

## Portfolio optimization with VaR approach: A comparative analysis for Japan, London, New York and India

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**Abstract.** *Risk managers use various types of techniques to estimate different kinds of risk and ways to minimize its impact. VaR which stands for Value at Risk is one of those techniques. Various new methods for calculation of VaR have been developed. In this study, four techniques of VaR estimations have been employed: i) Historical Simulation; ii) Variance Covariance Approach; iii) Monte Carlo Simulation, and iv) AR-GARCH method. The purpose of this study is to compare the different VaR estimation methods and draw conclusions based on the Back- Testing methods.*

*As per the analysis, historical method proved to be the best method for estimating value at risk. This method is widely preferred by risk managers and practitioners in the banking sector. Though the portfolios used in the study was diversified and contained stocks from different sectors, still the historical simulation method came out to be on the top as it was accepted for all the four portfolios. This method does have some limitations as the patterns generated from the past data may not hold true all the time.*

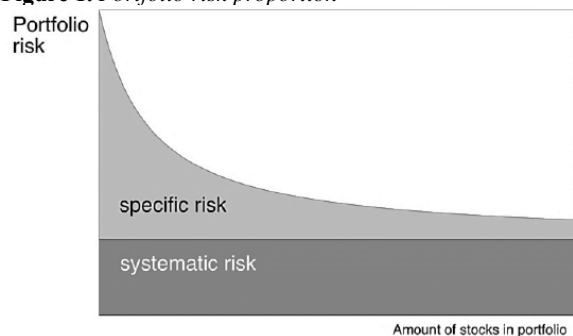
**Keywords:** Value-at-Risk (VaR); simulation model; variance-covariance matrix; Monte Carlo simulation; GARCH approach.

**JEL Classification:** D53, E17, E37, F65, G14, G17.

## Introduction

Uncertainties in the world of finance are inevitable. If an investor is willing to invest in various assets, then he can never avoid facing risk. Magnitude of risk varies across different domains of assets class. The primary role of risk managers is to hedge risk to avoid maximum loss and gain as much profit as possible. Risk managers use various types of techniques to estimate different kinds of risk and ways to minimize its impact. VaR which stands for value at risk is one of those techniques. It is one of the popular methods for estimating the market risk. “*VaR is a tool which gives a possibility to estimate the total risk of the portfolio*”, defined by John C. Hull. Total risk of a portfolio is categorized into two components: i) specific risk and ii) systematic risk. Specific risk refers to a kind of risk which can be diversified and thus be reduced or removed whereas Systematic risk is a type of risk which cannot be diversified and it depends on various factors like economic conditions of the country, inflation, political structure and their influence and on the stock market.

**Figure 1.** *Portfolio risk proportion*



In the past decade, VaR has become one of the most common techniques for risk management. In 1994, JP Morgan introduced *Risk Metrics*, a newly designed risk measured system which included various measures for risk over 300 financial instruments across 14 countries with proper defined methodologies of value at risk. The BCBS which stands for Basle Committee on Banking Supervision amended the Basel Capital Accord and later on obliged the banks which aligned with the Basel Committed to reserve capital based on the measurement of their market risk using the Value at Risk (Basle Committee, 1996). According to these norms, banks now must maintain three times of their VaR measurements in order to cover itself from potential risks. Other than that, banks can introduce their own internal methods and should be subjective to approval by Bank for International Settlements. While VaR is a standard tool for measuring risk, it has undergone various refinements. Obtaining accurate value of VaR has become the most important challenge for researchers and risk managers. Various new methods for calculation of VaR have been developed. In this project we will be covering the following techniques of VaR estimations: i) historical simulation; ii) Variance Covariance Approach; iii) Monte Carlo Simulation, and iv) AR-GARCH method.

## Value at Risk

*“Value at Risk is such a loss in market value of a portfolio that probability it will occur or even will be exceeded in a given period of time is equal to the predefined tolerance level”* (Jajuga, 2007). For example, a VaR value at 95% confidence interval of 10% risk depicts an expected loss of at least 10% one of every 20 days on average.

Mathematically, it can be represented as:

$$P[V \leq V_0 - VaR] = \alpha$$

Where:

V – value of the portfolio at the end;

V<sub>0</sub> – value of the portfolio at the beginning;

VaR – Value at Risk;

$\alpha$  – significance level.

Thus, VaR can be interpreted as the worst-case scenario loss over a specified time period.

Another way of representing value at risk mathematically would be:

$$P(r_t \leq F_{r,t}^{-1}(\alpha)) = \alpha,$$

$$VaR_{r,t}(\alpha) = F_{r,t}^{-1}(\alpha),$$

Where:

r<sub>t</sub> – rate of return on the portfolio;

F<sup>-1</sup><sub>r,t</sub>( $\alpha$ ) – quantile of loss distribution related to the probability of 1– $\alpha$ .

There are various approaches for measuring value at risk. This study has included four different methods: i) Historical Simulation; ii) Variance Covariance Approach; iii) Monte Carlo Simulation, and iv) AR-GARCH method.

## Historical simulation

Historical Simulation is a VaR technique which uses the historical data of returns in order to obtain the value at risk. A N<sup>th</sup> percentile is chosen as a target value which eventually determines the value at risk. For example, we want to determine the VaR by using the historical worst 1% of daily return as the targeted value. All the daily returns of the past data of the specified time period will be ranked from the lowest to highest and then the 1<sup>st</sup> percentile will be taken as the value at risk value. This value will then be multiplied by the current value of the asset to get the 1-day VaR estimate.

In case of a portfolio, daily returns for each stock is calculated and then multiplied by the respective weights assigned to it. Total return is calculated by adding the returns of all the stock. Further the daily returns thus calculated are ranked from lowest to highest and then the n<sup>th</sup> percentile is calculated and the required VaR is calculated. Also, it is again multiplied by the current value of the portfolio to get the 1-day VaR estimate.

The historical simulation technique is the most used VaR technique practiced by the risk managers. One of the best advantages of this technique is that it is easy to use and implement. It does not require any complex mathematical calculations or any software or platform to calculate. Also, the data need not be normally distributed. It can be of any distribution. It only uses the past data of returns and the past patterns irrespective of its distribution and provide estimates based on that.

However, the assumption of constant pattern of distribution may not be true or it may be true for short time horizon that the newer data explains the behavior of returns better than older data does. On the other hand, the method requires significant level of sample size because the  $N^{\text{th}}$  percentile being selected must come from large discrete data to make sense. *Hendricks* figured out that greater the sample size, lesser will be the variance in VaR estimation. He also found imprecise VaR estimate when the sampling period is too short. The major drawback of this method is the difficulty in selecting the proper sample size for its calculations.

### Variance covariance approach

Variance Covariance method also known as the Linear VaR or Delta Normal VaR is a simple and most used method for estimating value at risk. It uses some application of the Markowitz model given by Harry Markowitz, which uses the correlation factor among different assets. With the help of the historical data, daily returns can be calculated which further help in calculating the different parameters like mean, standard deviation and the correlation among them. It assumes that the data follows any specific distribution which is usually normally distributed. This helps in defining volatility in terms of the standard deviation of the portfolio. Since the data is normally distributed, this implies that it is symmetrical thus, it has a skewness 0 and a kurtosis 3. If we want to find position of a random variable (X) in a normal distribution, we use standard value of variable Z (Z-score).

Every variable can be transformed to standard variable with formula:

$$X = \mu + z \sigma$$

Where:

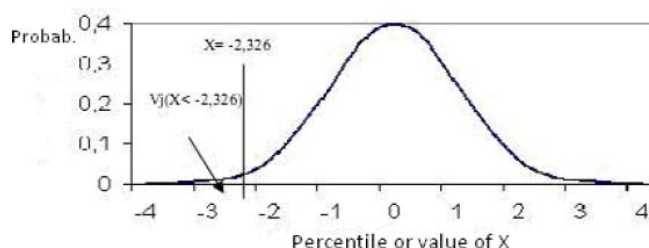
$$Z = (X - \mu) / \sigma;$$

$\mu$  – mean;

$\sigma$  – standard deviation.

Thus, VaR is calculated as the measure of the standard deviation.

**Figure 2.** Normal distribution depiction



The above figure shows normal distribution with value of  $x -2,326 \sigma$ . Probability for losses greater than  $x$  is shown as an area under the normal curve, left of  $x$ . This area is 1% of the total area under the curve, so there is a confidence of 99% that losses will not exceed  $-2,326 \sigma$ . During VaR calculation, the downward price changes or the price changes that exceeds some multiple of standard deviation.

VaR for the entire portfolio can be calculated by using the formula:

$$\text{VaR}_p = (Z \sqrt{V} P)$$

Where:

$Z$  – standard value;

$V$  – volatility or the standard deviation of the portfolio;

$P$  – portfolio position value.

Usually it is calculated by using the matrix formula:

$$\text{VaR}_p = (V C V^T)^{1/2}$$

Where:

$V$  – row vector of VaRs for each position;

$C$  – matrix of correlations.

### Monte Carlo simulation

The Monte Carlo simulation is a process which helps in generating random price paths to determine the approximate movements of future prices of the financial assets. In the application of VaR estimation, by Monte Carlo simulation, we can generate the probability distribution for rates of return of portfolios. In order to calculate a 1-day VaR for a portfolio, Monte Carlo simulation follows below procedures:

- Determine the current market value of the portfolio.
- Generate random samples of rates of return for all underlying assets of the portfolio from multivariate normal distribution.
- Apply the samples of returns for each asset and obtain their sampling values from current market values.
- Compose the portfolio value based on the sampling values of assets and find the rate of change from current market value of the portfolio.
- Repeat above steps many times to generate the probability distribution of rates of return of the portfolio.

Now the VaR of the portfolio is directly gathered from the distribution of simulated portfolio values. One way to implement step 2 is to estimate ST based on below equation:

$$S_T = S_0 \exp \left[ \left( \mu - \frac{\sigma^2}{2} \right) + \sigma \varepsilon \right]$$

Where:

$\varepsilon$  – random sample from a standard normal distribution.

A common way to generate the  $\varepsilon$ 's is from following function:

$$\varepsilon = \sum_{i=1}^{12} R_i - 6$$

Where:

$R_i$  – independent random numbers between zero and one.

In the world of the financial modelling, Monte Carlo simulation has proven to be a powerful tool due to its flexible approach. This approach can be used to generate random scenarios or scenarios by using historical data. One of the approach follows the normal distribution pattern for estimating the VaR using Monte Carlo simulation. This approach has undergone criticism as well as appreciation. Some practitioners believed that this approach is better than the historical simulation and produced better results while incorporating fat tails.

The biggest drawback of this method is that it generates iterations randomly which may rupture the results or could lead to biasedness in the results. The values obtained may not always align with the market. Also, compared to other approaches, the marginal cost for this method is higher than its benefits.

#### AR-GARCH method

VaR estimation using GARCH models is one of the common methods for estimating daily volatility. It doesn't mull over the normality of returns obtained. *"GARCH model is a model that estimates the conditional variance of an asset's returns as a function of past returns and conditional variances."*

Parameters of a GARCH(m, n) models for a portfolio can be calculated using the following equations:

$$r_t = \varepsilon_t \sqrt{h_t}, \quad \varepsilon_t \sim RB(0, \sigma^2)$$

$$h_t = \omega + \sum_{i=1}^m \alpha_i r_{t-i}^2 + \sum_{j=1}^n \beta_j h_{t-j}$$

The second equation has the following constraints associated with it:

$$\omega > 0, \alpha_i \geq 0, i = 1, \dots, m-1, \alpha_m \neq 0, \beta_j \geq 0, j = 1, \dots, n-1, \beta_n \neq 0$$

Where:

$\omega, \alpha_i, \beta_j$  – parameters of the model;

$h_t$  – conditional variance of the returns of the portfolio;

$\varepsilon_t$  – white noise.

In accordance with the above model, the conditional variance is given by the following equations:

$$\hat{h}_t[k] = E[h_{t+k} | \mathcal{F}_t]$$

Error term in the conditional variance forecast is given by:

$$e_t[k] = \sqrt{\hat{h}_t[k]}$$

And thus, the final equation for estimating the value at risk is:

$$VaR[k] = C\left(q(p)e_t[k]^*\right)$$

### Literature review

VaR is the most used tool for estimating market risk by the risk managers. It is a method which is used by the managers and practitioners daily to determine the possible worst-case scenario of the loss that they can incur (Stefaniak, 2018). Investors need to choose the method in accordance with the properties and characteristics exhibited by their portfolio (Corkalo, 2011). VAR methods are being commonly used by different banks, pension funds and mutual funds. Their weakness was examined by using five backtesting techniques which are test based on the time until first failure, test based on failure rate, test based on expected value, test based on autocorrelation and test based on mean absolute percentage error. Although GARCH approach was the best alternative for the VaR estimating technique. Results for different price indices were different using all the methods (Sinha and Chamu, 2000).

Back testing of the independence of first and higher orders were implemented for the 4 mentioned models above, for a forecast period of 1 and 10 days. The commitment depends on a more thorough criteria than those utilized in the writing for approving VaR models, as we performed backtesting for infringement autonomy of higher requests on conjecture time of 10 days. The final products indicated that just GARCH models were giving the satisfactory outcomes (Maluf and Asano, 2019). A new approach was used in this study which is the extreme value theory (EVT) and the kernel estimator technique. The data was taken for 13 years and the end results stated that the VaR estimates that were generated through the extreme value theory and kernel estimator techniques were far better than the traditional approaches of the value at risk (YiHou, 2000).

The mean VAR efficient frontier has been tested for its predictive abilities and has been found to be a good tool for determining value at risk for investors with which portfolio decisions may be made (Gaivoronski and Pflug, 2005). VAR may be helpful in predicting the behaviour of portfolios on the basis of their past observations. However, it has been suggested that to improve the ability of the model minimum VAR constraints may be adopted (Charpentier and Oulidi, 2009). VAR adjusted Sharpe ratio was proposed in the study in comparison to the traditional Sharpe ratio. It was found that worse-case scenario adjustment provided better results from VAR-Sharpe ratio than the later (Deng et al., 2013).

Optimization techniques have been used in three phases with simple solver to theoretical convergence. It has been found that the phase wise comparison of portfolios improvised the analytics for portfolio development and revision (Lim et al., 2010). A movement from traditional Markowitz model and Sharpe ratio requires introducing constraints on the

various portfolios. The study tried to assimilate the results from various traditional models and linear programming models for portfolio optimization (Ogryczak et al., 2017). The employment of column generating algorithm allows for better results from Copula based conditional value at risk models. A deeper analysis may be done with the help of these models for study of pessimistic scenarios (Krzemienowski and Szymczyk, 2016).

Normal distribution and historical simulations have been used to analyse the bond portfolios. Findings have suggested that these distributions provide reliable results to predict VAR. However, this technique has been found to be weak for showing results with constraints imposed on portfolios and in real life hidden risk may always be present (Winker and Maringer, 2007). Diversification strategies have been tested with various approaches like Markowitz, conditional VAR and back testing during the global financial crisis. It has been discovered that the simple diversification strategies have been generating the best estimates among others (Allen et al., 2016).

Linear programming models have started gaining popularity in estimating value at risk for various investment portfolios. The capability to introduce various constraints while appraising the risk profile of the portfolios allows for more comparisons (Mansini et al., 2007). The theory of Markowitz model has been studied with few violations and it has been found that a simple optimization technique may help individual investors to earn superior returns as compared to value-weighting approach (Dye and Groth, 2000).

## Methodology

### Sample size

The sample size considered for this study are four portfolios containing four stocks each from different sectors. They are selected based on their market capitalization. Stocks with the highest market capitalization are chosen for the study. Portfolios are diversified with reference to their geographical locations in order to increase the independence of the results and the influence of systematic risk on the results.

### Data

The data taken is 10 years' time series data from 2010-19 with varied observations representing the stock markets of Japan- 2447 (Nikkei225), London- 2522 (FTSE100), New York- 2519 (DJIA) and India- 2479 (Nifty50).

### Components of the portfolio

- Nifty50 (Reliance Industries, Tata Consultancy Services (TCS), Bharti Airtel, Sun Pharmaceuticals Ltd.)
- Nikkei225 (Toyota Motor Corporation, Mitsubishi UFJ Financial Group, Docomo; Soft Bank Group)
- DJIA (Microsoft, Exxon, Apple, Johnson and Johnson)
- FTSE100 (Royal Bank of Scotland, British American Tobacco, BP, Unilever)

All the stocks in each portfolio is assigned a weightage of 25%. VaR is estimated at a 95% confidence interval.



### Techniques used

After the data collection comparison of different VaR techniques is done. The techniques used are as follows:

- Historical Simulation.
- Variance-Covariance Approach.
- Monte Carlo Simulation.
- AR-GARCH approach.

For calculating the returns of the portfolio, log returns is used for calculating the returns generated by each stock in a portfolio and then they are multiplied by their weights assigned to determine the total returns of the portfolio.

Log returns is calculated by using the formula:

$$R_t = \ln(P_t/P_{t-1})$$

Where:

$R_t$  – log return of the stock;

$P_t$  – price of the stock at present day;

$P_{t-1}$  – price of the stock the previous day.

Lastly, these tests were backed by the backtesting methods in order to validate the results.

### Back testing

After a VaR model is developed, it is important to check how reliable the results are and what is the accuracy of the results obtained. The statistical method for estimating VaR accurately is called backtesting. The aim of backtesting is to determine whether the amount of losses calculated by VaR is correct. For this purpose, various tests are implemented. These tests check whether the frequency of exceptions, during the prescribed time interval, are in accordance with the confidence level chosen. The commonly used tests are:

- **POF Test:** It is one of the most common used backtesting methods used by the practitioners. It measures how many exceptions occur in the model i.e. when the actual return is less than the estimated VaR value. It is very simple to calculate as it does not require many variables. The only information required for this are i) number of observations, ii) number of exceptions and iii) the confidence level. The null hypothesis for the test is:

$$H_0: p = \hat{p} = \frac{x}{T}$$

The rationale behind this is figure out whether the observed failure rate is in sync with the actual failure rate. It works based on the likelihood ratio test.

$$LR_{POF} = -2\ln\left(\frac{(1-p)^{T-x}p^x}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^x}\right)$$

Null hypothesis for this is that the model is correct. If the value of the  $LR_{POF}$  is greater than the critical value, then the null hypothesis is rejected, and the model is inaccurate.

▪ **Loss function:** This method of backtesting does not take into consideration any hypothesis, instead, it is based on determining the magnitude of loss that can occur if any violation occurs. The loss function could be stated as:

$$L(VaR_t(\alpha), x_{t,t+1}) = \begin{cases} 1 + (x_{t,t+1} - VaR_t)^2 & \text{if } x_{t,t+1} \leq -VaR_t(\alpha) \\ 0 & \text{if } x_{t,t+1} > -VaR_t(\alpha) \end{cases}$$

This score increases with the magnitude of loss. For conducting the backtest method, a benchmark loss is taken and when the sample loss is greater than the benchmark loss then the model is rejected.

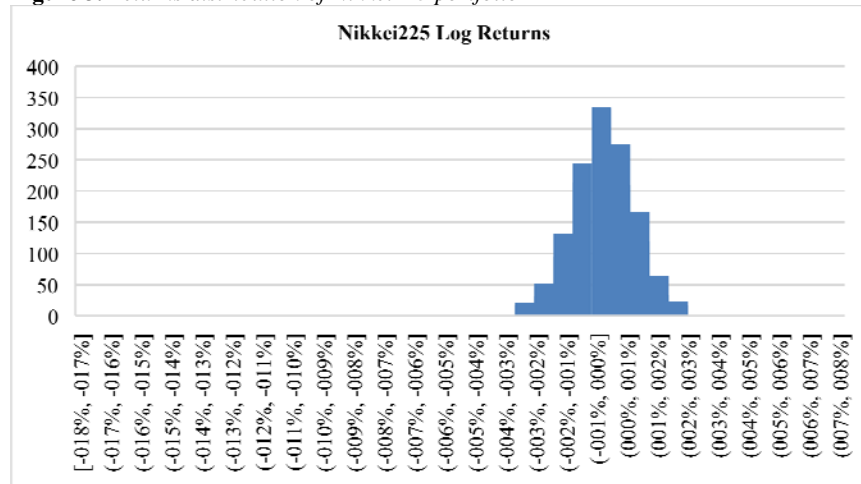
### Data analysis

After the data collection, their properties and characteristics are examined using descriptive statistics which is shown in Table 1 below:

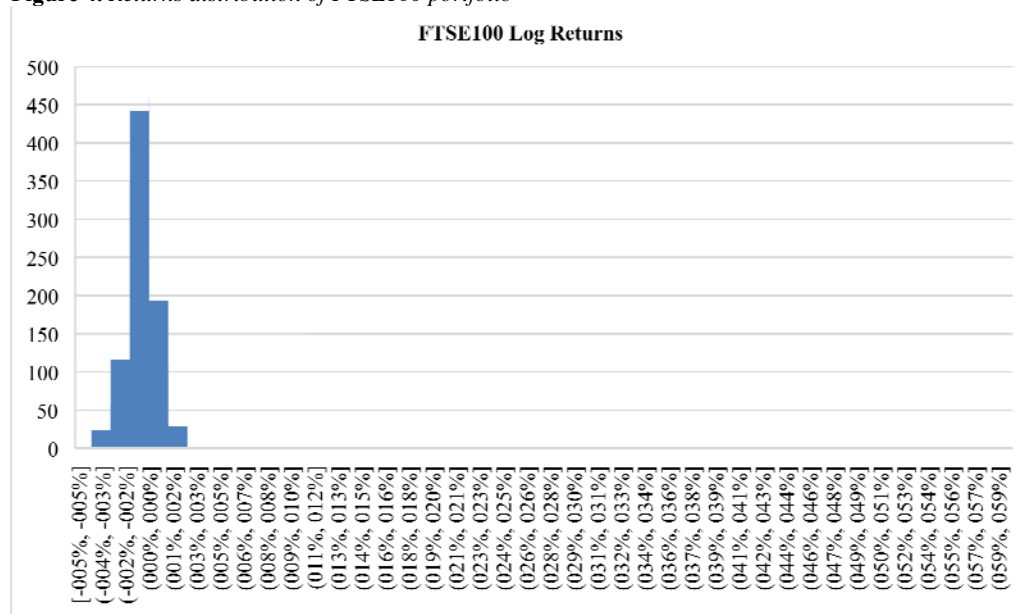
**Table 1.** Comparative Analysis of the returns of the portfolios

Indices	Nikkei225	FTSE100	DJIA	Nifty50
Mean	0.0002	0.0003	0.0005	0.0004
Minimum	-0.1750	-0.0529	-0.0471	-0.0595
Maximum	0.0727	0.5983	0.0530	0.0378
Count	2447	2522	2515	2479

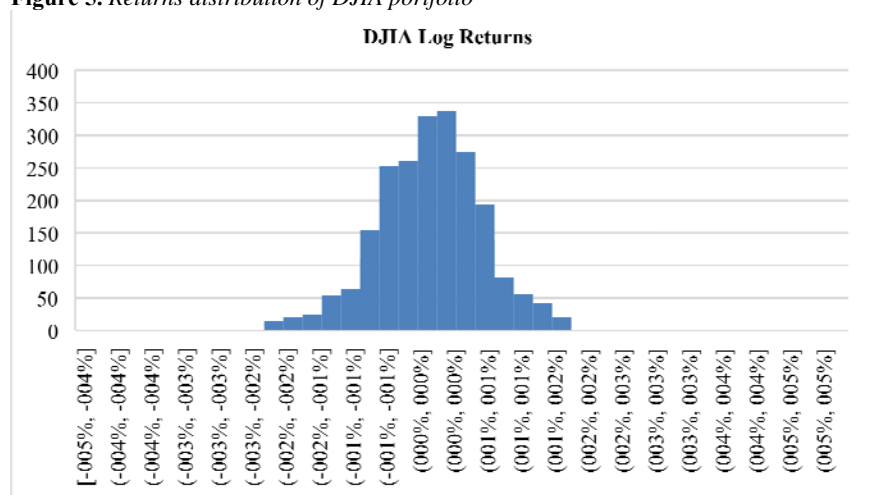
**Figure 3.** Returns distribution of Nikkei225 portfolio



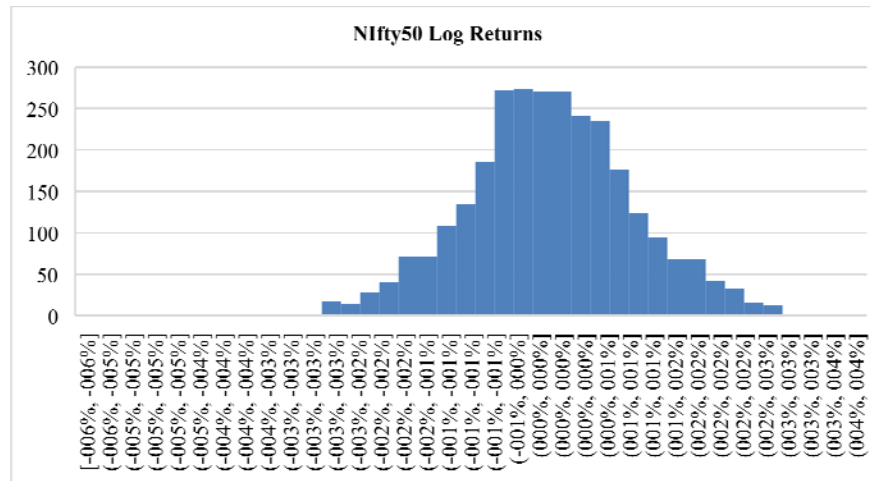
The returns generated by the portfolio shows that initially the frequency is close to zero and then it increases with the high returns generated and then again it closes to zero following a bell shape curve.

**Figure 4.** Returns distribution of FTSE100 portfolio

The returns generated by the portfolio shows that initially the frequency is close to zero and then it increases with the high returns generated and then again it closes to zero for high number of returns following a bell shape curve.

**Figure 5.** Returns distribution of DJIA portfolio

The returns generated by the portfolio shows that initially the frequency is close to zero and then it increases with the high returns generated and then again it closes to zero following a bell shape curve.

**Figure 6.** Returns distribution of Nifty50 portfolio

The returns generated by the portfolio shows that initially the frequency is close to zero and then it increases with the high returns generated and then again it closes to zero following a bell shape curve.

### Empirical results and findings

After the preliminary analysis is done, following results are generated out of testing the VaR Models:

**Table 2.** VaR calculations for Nikkei225

Nikkei225				
Model	Variance Covariance	Historical Simulation	Monte Carlo	AR-GARCH
Confidence Interval	95%	95%	95%	95%
VaR	-2.15%	-1.85%	-0.13%	-1.89%

**Note:** Own compilation done on MS-Excel.

The results from the above table indicates the calculations of the value at risk using different methods. From Table 2, for the Nikkei225 index the variance covariance approach gives the highest VaR estimate of -2.15% whereas the results from historical and AR-GARCH methods are almost identical. Monte Carlo simulation has produced the lowest VaR estimate for this portfolio.

**Table 3.** VaR calculations for DJIA

DJIA				
Model	Variance Covariance	Historical Simulation	Monte Carlo	AR-GARCH
Confidence Interval	95%	95%	95%	95%
VaR	-1.42%	-1.29%	-0.09%	-1.27%

**Note:** Own compilation done on MS-Excel.

From Table 3, for the DJIA index the variance covariance approach gives the highest VaR estimate of -1.42% whereas the results from historical and AR-GARCH methods are almost similar. Monte Carlo simulation has produced the lowest VaR estimate for this portfolio.

**Table 4.** VaR calculations for FTSE100

FTSE100				
Model	Variance Covariance	Historical Simulation	Monte Carlo	AR-GARCH
Confidence Interval	95%	95%	95%	95%
VaR	-2.77%	-1.74%	-0.13%	-2.00%

**Note:** Own compilation done on MS-Excel.

From Table 4, for FTSE100 index the variance covariance approach gives the highest VaR estimate of -2.77% whereas the results from historical simulation is -1.74% and AR-GARCH methods is -2.00%. Still the results produced by these two methods almost lie near to each other. Monte Carlo simulation has produced the lowest VaR estimate for this portfolio.

**Table 5.** VaR calculations for Nifty50

Nifty50				
Model	Variance Covariance	Historical Simulation	Monte Carlo	AR-GARCH
Confidence Interval	95%	95%	95%	95%
VaR	-1.78%	-1.69%	-0.11%	-1.65%

**Note:** Own compilation done on MS-Excel.

From Table 5, for Nifty50 index all the approaches except the Monte Carlo simulation is similar. Monte Carlo gives the lowest VaR estimate.

### POF-Test results

The abbreviations used in this tests are as follows:

T – total number of observations;

x – number of exceptions occurred;

CI – confidence interval, in this case, 95% confidence interval is used;

LR – Likelihood ratio following the chi-square distribution for n degree of freedoms.

As mentioned earlier if the exceptions occurrence percentage is less than or equal to the significance level, i.e. 1-CI, then the model is estimating the results accurately. Also, the LR ratio should be less than the critical value, which is 3.84 in our case, for the model to be accurate and accepted.

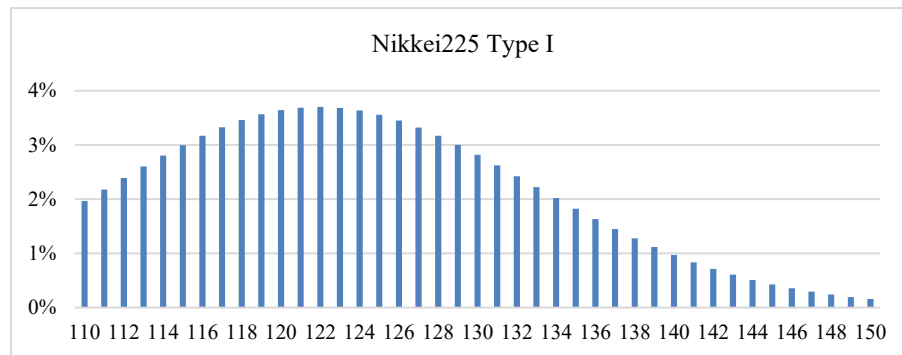
Type I error refers to rejecting a correct model and Type 2 refers to accepting an incorrect model. Suppose if the number of exceptions is selected as 110 for rejecting a model, then by using the binomial distribution, the probability of committing a type 1 error would be 2%.

**Table 6.** POF backtest results for Nikkei225

Nikkei225							
	T	x	T-x	x/T	CI	LR	Inference
Historical	2447	124	2323	5%	95%	0.0233	Accept
Variance-Covariance	2447	85	2362	3%	95%	3.3374	Accept
AR-GARCH	2447	2340	107	96%	95%	12.6588	Reject
Monte Carlo	2447	1044	1403	43%	95%	5.2479	Reject

**Note:** Own compilation done on MS-Excel.

**Figure 7.** Type I error of Nikkei225



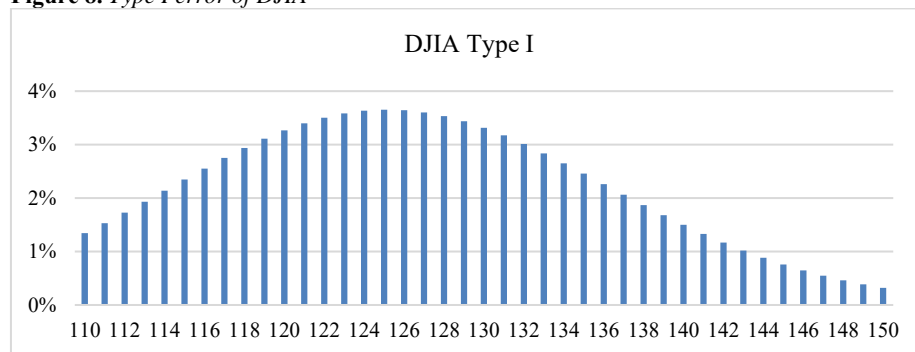
From Table 6, only historical and variance-covariance approach models were accepted for Nikkei225 portfolio. Assuming the exception for rejecting the model to be at 125, there is a 4% probability of committing a type I error for this portfolio.

**Table 7.** POF backtest results for DJIA

DJIA							
	T	x	T-x	x/T	CI	LR	Inference
Historical	2515	126	2389	5%	95%	0.0005	Accept
Variance-Covariance	2515	1130	1385	45%	95%	4.1987	Reject
AR-GARCH	2515	104	2411	4%	95%	3.1972	Accept
Monte Carlo	2515	1044	1471	42%	95%	14.1987	Reject

**Note:** Own compilation done on MS-Excel.

**Figure 8.** Type I error of DJIA

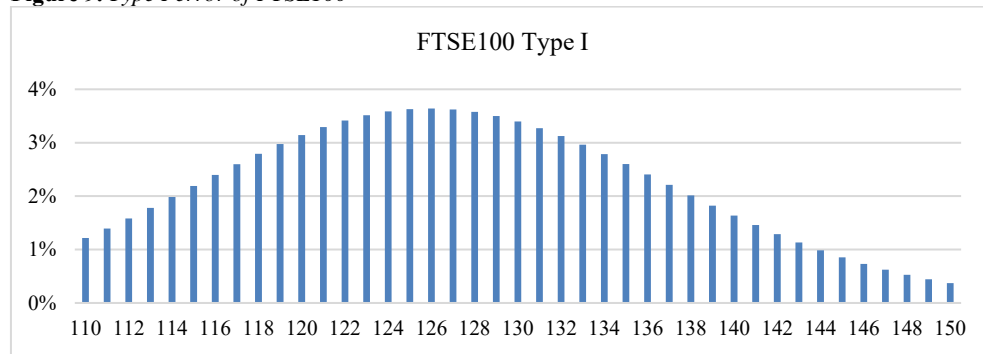


From Table 7, only historical and AR-GARCH approach models were accepted for DJIA portfolio. Assuming the exception for rejecting the model to be at 125, there is a 4% probability of committing a type I error for this portfolio.

**Table 8.** POF back test results for FTSE100

FTSE100							
	T	x	T-x	x/T	CI	LR	Inference
Historical	2522	127	2395	5%	95%	0.0067	Accept
Variance-Covariance	2522	1245	1277	49%	95%	13.3374	Reject
AR-GARCH	2522	67	2455	3%	95%	3.4907	Accept
Monte Carlo	2522	1046	1476	41%	95%	15.2479	Reject

**Note:** Own compilation done on MS-Excel.

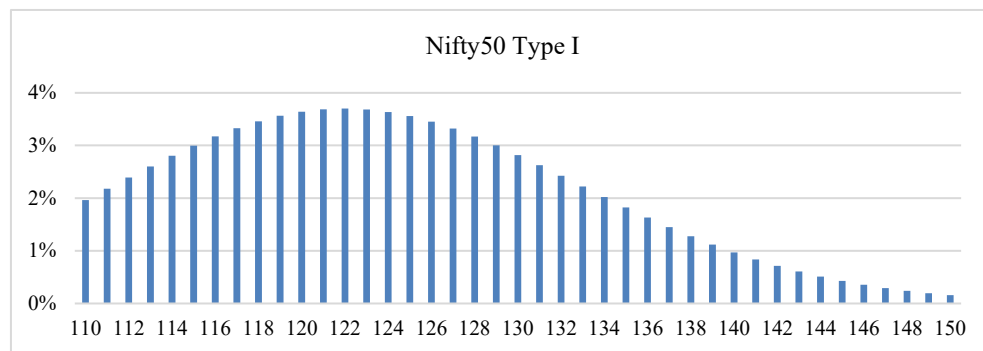
**Figure 9.** Type I error of FTSE100

From Table 8, only historical and AR-GARCH approach models were accepted for FTSE100 portfolio. Assuming the exception for rejecting the model to be at 125, there is a 4% probability of committing a type I error for this portfolio.

**Table 9.** POF back test results for Nifty50

Nifty50							
	T	X	T-x	x/T	CI	LR	Inference
Historical	2479	124	2355	5%	95%	0.0000	Accept
Variance-Covariance	2479	1218	1261	49%	95%	17.8757	Reject
AR-GARCH	2479	112	2367	5%	95%	1.2516	Accept
Monte Carlo	2479	1108	1371	45%	95%	8.9954	Reject

**Note:** Own compilation done on MS-Excel.

**Figure 10.** Type I error of Nifty50

From Table 9, only historical and AR-GARCH approach models were accepted for Nifty50 portfolio. Assuming the exception for rejecting the model to be at 125, there is a 4% probability of committing a type I error for this portfolio.

#### Loss function results

As mentioned, loss function considers the magnitude. For this the average sample loss is calculated and a benchmark loss figure is considered for its calculation. If the average sample loss is greater than the benchmark loss, then the model is rejected.

In this case the benchmark was taken as the maximum return produced on a single day for the period of 10 years. Reason being that if the investor has made a big return in a period

of 10 years say, then he will not be willing to lose that amount. So, if the VaR estimate is giving a loss greater than his maximum profit, the investor will not consider the implications of such a model and will reject it.

**Table 10.** Loss Function backtest results for Nikkei225

Nikkei225			
	Sample Loss	Benchmark Loss	Inference
Historical	5.03%	7.27%	Accept
Variance-Covariance	3.49%	7.27%	Accept
AR-GARCH	4.37%	7.27%	Accept
Monte Carlo	44.37%	7.27%	Reject

**Note:** Own compilation done on MS-Excel.

**Table 11.** Loss Function backtest results for DJIA

DJIA			
	Sample Loss	Benchmark Loss	Inference
Historical	5.01%	5.30%	Accept
Variance-Covariance	4.14%	5.30%	Accept
AR-GARCH	4.14%	5.30%	Accept
Monte Carlo	39.70%	5.30%	Reject

**Note:** Own compilation done on MS-Excel.

**Table 12.** Loss Function backtest results for FTSE100

FTSE100			
	Sample loss	Benchmark Loss	Inference
Historical	5.04%	5.30%	Accept
Variance-Covariance	1.03%	5.30%	Accept
AR-GARCH	2.66%	5.30%	Accept
Monte Carlo	41.48%	5.30%	Reject

**Note:** Own compilation done on MS-Excel.

**Table 13.** Loss Function backtest results for Nifty50

Nifty50			
	Sample Loss	Benchmark Loss	Inference
Historical	5%	3.78%	Reject
Variance-Covariance	4.20%	3.78%	Reject
AR-GARCH	4.52%	3.78%	Reject
Monte Carlo	44.70%	3.78%	Reject

**Note:** Own compilation done on MS-Excel.

From Tables 11, 12 and 13, except for the Monte Carlo simulation method, all the other models are accepted for Nikkei225, DJIA and FTSE100 portfolio. From Table 13 all the methods are rejected by the loss function backtesting technique.

## Conclusion

The purpose of this study was to compare the different VaR estimation methods and draw conclusions based on the backtesting methods. As per the analysis, historical method proved to be the best method for estimating value at risk. This method is widely preferred by risk managers and practitioners in the banking sector. Though the portfolios used in the study was diversified and contained stocks from different sectors, still the historical simulation method came out to be on the top as it was accepted for all the 4 portfolios. This method does have some limitations as the patterns generated from the past data may not hold true all the time.



Another alternative that prompted out of this study was the AR-GARCH approach of estimating the volatility. This method was accepted for 3 portfolios as well and could be the second alternative for the VaR estimation as per our study. Variance Covariance approach was also accepted in a few cases, but the overall implication of this method does not provide accurate results. Monte Carlo proved to be the most ineffective method in our study and was rejected for all the portfolios.

To conclude, one cannot determine a standard method for VaR estimation. It depends upon the properties and the characteristics possessed by the portfolio. Also, for different size of portfolios different results can be obtained using different methods. Risk managers should choose their method wisely after attaining proper information about the portfolio and the market.

#### Limitations and scope for further study

VAR does not measure worst case loss. 95% percent VaR means that in 5% of cases the loss is expected to be greater than the VAR amount. Value at Risk does not say anything about the size of losses within this 5% of cases and by no means does it say anything about the maximum possible loss. There are many alternative techniques available now like expected shortfall and CoVaR, which provides the magnitude of the losses and thus preferred more by the investors and managers to determine the risk. Different VaR models are appropriate for different portfolios depending upon their properties and characteristics of distribution of returns. This study was limited to only four portfolios and the results are limited to those only. The study only restricted to high-capitalization stocks only, scenarios for mid and small capitalization are not considered in this study. Only traditional approaches are used in this study for calculating VaR, new emerging methodologies like transformation-based approach and extreme value theory are not included in this study. These considerations may be used for studying the VAR strategies in portfolio diversification further with more indices and models.

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