the developed markets (Julija, 2017). Da Silva et al. (2003) computed VaR estimates using extreme value theory (EVT) for Asian stock markets and found that extreme value method is conservative than traditional methods to determine capital requirements. Gencay and Selcuk (2004) estimated VaR for nine emerging markets using EVT, variance-covariance approach and historical simulation and showed that EVT-VaR results are more accurate at higher confidence interval. Timotheos et al. (2004) evaluated the performance of ARCH family models in estimating the daily VaR in five developed stock indices and showed that leptokurtic distributions are able to produce a better result than the normal distribution models. Bao, Lee, and Saltoglu (2006) compared the predictive power of VaR models for five Asian economies and found that exponentially weighted moving average (EWMA) of RiskMetrics model is appropriate for a calm period, while EVT performed better in the crisis period. Mike and Philip (2006) used RiskMetrics, GARCH models, and two long memory GARCH models on developed and emerging market indices and found that asymmetric GARCH with t-error distribution gives superior estimates than other methods. Zikovic and Aktan (2009) explored the relative performance of a wide array of VaR models for Turkish and Croatian stock indices and found that during crisis period except for EVT and historical hybrid simulation all the VaR models underpredict the true level of risk. Andjelic et al. (2010) investigated the relative performance of Historical simulation (HS) and Delta normal VaR methods for four emerging markets and opined that these models may not be suitable. Under the GARCH framework to estimate VaR for BELEX15 index based on normal and student t distribution, Nikolic and Dragan (2011) found that GARCH models perform better than IGARCH models in evaluating the VaR for the index. Stavros and Christos (2012) compared three methods such as exponentially weighted moving average (EWMA) of RiskMetrics, symmetric and asymmetric GARCH to evaluate VaR in developed markets and found that ARCH model provided satisfactory forecasts of VaR than the other methods. Bucevska (2013) estimated VaR for Macedonian stock indices and showed that the EGARCH model with student's t distribution and GJR-GARCH model are more robust and adequate for estimating and forecasting VaR. Mirjana and Sinisa (2013) evaluated the performance of symmetric and asymmetric GARCH models in Serbian stock markets and found that EGARCH model with normal distribution and GARCH with t distribution have made an adequate estimation of VaR. Mirjana and Sinisa (2015) employed the GARCH type methods to estimate VaR for CEE emerging markets and showed that the GARCH model with t distribution gives better estimates than the normal distribution. Stavros and Artemis (2017) observed that the GARCH model is more suitable for evaluating VaR in developed markets. Julija et al. (2017) examined the adequacy of GARCH models in estimating the VaR of Montenegrin Stock market and found that these models with student-t and Johnson distribution produce relatively better VaR estimates.

With regard to BRICS stock markets, the literature has been very sparse. Mehmet and Bulent (2011) compared the GARCH models in estimating VaR for emerging (Brazil and Turkey) and developed (Germany and the USA) markets during the global financial crisis period and showed that GARCH(1,1) with student's t distribution performs better than the Normal. Leandro and Rosangela (2017) estimated VaR for S&P 500 and IBOVESPA index using the traditional GARCH and range based volatility GARCH models and showed that range based models provide more accurate VaR than the traditional GARCH models. Wilton et al. (2018) employed the GARCH models on Brazilian sectoral stock indices to estimate VaR and showed that the models with student-t distribution are more suitable. Jayanth R. Varma (1999) observed that the GARCH-GED performed well at all risk levels considered for Indian stock market. Indrajit Roy (2011) estimated VaR of Indian stock markets using historical simulation, GARCH models and found that the GARCH model produced better estimates of VaR. Sai Pranav et al. (2018) employed various GARCH models and observed that GJR-GARCH and EVT-t copula outperforms traditional VaR methods. Wai-Cheung Ip et al. (2006) compared the switching-regime ARCH model and the GARCH(1,1) in estimating the VaR for China stock markets and found that ARCH models preferred to GARCH(1,1). Guangquang Liu et al. (2018) showed that Heterogeneous autoregressive quarticity (HARQ) models estimate the VaR better than the HAR-type models in the Chinese stock market. McMillan and Thupayagale (2010) evaluated the performance of alternative volatility models in forecasting volatility and VaR in South Africa market and showed that asymmetric GARCH and long memory models outperform other models considered for the VaR estimation. Raghavan et al. (2017) estimated VaR for BRIC stock markets using historical, Monte Carlo and GARCH simulations and found that GJR-GARCH is more suitable for Brazil and China while the historical simulation is most appropriate for Russia and Indian stock markets. Lumengo and Lebogang (2018) compared the performance of three multivariate GARCH models, the DCC, ADCC and CCC GARCH models in estimating VaR for BRICS stock markets and found that the DCC performs the best among three GARCH models.

3. Methodology

GARCH-based VaR estimation

Mainly there are two fundamental approaches for VaR estimation viz. parametric and non-parametric. One of the most popular non-parametric methods among practitioners is the historical simulation (HS) because it works well for nonlinear components. However, the limitation of the HS is that it responds very late to the big movements. As far as the parametric approach is concerned the GARCH based methods are chosen because they overcome the limitations of HS and ability to capture the volatility clustering in the series which is very common in stock returns. Considering that modeling time-varying risk is vital for the measurement of VaR, GARCH-based methodology provides valuable information about the forthcoming risk. If the GARCH model satisfies the volatility persistence or the sum of $ARCH\ and\ GARCH\ terms$ closer to unity it means that the model is able to describe the conditional volatility which is the basis for VaR estimation.

The GARCH (p, q) model

The time series econometric model with an autoregressive moving-average model (ARIMA) (m, d, n), for VaR estimation can be represented as:

$$r_t = \mu + \sum_{i=1}^{m} r_{t-i} + \sum_{j=1}^{n} \varepsilon_{t-j} \quad (1)$$

The conditional variance of returns r_t are estimated using the symmetric as well as asymmetric volatility models such as SGARCH, EGARCH and GJR GARCH.

To model the changes in the variance of time series Engle (1982) introduced the ARCH model, later Bollerslev (1986) extended this model and is given by

$$\sigma_t^2 = \Omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (2)$$

This model includes p ARCH terms (ε_{t-i}^2) and q GARCH $\left(\sigma_{t-j}^2\right)$terms. The sum of ARCH and GARCH (α + β) terms indicates the level of persistence of volatility of the series. If the sum is close to one (unity) then the volatility is said to be persistence.

EGARCH (p, q) model

In order to accommodate the asymmetric response, a new class of models was introduced by Nelson (1991) and is known as exponential GARCH or EGARCH (p, q). The model is represented as

$$\log(\sigma_t^2) = \Omega + \sum_{i=1}^{q} \alpha(|Z_{t-i}| - E|Z_{t-i}|) + \gamma Z_{t-i} + \sum_{j=1}^{p} \beta \log(\sigma_{t-j}^2) \quad (3)$$

Where $Z_{t-i} = \frac{\varepsilon_{t-i}}{\sigma_{t-i}}$

For $\gamma < 0$ negative shocks will have a bigger impact on future volatility than positive shocks of the same magnitude. Furthermore, the sum of α and β governs the persistence of volatility shocks in the GARCH (1,1) model, whereas only parameter β governs the persistence volatility shocks in EGARCH(1,1) model.

GJR GARCH (p, q) model

Another variant of the asymmetric GARCH model to capture the leverage effect is the Glosten, Jagannathan, and Runkle (1993) model also known as the GJR GARCH (p, q) model. The Conditional variance of this model is specified as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \gamma_i I_{t-i} \varepsilon_{t-i}^2 + \sum_{k=1}^{r} \beta_i \sigma_{t-i}^2 \quad (4)$$

In Eq (4), γ_i is the asymmetric or leverage effect and $I_{t-1}$is the dummy variable used to differentiate the good and bad news i.e., $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ indicating bad news, and $I_{t-1} = 0$ if $\varepsilon_{t-1} \geq 0$ indicating good news. The GJR GARCH model specification assumes that unexpected changes in the market returns or $\varepsilon_t$will have a different effect on the volatility of stock return σ_t^2. Good news will lead to higher return; hence it is associated with higher variance through γ. A non-zero value of γ indicates the asymmetric nature of the returns. On the other hand, when γ is zero, the model reduces to symmetric GARCH model.

VaR estimation and Backtesting

In order to estimate VaR we have divided the entire data into the estimation window and a test window. The first 1000 observations have been used for estimation window and the remaining observations are used as an out-of-sample testing window for the 1-day VaR estimates at a 99% confidence level.

It is necessary to check the reliability and accuracy of the model once we estimate the VaR model. Using a statistical procedure for examining the appropriateness of the VaR model is called Backtesting. The backtesting enables us to ascertain the robustness of VaR estimates. There are two statistical tests namely Kupiec and Christoffersen tests for unconditional and conditional coverage respectively to know the number of exceedances. The unconditional coverage test checks whether in a given time interval the number of exceptions is equal to the number of theoretically estimated exceptions at a specified confidence level. Also, the unconditional coverage test assumes that the exceptions are evenly spread out across the period. However, in the real world the exceptions are very much bunched up and are not evenly spread out signaling they are conditioned or based on time variation(2). The conditional coverage tests check whether the exceptions are time-varying and independent.

Kupiec (1995) test

The Kupiec test for unconditional coverage is based on the Binomial approach. It uses a likelihood ratio test to check whether the number of actual exceptions from the calculated VaR is equal to the theoretically expected exceptions number at a given confidence level. If the data suggests that the probability of exceptions is different than *p*, the VaR model is rejected. The Kupiec test statistic is computed from the following equation.

$$LR_{uc} = -2\log\left(\frac{(1-p)^{N-x}p^x}{\left(1-\frac{x}{N}\right)^{N-x}\left(\frac{x}{N}\right)^x}\right) \tag{5}$$

Where *x* is the number of failures, *N* the number of observations and *p = 1- VaR* level. This statistic is asymptotically distributed as a chi-square variable with 1 degree of freedom. The VaR model fails the test if this likelihood ratio exceeds a critical value. The critical value depends on the test confidence level.

Christoffersen's (1998) test

Christoffersen (1998) proposed a test for conditional coverage estimation. This test measures the dependency between consecutive days and the test statistic for independence is given by

$$LR_{cc} = -2\ln[\ln(1-\mathrm{p})^{\mathrm{N-x}}\mathrm{p}^{\mathrm{x}}] + 2\ln\left[(1-\pi_{01})^{\mathrm{n}_{00}}\pi_{01}^{\mathrm{n}_{01}}(1-\pi_{11})^{\mathrm{n}_{10}}\pi_{11}^{\mathrm{n}_{11}}\right] \qquad (6)$$

n_{00} = Number of periods with no failures followed by a period with no failures.
n_{10} = Number of periods with failures followed by a period with no failures.
n_{01} = Number of periods with no failures followed by a period with failures.
n_{11} = Number of periods with failures followed by a period with failures.
and
π_{01} – Probability of having a failure on period t, given that no failure occurred on period $t-1 = n_{01} / (n_{00}+ n_{01})$
π_{11} – Probability of having a failure on period t, given that a failure occurred on period $t-1 = n_{11} / (n_{10}+ n_{11})$

This statistic is asymptotically distributed as a chi-square with 1 degree of freedom. You can combine this statistic with the frequency of unconditional coverage test to get a conditional coverage (CC) mixed test:

$LR_{CC} = LR_{uc} + LR_{ind}$

This test is asymptotically distributed as a chi-square variable with 2 degrees of freedom.

The null and alternative hypothesis of both the tests are given as follows

For Kupiec unconditional coverage test

H_0: Correct unconditional coverage.

H_1: Incorrect unconditional coverage.

Similarly, for Christoffersen conditional coverage.

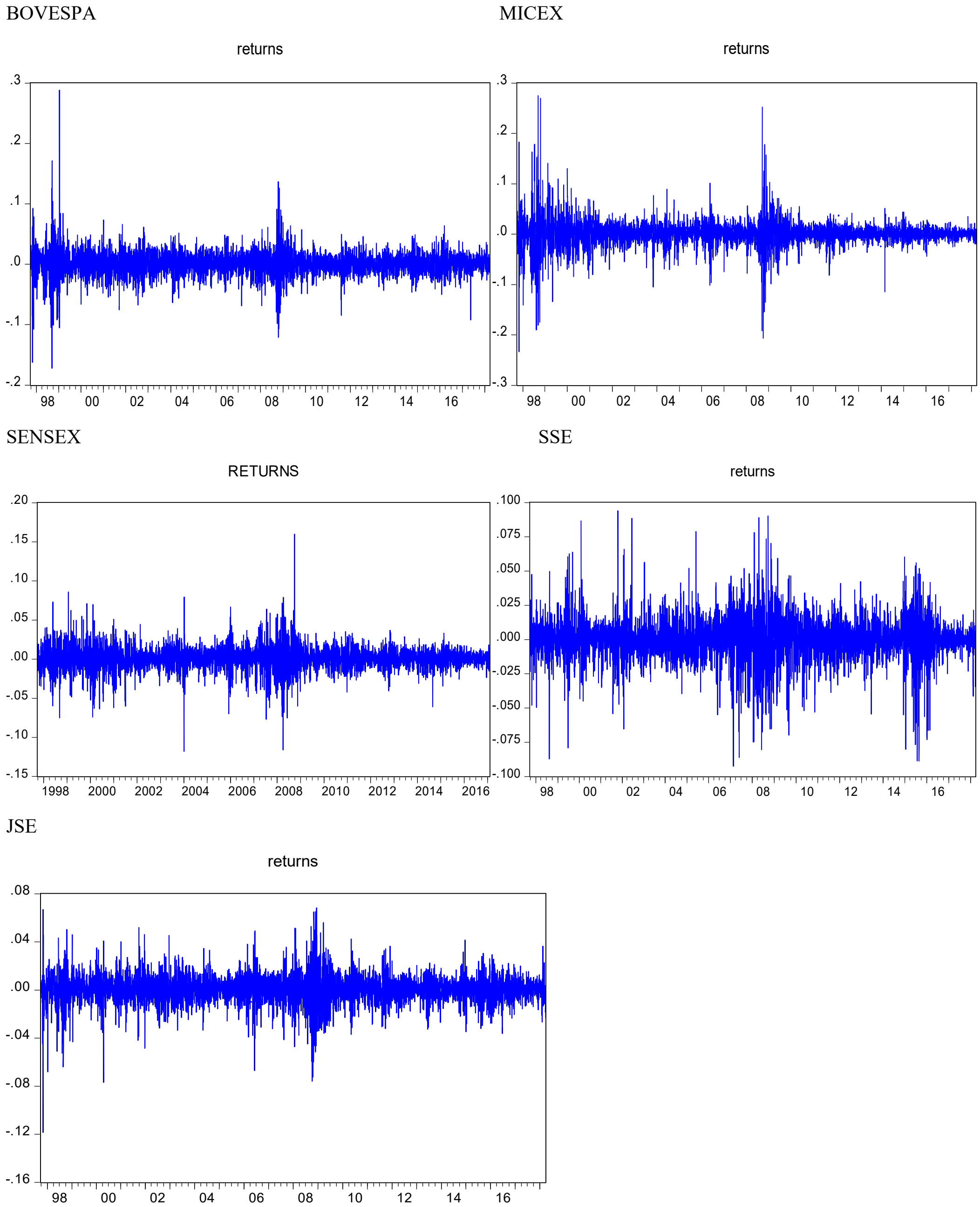
H_0: Correct exceedances and independence of failures.

H_1: Incorrect exceedances and independence of failures.

4. Data and results

This study uses the daily returns calculated from closing prices of BRICS stock markets for the period 25th September 1997 to 30th March 2018. The stock indices considered here are BOVESPA (Brazil), MICEX (Russia), SENSEX (India), SSE (China) and JSE (South Africa). The closing prices data for Brazil, Russia, India, and China are obtained from the Yahoo finance (www.yahoofinance.com) and South Africa prices are from the Wallstreet Journal (www.wsj.com). Here we consider all local currency denominated stocks for this analysis. The returns of these indices are calculated from the closing prices by using formula, $r_t = \log\left(\frac{p_t}{p_{t-1}}\right) \times 100$, where p_t and p_{t-1} are the closing prices at period (t) and (t-1) respectively. It is observed from the log return series of BRICS stock markets that all the indices have volatility clustering[(3)] (Fig 1) which is also the prerequisite for GARCH analysis.

Figure 1: *Daily log return series of BRICS Stock Markets*



To know the important characteristics of BRICS stock market returns we calculated the summary statistics measures and provided in Table 1.

Table 1. *Descriptive Statistics of returns of BRICS Stock Markets*

Stock Indices	Mean	Std. Dev	Skewness	Kurtosis	JB Statistic	ARCH LM(Lag=5)
BOVESPA	0.039	2.032	0.320	16.91	40975.60(0.00)	447.68(0.00)
MICEX	0.061	2.574	0.123	19.63	58944.26(0.00)	782.12(0.00)
SENSEX	0.043	1.524	-0.108	9.58	9128.35(0.00)	464.67(0.00)
SSE	0.021	1.585	-0.318	7.85	4956.30(0.00)	379.94(0.00)
JSE	0.040	1.229	-0.427	8.29	6125.67(0.00)	719.00(0.00)

Note: p values are given in parenthesis.

From Table 1, it is observed that the mean returns of all the BRICS markets are positive indicating on an average all these markets have experienced profits during the study period. The larger value of standard deviation and mean of returns in case of Russian market indicating that higher the risk and higher the return. The skewness for all the return series is asymmetric with both positive and negative asymmetric values. The Kurtosis values indicate that the return series having fat tails which is a common phenomenon of stock returns. The JB statistic for all the markets is statistically significant, thereby indicating the return distributions are non-normal. The ARCH-LM test indicates that all the series have rejected the null hypothesis of no ARCH effect specifying that GARCH models can be employed.

In order to select the ARMA lag order and GARCH specification, we used the AIC criterion. The results of SGARCH, EGARCH and GJRGARCH models along with the error distributions: normal, students t and generalized error distribution are estimated based on AIC of BRICS returns series and are provided in following tables.

Table 2. *Estimation of SGARCH with Normal error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0007 (3.33)	0.001 (5.163)	0.0008 (5.310)	0.0002 (1.638)	0.0007 (5.395)
AR(1)	0.705 (1145.76)	-1.030 (-71.07)			
AR(2)	-0.997 (-1835.68)	-0.953 (-65.16)			
MA(1)	-0.707 (-1654.79)	1.046 (75.28)	0.083 (5.47)		0.064 (4.181)
MA(2)	0.998 (2627.21)	0.954 (66.88)	-0.012 (-0.89)		
Ω	0.000007 (7.560)	0.000004 (13.876)	0.000001 (7.14)	0.000001 (7.170)	0.000002 (7.771)
α	0.093 (16.51)	0.111 (19.75)	0.089 (18.98)	0.079 (19.87)	0.113 (17.675)
β	0.885 (118.47)	0.883 (172.16)	0.905 (211.07)	0.917 (248.87)	0.869 (133.79)
α+β	0.978	0.994	0.994	0.996	0.982
AIC	-5.2701	-5.1575	-5.8633	-5.7423	-6.2317

Table 3. *Estimation of SGARCH with Students t error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0007* (3.33)	0.001* (5.01)	0.0008* (5.05)	0.0002 (1.64)	0.0007* (5.36)
AR(1)	1.528* (46.89)	-0.99* (-20.09)			
AR(2)	-0.919* (-17.57)	-0.849* (-6.8)			
MA(1)	-1.518* (-42.74)	1.02* (18.55)	0.083* (5.55)		0.064* (4.36)
MA(2)	0.903* (15.85)	0.85* (7.5)	-0.012 (-0.86)		
Ω	0.000008 (3.62)	0.000005 (4.08)	0.000002 (1.36)	0.000002 (1.36)	0.000003 (2.31)
α	0.079* (22.69)	0.099* (12.29)	0.090* (5.84)	0.075* (6.3)	0.104* (8.36)
β	0.903* (149.02)	0.900* (94.32)	0.903* (59.43)	0.921* (75.19)	0.878* (62.33)
α+β	0.982	0.999	0.993	0.996	0.983
AIC	-5.2960	-5.2294	-5.9054	-5.8221	-6.2561

Table 4. *Estimation of SGARCH with GED error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0008 (4.109)	0.0009 (5.186)	0.0008 (5.502)	0.0005 (3.796)	0.0008 (5.942)
AR(1)	1.491 (10.524)	-1.031 (-68.95)			
AR(2)	-0.706 (-5.592)	-0.955 (-66.28)			
MA(1)	-1.491 (-10.267)	1.042 (67.29)	0.082 (5.710)		0.054 (5.708)
MA(2)	0.692 (5.241)	0.954 (64.53)	-0.015 (-1.133)		
Ω	0.000007 (5.382)	0.000003 (5.35)	0.000001 (4.537)	0.000001 (3.83)	0.000002 (5.708)
α	0.088 (11.451)	0.099 (12.037)	0.089 (11.728)	0.075 (10.19)	0.109 (12.447)
β	0.890 (90.958)	0.897 (119.50)	0.904 (124.37)	0.921 (132.06)	0.872 (93.94)
α+β	0.978	0.996	0.993	0.996	0.981
AIC	-5.2899	-5.2144	-5.8976	-5.8252	-6.2493

Table 5. *Estimation of EGARCH with Normal error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0002 (1.048)	0.0008 (4.459)	0.0004 (2.526)	0.0001 (1.017)	0.0003 (2.793)
AR(1)	0.165 (8.946)	0.257 (0.266)			
AR(2)	-0.941 (-48.733)	0.050 (0.087)			
MA(1)	-0.162 (-9.238)	-0.222 (-0.230)	0.094 (6.263)		0.067 (4.71)
MA(2)	0.950 (53.422)	-0.078 (-0.129)	-0.003 (-0.225)		
Ω	-0.315 (-11.437)	-0.306 (-19.581)	-0.339 (-15.75)	-0.239 (-14.681)	-0.426 (-15.956)
α	0.154 (15.083)	0.234 (27.530)	0.194 (21.24)	0.178 (21.98)	0.202 (23.803)
β	0.975 (344.81)	0.983 (661.21)	0.978 (470.95)	0.987 (604.27)	0.970 (343.31)
γ	-0.081 (-13.758)	-0.038 (-8.99)	-0.080 (-13.78)	-0.025 (-5.988)	-0.093 (-15.90)
AIC	-5.2885	-5.1557	-5.8803	-5.7562	-6.2529

Table 6. *Estimation of EGARCH with Students t error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0004* (2.5691)	0.0008 (5.13)	0.0005 (3.59)	0.0004 (3.18)	0.0005 (3.50)
AR(1)	1.559* (996.29)	0.299 (0.68)			
AR(2)	-0.995* (-712.77)	-0.021 (-0.21)			
MA(1)	-1.555* (-980.54)	-0.258 (-0.59)	0.095 (6.81)		0.058 (3.71)
MA(2)	0.988 (76029.59)	-0.016 (-0.17)	-0.010 (-0.78)		
Ω	-0.167 (-20.35)	-0.077 (-10.34)	-0.188 (-14.53)	-0.102 (-10.22)	-0.194 (-18.13)
α	0.143 (9.18)	0.199 (38.17)	0.194 (12.96)	0.171 (91.69)	0.161 (8.41)
β	0.979 (929.23)	0.990 (998.31)	0.978 (659.73)	0.988 (814.21)	0.978 (814.80)
γ	-0.078 (-9.84)	-0.033 (-4.01)	-0.188 (-14.53)	-0.025 (-3.342)	-0.087 (-10.73)
AIC	-5.3108	-5.2299	-5.9191	-5.830	-6.2730

Table 7. *Estimation of EGARCH with GED error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0004 (1.995)	0.0007 (4.341)	0.0005 (3.535)	0.0004 (3.453)	0.0005 (3.725)
AR(1)	0.273 (18.987)	-1.033 (-68.202)			
AR(2)	-0.948 (-57.731)	-0.954 (-65.54)			
MA(1)	-0.271 (-19.209)	1.045 (67.14)	0.090 (6.243)		0.058 (4.046)
MA(2)	0.954 (60.652)	0.954 (64.21)	-0.009 (-0.662)		
Ω	-0.292 (-8.595)	-0.243 (-10.70)	-0.336 (-10.206)	-0.229 (-7.891)	-0.394 (-11.009)
α	0.147 (10.867)	0.203 (14.50)	0.194 (13.40)	0.171 (11.816)	0.190 (16.134)
β	0.978 (281.66)	0.988 (470.77)	0.978 (307.33)	0.988 (349.16)	0.973 (265.83)
γ	-0.079 (-10.317)	-0.033 (-4.656)	-0.083 (-9.452)	-0.025 (-3.33)	-0.090 (-11.859)
AIC	-5.3041	-5.2139	-5.9104	-5.8325	-6.266

Table 8. *Estimation of GJR-GARCH with Normal error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0003 (1.373)	0.0007 (3.655)	0.0005 (3.401)	0.0001 (0.926)	0.0004 (2.989)
AR(1)	0.264 (13.731)	-1.034 (-74.64)	0.092 (5.901)		
AR(2)	-0.929 (-58.253)	-0.954 (-66.01)	-0.005 (-0.356)		
MA(1)	-0.259 (-14.203)	1.049 (79.10)			0.069 (4.551)
MA(2)	0.938 (66.38)	0.956 (67.23)			
Ω	0.00009 (9.091)	0.000005 (13.320)	0.000002 (8.855)	0.000001 (7.340)	0.000003 (9.549)
α	0.019 (3.254)	0.082 (13.253)	0.045 (8.835)	0.066 (13.203)	0.039 (7.243)
β	0.892 (114.61)	0.883 (174.12)	0.895 (169.89)	0.916 (243.26)	0.879 (147.61)
γ	0.122 (12.822)	0.055 (7.488)	0.100 (11.25)	0.028 (4.657)	0.120 (12.039)
AIC	-5.2915	-5.1627	-5.8765	-5.7443	-6.2506

Table 9. *Estimation of GJR-GARCH with Students t error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0003 (1.47)	0.0007* (3.40)	0.0005* (3.291)	0.0001 (0.890)	0.0004* (2.904)
AR(1)	1.561* (1552.24)	0.346 (0.524)			
AR(2)	-0.995* (-1075.24)	0.163 (0.499)			
MA(1)	-1.557* (-8909.31)	-0.294 (-0.446)	0.092* (6.090)		0.068* (4.678)
MA(2)	0.991* (27002.27)	-0.198 (-0.570)	-0.0048 (-0.337)		
Ω	0.000009 (26.31)	0.000005 (4.067)	0.000003 (2.403)	0.000002 (1.554)	0.000003 (4.636)
α	0.0208* (6.70)	0.082* (8.84)	0.045* (5.950)	0.065* (6.397)	0.035* (4.636)
β	0.892* (159.21)	0.879* (99.53)	0.895* (82.72)	0.916* (83.71)	0.889* (78.67)
γ	0.117* (10.34)	0.061* (5.044)	0.099* (6.461)	0.028* (3.151)	0.113* (7.995)
AIC	-5.3119	-5.2315	-5.9719	-5.8234	-6.2695

Table 10. *Estimation of GJR-GARCH with GED error distribution*

	BOVESPA	MICEX	SENSEX	SSE	JSE
μ	0.0004 (2.203)	0.0008 (4.482)	0.0006 (4.06)	0.0004 (3.523)	0.0005 (3.957)
AR(1)	0.283 (15.279)	-1.005 (-23.840)			
AR(2)	-0.941 (-50.831)	-0.884 (-29.74)			
MA(1)	-0.279 (-15.600)	1.025 (23.73)	0.088 (6.029)		0.059 (3.990)
MA(2)	0.947 (54.368)	0.883 (29.36)	-0.010 (-0.72)		
Ω	0.000008 (6.55)	0.000003 (5.48)	0.000002 (5.871)	0.000001 (3.941)	0.000002 (6.943)
α	0.018 (2.363)	0.075 (7.775)	0.042 (5.007)	0.062 (6.96)	0.037 (5.135)
β	0.898 (92.198)	0.896 (118.81)	0.891 (107.57)	0.918 (129.77)	0.882 (108.81)
γ	0.115 (9.574)	0.046 (3.938)	0.109 (7.983)	0.027 (2.529)	0.116 (9.141)
AIC	-5.3060	-5.2168	-5.9085	-5.8261	-6.2635

The ARIMA(m, d, n)-GARCH(p, q) specification for each market is selected based on the AIC criterion. From the above Tables 2-10 it is observed that the symmetric GARCH models of all the markets show that there is strong volatility persistence in all the markets as the lagged squared residuals parameter(α) and the lagged conditional variance parameter(β) are significant and the sum of both is close to unity. Also, both the asymmetric GARCH models reveal that the asymmetric or leverage effect (γ) is statistically significant in all the markets for both the models.

For the VaR estimation, here we consider all the three models namely SGARCH, GJR-GARCH, and EGARCH models with the error distributions as normal, students t, and GED distributions. From the above results, we can see that the Brazil, Russia, India, and South Africa markets are fitting with students t distribution and GED distribution for China as these distributions having minimum AIC. The same is observed by Mehmet and Bulent (2011) for Brazil. However, each market following different orders of ARIMA-GARCH orders and we can observe for Brazil, ARIMA(2,0,2)-GARCH(1,1), Russia ARIMA(2,0,2)-GARCH(1,1), India ARIMA(0,0,2)-GARCH(1,1), China ARIMA(0,0,0)-GARCH(1,1), South Africa, ARIMA(0,0,1)-GARCH(1,1).

The accuracy of the model considered for the estimation of VaR is measured comparing the number of actual or realized exceedances with the frequency of expected exceedances for a level of significance. The best model is the one which has the realized return exceedances closer to the expected exceedances. If the actual exceedances are more than the expected exceedances it indicates the model has underestimated the risk. On the contrary, if the expected exceedances are more than the actual exceedances it denotes that the model has overestimated the risk. When actual and expected exceedances are closer we can say that the model is properly estimated the risk. However, since we are interested in the left tail of the distribution (losses), corresponding to a long-position portfolio, an actual failure rate lower than the expected is "good" enough.

We perform the backtesting using unconditional and conditional converge tests for all the models considered in the study. The unconditional coverage test checks whether the number of actual exceedances is closer to the expected exceedances while the conditional coverage test checks whether the exceptions are time-varying and serially independent. The Kupiec and Christoffersen tests are used for unconditional and conditional coverage tests at 1% level of significance and the results are given in Table 11.

Table 11. *Backtesting results of VAR models*

Stock Indices		SGARCH	EGARCH	GJR GARCH
BOVESPA	Expected Exceedance	40.7	40.7	40.7
	Actual Exceedance	30	32	27
	The Kupiec test 99%	3.116	2.019	5.272
	p-value	0.07	0.155	0.022
	The Christoffersen test 99%	3.562	2.526	6.633
	p-value	0.168	0.283	0.06
MICEX		SGARCH	EGARCH	GJR GARCH
	Expected Exceedance	41	41	41
	Actual Exceedance	37	39	38
	The Kupiec test 99%	0.427	0.11	0.242
	p-value	0.513	0.74	0.623
	The Christoffersen test 99%	1.1	0.858	0.952
	p-value	0.577	0.651	0.621

Stock Indices		SGARCH	EGARCH	GJR GARCH
SENSEX		SGARCH	EGARCH	GJR GARCH
	Expected Exceedance	40.5	40.5	40.5
	Actual Exceedance	35	35	33
	The Kupiec test 99%	0.799	0.799	1.509
	p-value	0.371	0.371	0.219
	The Christoffersen test 99%	1.825	1.825	2.707
	p-value	0.402	0.402	0.258
SSE		SGARCH	EGARCH	GJR GARCH
	Expected Exceedance	39.6	39.6	39.6
	Actual Exceedance	58	57	54
	The Kupiec test 99%	7.535	6.781	4.735
	p-value	0.006	0.009	0.03
	The Christoffersen test 99%	7.561	6.819	4.823
	p-value	0.023	0.033	0.09
JSE		SGARCH	EGARCH	GJR GARCH
	Expected Exceedance	41.2	41.2	41.2
	Actual Exceedance	40	42	44
	The Kupiec test 99%	0.078	0.016	0.188
	p-value	0.78	0.901	0.665
	The Christoffersen test 99%	0.986	0.584	1.138
	p-value	0.611	0.747	0.566

Note: The Kupiec and Christoffersen critical values at 99% are 6.634 and 9.210, respectively.

From Table 11, we can see that Brazil, Russia, India, and South Africa markets have passed both the tests for all the three models at a 99% confidence level. This indicates that these models give correct VaR estimates regarding unconditional coverage (expected and actual exceedances are closer) and conditional coverage. However, in the case of South Africa, though the models have failed to reject the null hypothesis, both the asymmetric GARCH models have underestimated the VaR value as the actual exceedance is more than the expected exceedance. The GARCH models for China have failed both the test at a 99% confidence level indicating that these models have rejected the null hypothesis of correct exceedance and independence of failures. We can conclude that the GARCH based VaR are suitable for Brazil, Russia, India, and South Africa markets but not for China stock markets.

Summary & Conclusions

Ever since the emerging economies initiated reforms in their financial sector the stock markets experienced a remarkable growth due to an increase in foreign capital inflows. However, the asymmetric information in emerging markets results in a high degree of volatility than the developed markets. The presence of volatility makes the investments more risky for both domestic and foreign investors. Thus, it is important to study the volatility behavior of emerging markets and also estimation which arises due to excessive volatility in these markets. In this context, this study aims at examining the volatility behavior and estimation of the risk in BRICS emerging markets by considering the daily stock indices from 25th September 1997 to 30th March 2018.

This study examined the adequacy of GARCH models in estimating the Value at Risk (VaR) by employing symmetric and asymmetric GARCH models to estimate the VaR for BRICS stock markets. The results of the study reveal that for Brazil, Russia, India, and South Africa markets both symmetric and asymmetric GARCH models with student t distribution have passed the Kupiec and Christoffersen with a 99% level of confidence. Whereas in the case of China, the models (with GED error distributions) have failed the unconditional and independence test with a 99% level of confidence.

Notes

(1) David McMillan and Pako Thupayagale (2010).
(2) Some point in time the exceptions are higher and sometimes lower.
(3) Mandelbrot (1963) noted: "...large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes..."

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