Theoretical and Applied Economics
Volume XXXI (2024), No. 4(641), Winter, pp. 53-66

A simple model of online marketing, production, profit and growth

Biswajit MANDAL
Visva-Bharati University, India
bisuman@gmail.com
Labani Roy CHAKRABORTY
Visva-Bharati University, India
niroja.lab2018@gmail.com
Alapan SANYAL
Visva-Bharati University, India
subhashree.sanyal@visva-bharati.ac.in

Abstract. *We build an optimization model to decipher the effects of digitalization in trading or marketing. The basic model uses Cobb -Douglas production function to argue that a shift from offline to online may induce increase in profit and subsequent economic growth. The factor-intensity of trading has a role in this context. Additionally, night time utilization due to digitalization further strengthens our results. The extended model with a CES production function gives similar results. However, revenue and cost implications are not same in these two cases. Nevertheless, the essence of the basic results holds true even in a more generalized case.*

Keywords: digitalisation, selling, output, profit, growth.

JEL Classification: D0, D3, D23, D24, O14, O40.

1. Introduction

A very interesting change took place after digitalization crept in the world. All the consumers have gradually become accustomed with digitalization which has caused a part of the production-transaction combo go online. The production part, however, takes place offline but the trading part has shifted online in our digitalized regime. Due to such shift from offline to online, it has become more capital-intensive. Globalization has enabled digitalization to be strengthened all across the world with the help of high bandwidth internet service. The structural change in the pattern of marketing which has been brought about by the emergence of digitalization has become so adoptable by both of the market agents: Producer and consumer that it has become an integral part of our daily life.

The very concept of digitalization is widely extended and it has been building its dominance over the past few years. In recent times, to survive a competitive market structure, brands need to increase their online visibility and this has led open to a new chamber of online services strengthening the IT sectors. Talking from a very miniscule level of understanding, we can notice that by increasing the online visibility of a business, it acquires greater probabilities of generating greater revenues. It is so, because the online market is open 24x7; thereby increasing the accessibility of availing the product by the consumers.

Officially the digitalization in India was boosted in the year 2015, with the launching of the Digital India programme. But the roots of the digitalization in India dates back to the late 90s, when globalization was introduced in India. The expansion in the IT sectors and the easy accessibility of internet after the Digital India programme further shaped the future of digitalization in India. Digitalization has helped the businesses of various sectors to be widely visible and accessible to people regardless of their age, gender, social background etc. There are several reasons for digitalisation of sales becoming so well received. Due to digitalization, the products are available online to a wide range of consumers. Business firms of different scales find the same platform to represent themselves thus infusing a healthy competition. Market interactions do not need to be offline anymore after digitalization thus saving a lot of time. Less chances of price discrimination as information available online is same for all. Digitalisation has led open a vast area of research namely, the concept of night time accessibility to market. This phenomenon is often used in recent trade literature [(Marjit, Mandal and Nakanishi (2020)] to bring some interesting upshots owing to virtual transaction.

In addition to this, the Covid -19 period witnessed an upsurge in digitalization. After the governments of different countries announced lockdowns, the only way these countries could survive was to access others through digital platforms. In India also, the lockdown phase left people with no other option but to go digital for every possible activity. On the other hand, we have also seen how the global pandemic has laid the foundation of digitalized marketing structure. Covid-19 is surely one of the primary reasons why digitalization has been extensively used by the people. The consumers were to avail online marketing options during Covid -19 lockdown as no more options were left. The contactless feature of the digital marketing played a pivotal role in containing the spread of the disease. For the producers, the scope to sell their products in offline markets was diminished, which led them to shift their products online. To survive, the influx of consumers in the available structure of digitalization, there has been expansion in the IT sectors as well. This is how Covid-19 fuelled digitalization. On the other hand, digitalization is the reason why the world has survived the pandemic situation. So, in a way Covid-19 and the subsequent lockdown acted like a catalyst for the proliferation of digitalization all across the globe.

With the shift of the trading to online mode, a whole new sphere of trading has developed. Each and every brand ranging from small-scale to large-scale can trade 24x7 and even on night times. This has not only led to a wider scope of earning revenues for the small-scale businesses but also has given them the opportunity to be accessible to a wide range of consumers. And, we all know that an all-round digitalization made this possible. Hence, we capture, this process of transition from offline type of trading to online-offline hybrid mode of trading. Such phenomenon isn't only contemporary but also widely followed in every aspect of our daily life. Therefore, it calls for a formal model for underpinning the theoretical rationales behind such remarkable transformation.

Earlier both the production of goods and its selling used to be held offline but with times digitalization has departmentalized the concept of production into two components. The first part, production remains unchanged in its offline domain involving both capital and labour. The later part as that of the trading/marketing is where the real transition occurs. With digitalization, there has been a shift in the domain of trading from offline to online. As stated earlier, trading or the marketing part of the production can be made online with just few taps on our internet bearing devices. Therefore, marketing or selling doesn't require an offline market anymore. The online market is enough to suffice to the demands of a wide range of consumers, consuming a wide range of products. Such trading or transaction of goods or services isn't a new phenomenon. The issue is already there in the literature. The only difference is that it was not so much technology driven few decades ago. Falvey (1976) is a pioneering paper that introduced trading as a separate activity as a non-traded domestic good. The idea of non-traded domestic good is also very aptly pointed out in a paper by Marjit and Mandal (2012). Deardorff (2004), Laussel and Riezman (2008) are among the noted contributors in this line of research. Another paper by Mandal and Marjit (2010) also takes care of similar kind of transaction cost in a typical trade theoretic model to shed light on the wage inequality.

Taking motivation from this perceptiveness we move forward to check how digitalization of trading can impact the output and the profit. For that purpose, we consider two different types of production function: One is the Cobb-Douglas (C-D) production function and the other one is the production function which is from the group of constant elasticity of substitution (CES). Then, we try to go deeper if such transition from offline to online can lead to capital accumulation in the economy. After that we attempt to capture the consequences of digitalization if night time is exploited properly with the assistance of virtual marketing mechanism. The rest of the paper continues as follows. Section 2 constructs the environment and the basic model. Section 2.A and Section 2.B emphasizes on the effect on capital accumulation and the night time exploitation with the help of C-D function. Section 3 focuses on the same issues but with a different kind of production function, which is CES in nature. Some concluding remarks are there in the end.

2. Environment and the Basic Model

Traditionally, the principal component of the production process was organised in offline mode which is usually labour intensive in nature. The process of producing that principal component of any good has to take place physically or in offline. But the advent of digitalisation has almost replaced the conventional shop-based trading and has introduced a less contact intensive or sometimes close to contactless direction of it. As we know trading or selling of a good has two different stages to follow: One is ordering the good and the payment for it, another is delivery of that good to its consumer. Before the virtual mode of marketing or selling was introduced, both the buyer and the seller had to meet at a store. The buyer used to order his or her essentials and pay for it according to its availability. And the seller's job was to deliver the good as soon as the payment was done. Here, the whole trading process needs physical presence of both of the market agents. But thanks to the transition from offline to online, the means of trading of a good is gradually moving from contact-intensity to less contact-intensity. The less-contact intensive avenue of trading indicates towards a mode of selling which is machine-dependent or in other words more capital intensive. Here machine dependency or capital intensity refers to the high bandwidth connection of internet and other technological innovations which has paved the way of this kind of transition. Therefore, looking at the traditional definition of factor intensity, we can say that this paradigm shift from offline to online makes the mode of trading more capital intensive. As more and more capital or machine is required for the selling purpose, the use of labour or man is getting more shrunken. The emergence of various online marketing platform has eliminated the requirement of physical presence of both the producer and consumer. Consumers are now able to check the availability of their necessities and also can order for their desirable products from the comfort of their home. So, from the view of a consumer the reason behind this type of transition in sales is now easily understandable. A producer is only concerned with two things: One is customer and the other is the profit. Therefore, a rational producer is always at stake with the customer's preference and also wants to maximize the profit at the same time. And this structural shift from offline to online takes care of both the concerns.

In tune with the argument, we stated in the introduction, here we would develop an optimization problem for the producer. At the very outset of developing the structure of the model let us confess one thing. It is very difficult to theoretically contemplate such full or partial transition from offline to online mechanism. We try to define this situation by changing the factor requirement or requirement ratio in favour of capital for that use of artificial intelligence or internet-based marketing technique undoubtedly requires more capital than labour. We will discuss more on this later while defining the formal model. Till date we have not seen much work dealing with such issue except Gries and Naude (2018), Aghion, Jones & Jones (2018), Marjit and Das (2022), Acemoglu and Restrepo (2018), Gordon (2018). Therefore, our attempt in this essay is a tiny effort to add some value to the existing stuff.

Let us now quickly define the mathematical optimization problem. We consider the final outcome, F which has two components: G and Q. The price of F is denoted by P_F which is positive ($P_F > 0$)The former part of F consists of the very production of the good, which we assume here as readymade or exogenously given. We also assume that $G > 0$ .The cost of production of *G* is straightaway equal to P_g. But the produced good has to be consumed by the buyer. Therefore, until and unless it reaches the consumers, there is no relevance of producing *G.Q*, the later part of F, therefore, deals with the marketing or selling of *G*. We attempt to model a Cobb-Douglas production function for this *Q*, where factors of production are capital *(K)* and labour *(L)*. Also note that standard assumptions of production functions, marginal productivities, convexity etc. are considered here.

The production function for the final outcome product *F* and *Q* are represented by the following equations:

$$F = G.Q \tag{1}$$

$$Q = (\mu K)^{\alpha}\{(1-\mu)L\}^{1-\alpha} \tag{2}$$

Here, μ stands for the factor requirement ratio which can be used as a proxy for factor intensity comparison in two different situations of offline and online. α implies the output elasticity or the factor responsiveness to the output. Thus, the final equation for *F* looks like:

$$F = G.(\mu K)^{\alpha}\{(1-\mu)L\}^{1-\alpha} \tag{3}$$

Where $0 < \mu, \alpha < 1$

The total cost function for *F* is:

$$TC = P_g + wL + rK$$

Where P_g is exogenously given and fixed. The return to labour and capital are, respectively, denoted by *w* and *r*.

Now, producer's objective is to maximize profit. Hence, we can write the profit function as follows:

$$\pi = P_F F - P_g - wL - rK = P_F.G.(\mu K)^{\alpha}\{(1-\mu)L\}^{1-\alpha} - P_g - w.L - r.K$$

Following the techniques of optimization, we know that profit maximization requires,

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

Therefore, $\frac{(\mu K)^{\alpha}(1-\alpha)L^{1-\alpha-1}(1-\mu)^{1-\alpha}}{\mu^{\alpha}\,\alpha K^{\alpha-1}\{(1-\mu)L\}^{1-\alpha}} = \frac{w}{r}$

Manipulating above equations, we arrive at the values of *K* and *L* as

$K = \frac{\alpha}{1-\alpha}\frac{Lw}{r}$ and $L = \frac{1-\alpha}{\alpha}\frac{rK}{w}$

Plugging, $K = \frac{\alpha}{1-\alpha}\frac{Lw}{r}$ and $L = \frac{1-\alpha}{\alpha}\frac{rK}{w}$ in (2) we get

$$L = Q\mu^{-\alpha}(1-\mu)^{-(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} r^{\alpha} w^{-\alpha}$$

and

$$K = Q\mu^{-\alpha}(1-\mu)^{-(1-\alpha)}(\frac{1-\alpha}{\alpha})^{(1-\alpha)}r^{-(1-\alpha)}w^{(1-\alpha)}$$

We will now use these values of L and K in the profit function:

$$\pi = (P_F F - P_g) - \left[\left\{wQ\mu^{-\alpha}(1-\mu)^{-(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} r^{\alpha}w^{-\alpha}\right\} + \left\{rQ\mu^{-\alpha}(1-\mu)^{-(1-\alpha)}\left(\frac{1-\alpha}{\alpha}\right)^{-(1-\alpha)} r^{-(1-\alpha)}w^{(1-\alpha)}\right\}\right]$$

$$\pi = (P_F F - P_g) - \{Qw^{1-\alpha}r^{\alpha}\alpha^{-\alpha}(1-\alpha)^{\alpha}\mu^{-\alpha}(1-\mu)^{-(1-\alpha)}\}\left(1+\frac{\alpha}{(1-\alpha)}\right)$$

$$\pi = (P_F F - P_g) - Qw^{1-\alpha}r^{\alpha}\left(\frac{1}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)\left(\frac{1}{\mu^{\alpha}(1-\mu)^{(1-\alpha)}}\right) \quad (4)$$

It is perceptible from (4) that $(P_F F - P_g)$ is constant. Q is unchanged as the C-D production function assumes that inputs are perfect substitute for each other. Therefore, whatever change happens to π is only because of the remaining part of the function. In this remaining part, a part is also constant, $w^{1-\alpha}r^{\alpha}$. For brevity we will just focus on the part which deals with μ. And we find that the relationship between μ and the total cost or profit isn't unidirectional. It has some interesting implications. This phenomenon is described in Figure – 1 where we consider three separate values of μ and the changes in profit.

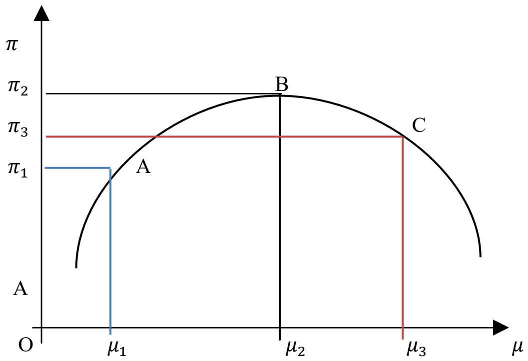
Let us assume, $\alpha = 0.5\ and\ (1-\alpha) = 0.5$. We have taken few arbitrary values of μ to capture the effect of it on profit. The following table shows us the results:

Table 1. *Some numerical values regarding μ in C-D case*

μ	(1-μ)	$\left(\frac{1}{\mu^{\alpha}(1-\mu)^{(1-\alpha)}}\right)$
0.2	0.8	2.56 (approx.)
0.4	0.6	2.08(approx.)
0.5	0.5	2.04(approx.)
0.7	0.3	2.22(approx.)
0.8	0.2	2.56(approx.)

From Table-1 it is apparent that with each successive increase in the value of μ, the value of $\left(\frac{1}{\mu^{\alpha}(1-\mu)^{(1-\alpha)}}\right)$ falls initially up to $\mu = 0.5$, after that if μ rises it starts rising. This result leads us to change in cost of Q which is directly related with the value of $\left(\frac{1}{\mu^{\alpha}(1-\mu)^{(1-\alpha)}}\right)$. Therefore, due to rise in μ initially the cost of Q falls, then it reaches the minimum when μ takes the value of 0.5, and eventually the cost goes up with increasing μ. So, without any change in Q, profit increases with the rise in μ, then falls beyond a threshold value of μ which is 0.5 in our case. Therefore, following the above results we get a U-shaped total cost curve. In Figure-1, it is the change in profit curve which we get from three different levels of profit corresponding to three different values of μ, those are μ_1, μ_2 and μ_3, respectively. We can see the change in profit as an inverted U-shaped curve, which is initially rising and after reaching the peak it starts falling.

Figure 1. *Effect of μ on π in the C-D case*



Thus, transition from offline to online which is captured by an increase in μ*) causes profit to rise at first, reaches the maximum at μ=0.5 and then starts declining with further rise in μ. Hence, we have our first proposition.

*) The point of interest is that the production technology is already sufficiently capital-intensive (μ > 0·5), a more capital intensity of the production process will not turn out to be more profitable. This, in turn implies that uses of capital and labour would be optimum in this type of production and cost function when $\frac{K}{L} = 1$ which we have captured here through the changes in μ or the share parameter of capital indicating the transition from offline to online mode of marketing process. The main reason behind such an intriguing outcome is the presence of the conventional diminishing marginal productivity principle and the law of variable proportions.

Proposition I: *An increase in μ or factor intensity indicating digitalization in marketing leads to an increase in profit of the final good produced. But the relation isn't uniform and not always positive.*

Proof: See discussion above.

2.A. Effect of μ on Capital Accumulation

In this section we focus on the effect of **μ** on capital accumulation. We have considered a constant savings rate γ and φ as the rate of depreciation of the capital, so that we have the capital accumulation equation as:

$$\Delta K = \gamma\pi - \varphi K$$

$$\Delta K = \gamma\left\{\left(P_F F - P_g\right) - Qw^{1-\alpha}r^{\alpha}\left(\frac{1}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)\left(\frac{1}{\mu^{\alpha}(1-\mu)^{(1-\alpha)}}\right)\right\} - \varphi K \qquad (5)$$

From (5) it is clear that up to μ=0.5 as profit rises, it leads to capital accumulation. After that as profit declines there is capital decumulation. Therefore, our next proposition is:

Proposition II: *Digitalization causes an increase in capital accumulation up to a certain value of μ, beyond that it hurts the process of capital accumulation.* ■

2.B. Night Time Exploitation and δ

As we have mentioned earlier, we now attempt to extend the model further to check the impact of utilization of night time in online selling. Since, buyers have the access of various online shopping platform even at night, the process of ordering a product and the payment may take place during night time. So, the exploitation of the night time may lead to an early morning delivery. An early delivery is highly desired by any customer so it will be reflected on the effective price of the final good, P_F . Let us denote the time preference by □. As the early delivery is positively related with consumer’s willingness to pay for the final product, the effective price should now be δP_F with $\delta \leq 1$. δ is less than unity ($\delta < 1$) when night time isn’t used for marketing which is similar as offline. δ is equal to unity ($\delta = 1$), when we move from offline to online mode of marketing. Therefore:

$$\delta P_{F_{on}} > \delta P_{F_{off}}$$

As effective price of the final good increases it is obvious that it would lead to a higher level of profit if night time is utilised properly. So, the profit equation becomes:

$$\pi_{online} = \left(P_F \delta F - P_g\right) - Qw^{1-\alpha} r^{\alpha} \left(\frac{1}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)\left(\frac{1}{\mu^{\alpha}(1-\mu)^{(1-\alpha)}}\right)$$

From section 2. A, we arrived at a relation between profit and capital accumulation which is direct. So, in the case of night time exploitation, with a higher level of profit the level of capital accumulation will also be higher. The result is very easy to be understood:

$$\Delta K_{off} = \gamma \delta \pi - \varphi K (\delta < 1)$$

$$\Delta K_{on} = \gamma \delta \pi - \varphi K \ (\delta = 1)$$

$$\Delta K_{on} \text{-} \Delta K_{off} = \gamma(\delta)_{on} \pi - \gamma(\delta)_{off} \pi$$

So, we propose that:

Proposition III: *Exploitation of night time through digitalization yields double benefit through μ and P_F and hence the basic results get strengthened further.*

Proof: See discussion above

3. Extended Model with CES Production Function

In the previous section of our essay, we have developed a basic model which is Cobb-Douglas in nature. We all know that C-D production function is a special case of the group of the production function with constant elasticity of substitution property. So, it is rational to develop a production structure which is CES in nature. Hence, in this section our focus is to extend the basic model in order to capture the concept of CES. The basic structure of the final production function is same as before. But in this case, Q is being considered as a CES production function which also requires capital *(K)* and labour *(L)*. The two factor CES production function was introduced in a paper by Solow (1956). Arrow, Chenery, Minhas and Solow (1961), and McFadden and Daniel (1963) are among the noted contributors who made it popular later. But there is subtle difference between CES and C-D production function that we developed before. We will come to those distinctive features later.

The production function for the final output F can be written as:

$$F = G.Q \tag{6}$$

Where F and G holds the same implications like what we had in the basic model. Q also stands for the marketing part of the produced good but the production function for Q takes a different shape:

$$Q = [(\mu k)^{\rho} + \{(1-\mu)L\}^{\rho}]^{\frac{1}{\rho}}$$

$$Q = [(\mu k)^{\rho} + \{(\theta)L\}^{\rho}]^{\frac{1}{\rho}} \tag{7}$$

(Assume that $(1-\mu) = \theta \Rightarrow (\mu + \theta) = 1$)

Therefore, the final expression for F is:

$$F = G[(\mu k)^{\rho} + \{(\theta)L\}^{\rho}]^{\frac{1}{\rho}} \tag{8}$$

Here, μ represents the factor intensity or the share parameter, and ρ is the substitution parameter. Assume that $0 < \mu, \rho < 1$. Hence, reciprocal of ρ or $\frac{1}{\rho} > 1$.

The cost function for F is same as before, where P_F is the price of F, P_g is the price of G and both are assumed to be positive.

The producer's objective function is slightly different from what we had in section 2. We can write the new profit function as follows:

$$\pi = P_F F - P_g - wL - rK = P_F.G[(\mu k)^{\rho} + \{(\theta)L\}^{\rho}]^{\frac{1}{\rho}} - P_g - w.L - r.K$$

Following the conventional profit maximizing condition, we arrive at the result which is as follows:

Hence, $$\frac{\frac{1}{\rho}[(\mu k)^{\rho}+\{\theta L\}^{\rho}]^{\frac{1-\rho}{\rho}}\rho(\theta L)^{\rho-1}\theta=w}{\frac{1}{\rho}[(\mu k)^{\rho}+\{\theta L\}^{\rho}]^{\frac{1-\rho}{\rho}}\rho(\mu k)^{\rho-1}\mu=r} = \frac{w}{r}$$

From the above we get the values of L and K, as follows:

$$L = \bar{Q}.\frac{\theta^{\frac{\rho}{1-\rho}}}{w^{\frac{1}{1-\rho}}} = \bar{Q}\,.\theta^{\frac{\rho}{1-\rho}}w^{\frac{1}{\rho-1}} \tag{9}$$

Similarly, we can get:

$$K = \bar{Q}.\frac{\mu^{\frac{\rho}{1-\rho}}}{r^{\frac{1}{1-\rho}}} = \bar{Q}\,.\mu^{\frac{\rho}{1-\rho}}r^{\frac{1}{\rho-1}} \tag{10}$$

Here, $\bar{Q}$ = initial level of output.

Substituting the values of L and K in (7) we get:

$$Q = \left[\left(\mu\bar{Q}\mu^{\frac{\rho}{1-\rho}}r^{\frac{1}{\rho-1}}\right)^{\rho} + \left\{\left(\theta\bar{Q}\theta^{\frac{\rho}{1-\rho}}w^{\frac{1}{\rho-1}}\right)\right\}^{\rho}\right]^{\frac{1}{\rho}}$$

$$Q = \left\{\left(\bar{Q}^{\rho}\mu^{\frac{\rho}{1-\rho}}r^{\frac{\rho}{\rho-1}}\right) + \left(\bar{Q}^{\rho}\theta^{\frac{\rho}{1-\rho}}w^{\frac{\rho}{\rho-1}}\right)\right\}^{\frac{1}{\rho}}$$

$$Q = \bar{Q}\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{\theta}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}}$$

$$Q = \bar{Q}\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}} \tag{11}$$

(Since $1 - \mu = \theta$)

Plugging the values of K, L and (11) in the profit equation we get:

$$\pi = P_F G\bar{Q}\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}} - P_g - r\left(\bar{Q}\mu^{\frac{\rho}{1-\rho}}r^{\frac{1}{\rho-1}}\right) - w\left(\bar{Q}\theta^{\frac{\rho}{1-\rho}}w^{\frac{1}{\rho-1}}\right)$$

$$\pi = P_F G\bar{Q}\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}} - P_g - \bar{Q}\left\{\left(\mu^{\frac{\rho}{1-\rho}}r^{-\left(\frac{\rho}{1-\rho}\right)}\right) + \left(\theta^{\frac{\rho}{1-\rho}}w^{-\left(\frac{\rho}{1-\rho}\right)}\right)\right\}$$

$$\pi = P_F \,.\, G \,.\, \bar{Q}\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}} - P_g - \bar{Q}\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\} \tag{12}$$

From (12) it is apparent that the result of this section isn't similar with the results that we got in the basic model. In case of CES production function the eventual impact of μ becomes effective through two channels: One is through revenue and the other is through cost. It should be noted that both the revenue and the cost are getting influenced evidently by any change in μ at the same time. Therefore, one must take into account both these effects together to arrive at the conclusion regarding the eventual effect of change in μ. Also, note that $P_F G$ and P_g are constants. So, all the changes that we will have in this extended model are solely because of the change in μ. The effect on revenue part would be much higher than the cost part because of what we have assumed in the very beginning that $P_F > 0$, $G > 0$ and $\frac{1}{\rho} > 1$.

We have attached another table to the purpose in the appendix. From that table, it is very much comprehensible that in this case of CES production function, the revenue and the cost follow the same trend: they increase as μ rises, reach the maximum at $\mu = 0.5$ and then start falling if further rise in μ takes place. But as we have mentioned before that the effect on revenue will be much higher than the effect on cost, so the profit will always be positive. The profit increases initially till the level where $\mu = 0.5$, and just after reaching the maximum if μ rises any further the profit starts to decline. The trend of the profit is drawn in the diagram (Figure – 3) given in the appendix.

Following section 2.A, we now consider a capital accumulation equation. It is as follows:

$$\Delta K = \lambda\pi - \varphi K$$

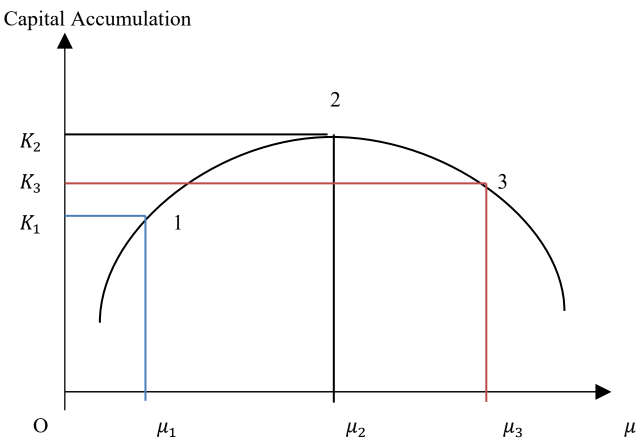
Here λ represents the constant saving rate and φ refers to the rate of depreciation of the capital. Therefore:

$$\Delta K = \lambda P_F \mathrm{G}Q\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}} - P_g - Q\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\} - \varphi K \qquad (13)$$

From (13) we understand that as profit increases up to $\mu = 0.5$, an increase in capital accumulation occurs. However, after reaching the maximum it starts declining. However, it is unlikely to be negative contrary to what we had in C-D production case. In case of C-D the profit could turn out to be negative because the revenue remains same throughout whereas the cost component changes. So, it isn't unthinkable to have a negative profit, at least theoretically. unlike C-D production function - it doesn't become negative. Due to the stronger effect of revenue, the profit is always positive whatever be the value of μ. So, ideally the process of capital accumulation will take place, but the rate of accumulation may be lower as μ increases further compared to the rate of accumulation when the profit was increasing.

In Figure-2, we try to illustrate the effect of μ on capital accumulation in the CES case. Here in the figure, we can see three different levels of capital accumulation or quantity of capital which are $K_1, K_2, and\ K_3$ corresponding to three different values of μ which are $\mu_1, \mu_2\ and\ \mu_3$, respectively.

Figure 2. *Effect of μ on capital accumulation in the CES case*



By this time, it is perhaps clear that compared to the C-D function case, the profit is much higher in the case of a CES production function. It is already observed earlier in the section 2.B that the exploitation of the nigh time in online marketing induces the effective price of the final output, δP_F to rise. So, without any further clarification one can argue that, increase in the effective price of the final output makes the revenue much higher, hence the profit is much larger if we move from offline to online marketing. The profit equation with night time utilization is therefore:

$$\pi_{online} = \delta P_F \mathrm{G}Q\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}} - P_g - Q\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\} \qquad (14)$$

Hence, the following proposition is immediate:

Proposition IV:

A. As μ increases, the revenue and the cost both increases simultaneously, but the revenue having the stronger effect of μ leads to an increase in profit. After a certain value of μ, the profit starts declining.

B. With the rise in μ, capital accumulation takes place but beyond a certain value of μ capital accumulation decreases.

C. Due to the utilisation of night time μ and δP_F both increases at a time. It leads to much higher profit compared to the C-D production function.

Proof: See discussion above.

4. Concluding Remarks

In this essay, we have constructed a theoretical model using both Cobb-Douglas and CES production function. It has been shown that the transition of the marketing process from offline to online has brought about a remarkable change in the economy as a whole. Here, we have tried to establish the relationship between digitalization, capital-intensity, output, profit etc. This capital-intensive nature of the trading part of the production clearly indicates towards less-contact intensity and hence a move from offline to online. It is explicit that the transition of the mode of marketing which can be captured by the factor intensity, has direct effect on the profit of the final good produced. But the trend or the pattern of the effect depends on the type of the production function we are using. We have arrived at very same results for two different kind of production functions which may have some interesting implications which deserve to be looked at further. Another important thing the policy makers must be careful about is that the relationship between capital accumulation and subsequent growth prospect of the economy due to digitalization isn't unidirectional and preprogramed. It largely depends on the overall economy wide degree of capital-intensity of the production process. If it is already high, we may think of not going for further rounds of digitalization. So, for a developing economy with lower level of capital-intensity in production one can utilize the opportunity rendered by the digitalization to appropriate some more profits and ensure subsequent growth possibilities

Appendix

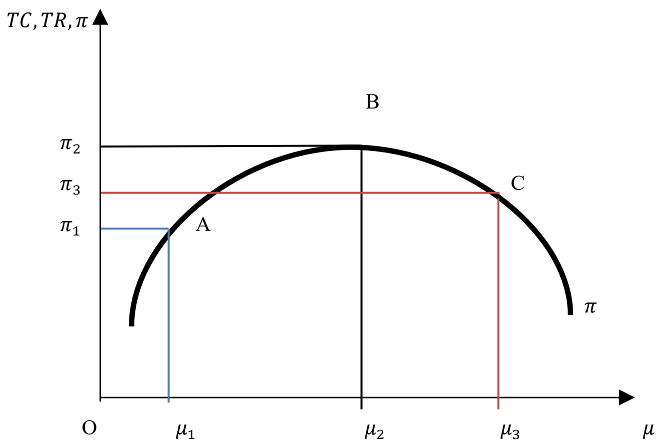
We have taken some arbitrary values of μ to check the eventual change in profit. For ease of calculation, we presume $\rho = \frac{1}{3}, r = 1 \; and \; w = 1$.

Table 2. *Some numerical values regarding μ in CES case*

μ	$(1 - \mu)$	$\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}}$	$\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}$
0.2	0.8	2.41 (approx.)	1.34(approx.)
0.4	0.6	2.79(approx.)	1.407(approx.)
0.5	0.5	2.83(approx.)	1.41(approx.)
0.7	0.3	2.65(approx.)	1.38(approx.)
0.8	0.2	2.41(approx.)	1.34(approx.)

Table 2 shows the values of $\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}}$ and $\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}$ for different values of μ. It is apparent that as μ increases, the value of $\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}}$ keeps increasing till $\mu = 0.5$, then it starts declining as μ increases further. Similar kind of result we get for the value of $\left\{\left(\frac{\mu}{r}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\mu}{w}\right)^{\frac{\rho}{1-\rho}}\right\}$ where it initially rises and falls afterwards with each successive increase in the value of μ.

Figure 3. *Effect of μ on π in the CES case*



In Figure 3, the change in profit curve is shown. As we mentioned earlier that in the CES case the revenue will always be greater than the cost so that the change in TC curve will lie below the TR curve for any value of μ. For that reason, the profit must be always positive. We derive it with response to three successive levels of μ: $\mu_1, \mu_2 \; and \; \mu_3$, respectively. The curve is initially upward sloping and after reaching the maximum it becomes downward sloping.

References

Acemoglu, D. and Restrepo, P., 2018. Artificial intelligence, automation, and work. In *The economics of artificial intelligence: An agenda* (pp. 197-236). University of Chicago Press.

Aghion, P., Jones, B.F. and Jones, C.I., 2017. *Artificial intelligence and economic growth* (Vol. 23928). Cambridge, MA: National Bureau of Economic Research.

Arrow, K.J., Chenery, H.B., Minhas, B.S. and Solow, R.M., 1961. Capital-labor substitution and economic efficiency. *The review of Economics and Statistics*, pp. 225-250.

Cheng, M.L., 2010. China forecasting model of economic growth based on production function. *Statistics and Decision*, *20*, pp.34-36.

Cheng, M., 2016. A generalized constant elasticity of substitution production function model and its application. *Journal of Systems Science and Information*, *4*(3), pp. 269-279.

Deardorff, A.V., 2014. Local comparative advantage: trade costs and the pattern of trade. *International Journal of Economic Theory*, *10*(1), pp. 9-35.

Dixit, A.K. and Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. *The American economic review*, *67*(3), pp. 297-308.

Falvey, R.E., 1976. Transport costs in the pure theory of international trade. *The Economic Journal*, *86*(343), pp. 536-550.

Gordon, R.J., 2018. *Why has economic growth slowed when innovation appears to be accelerating?* (No. w24554). National Bureau of Economic Research.

Gries, T. and Naudé, W., 2018. Artificial intelligence, jobs, inequality and productivity: Does aggregate demand matter?

Hang, B.L., Wang, W.R. and Ding, L.Q., 1997. An expand application to the CES production function. *Quantitative & Technica Economics*, *8*, pp. 52-55.

Khan, R.K., Mahata, S. and Nag, R.N., 2023. Pandemic crisis, contact intensity and gender disparity in a developing economy. *Economic Papers: A journal of applied economics and policy*, *42*(1), pp. 30-53.

Laussel, D. and Riezman, R., 2008. Chapter 5 Fixed Transport Costs and International Trade. In *Contemporary and Emerging Issues in Trade Theory and Policy* (pp. 91-107). Emerald Group Publishing Limited.

Mandal, B., 2015. Distance, production, virtual trade and growth: A note. Economics, 9(1), p. 20150001.

Mandal, B. and Marjit, S., 2010. Corruption and wage inequality? *International Review of Economics & Finance*, *19*(1), pp. 166-172.

Marjit, S. and Mandal, B., 2012. Domestic trading costs and pure theory of international trade. *International Journal of Economic Theory*, *8*(2), pp. 165-178.

Marjit, S., Mandal, B. and Nakanishi, N., 2020. *Virtual Trade and Comparative Advantage*. Springer Singapore.

Marjit, S. and Das, G.G., 2023. *Finance, trade, man and machines: a new-ricardian heckscher-ohlin-samuelson model* (No. 1218). GLO Discussion Paper.

Marjit, S. and Das, G.G., 2021. *Contact-intensity, collapsing entertainment sector and wage inequality: A finite change model of covid-19 Impact* (No. 9311). CESifo Working Paper.

McFadden, D., 1963. Constant elasticity of substitution production functions. *The Review of Economic Studies*, *30*(2), pp. 73-83.

Solow, R.M., 1956. A contribution to the theory of economic growth. *The quarterly journal of economics*, *70*(1), pp. 65-94.