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Business cycle synchronization of Tunisia and Euro Zone countries: a parametric and non-parametric analysis

Wissem RABHI
University of Sfax, Tunisia
wissem.rabhi@yahoo.fr

Abstract. *In this article, we used a parametric Hamilton approach and a non-parametric Bry-Boschan method to characterize the business cycles of Tunisia and the Euro zone between 1990 and 2017. The estimation results show that Tunisia's growth cycle is very close to that identified for the Euro zone. Furthermore, based on the transition probability matrix, we find that these coefficients are highly significant and differentiated, ranging from 0.739 to 0.989. Consequently, these probabilities illustrate the state of the Tunisian economy, which is characterized by major recessions. As for the Euro zone economy, the smoothed and filtered probabilities identify five major recessions.*

Keywords: business cycle, Markov Switching Model, Bry-Boschan method, Tunisia, Euro zone.

JEL Classification: E32, F43, F47, C32.

1. Introduction

Although the subject of business cycles has been observed by economists since the 19th century. With the great depression of 1929, the study of cyclical phenomena was inspired by the credit theory initiated by Hayek (1933) and Von Mises (1933). However, after the Second World War, many theories coexisted in the analysis of business cycles. In this respect, pre-Keynesian analyses based on the work of (Wicksell, 1898; Frisch, 1933 and Slutsky, 1937) considered that cyclical movements result from the interaction between exogenous impulses and the endogenous dynamics of the economic system. This idea has been advanced since the 1970s, with the birth of New Classical Economics (Lucas, 1972; 1973 and 1977), which gave a fundamental role to the problem of imperfect information and rational expectations in explaining economic fluctuations. In fact the economic fluctuations then deserve to be clarified on the basis of a rigorous and coherent real-time structure to facilitate better political, economic and financial decision-making.

From the 1980s onwards, as an extension of New Classical Economics, the theory of the real business cycle (TCR) was developed with the work of (Kydland and Prescott 1982; Long and Plosser 1983). This theory aims to show that cyclical movements in economic activity result from the optimal responses of economic agents to real shocks, essentially technological shocks, in a context of rational expectations. With the above-mentioned dependence, the question of business cycle synchronization became of major interest worldwide in the 1980s. As a result, it seems that business cycle theories have gradually evolved in different directions. Indeed, theoretically, the business cycle has been the subject of a number of recent works, notably (Grigoras and Stancieu, 2016; Mattera and Franses 2023; Stiblarova, 2023) who have defined the business cycle as "*an aggregate phenomenon that has led to recurrent and persistent variations in economic activity as a whole*". This does not mean pretending that there is a given periodicity and amplitude of expansion and recession phases, but simply certain regularities in the variations of economic activity.

Although, the question of business cycles in general, has been accompanied and sometimes initiated by its growing interest. Recently, the scope of business cycle synchronization has expanded considerably in both developed and developing countries (Yuwen Dai, 2014; Benhida, 2015; Matesanz and Ortega, 2016; Dua, and Tuteja, 2023). The economic crises and changes that have hit the global economy have given particular attention to the search for points of reflection on the origins of economic fluctuations and the identification of the characteristics of business cycles, with an emphasis on the explanatory factors of the cyclical transition mechanism between countries.

In this sense, the contribution of this article is to answer the central question of the cyclical synchronization of the Tunisian economy on the one hand, and to compare it with that of Euro zone countries such as Germany, Italy, France and Spain on the other. To answer this question, several statistical and econometric methods based on classical and modern approaches compete to establish a satisfactory empirical description of business cycle synchronizations.

The article proceeds as follows: Section 2, describes the characteristics of the Tunisian economy and the Euro zone. Section 3, presents a brief review of the empirical literature. Section 4 presents the model and describes the database. Section 5 contains the empirical results, and Section 6 concludes.

2. Literature review

In the recent empirical literature, numerous studies have tackled the subject of business cycles, more specifically the concept of business cycle synchronization, which is of major interest to economists and conjuncturists. In this context, the answer to the question of business cycle synchronization relies essentially on understanding economic fluctuations and comparing cycles between countries, which are the subject of numerous studies such as (Dufrénot and Keddad, 2014; Benhida, 2015; Grigoraş and Stanciu, 2016; Monnet and Puy, 2016; Elgahry, 2016; Compos et al. 2019; Beck, 2021).

Research into cyclic timing analysis has increased significantly over the last decade. Indeed, among the most recent studies, we can find the works of (Beck, 2021; Arčabić and Škrinjarić, 2021), which often reach very different conclusions. The difference in the variables used and the methods of assessing synchronization is the most important part that can explain this difference. In this respect, Arčabić and Škrinjarić, (2021) show that 50% and 90% of the variation in output explains the Euro zone business cycle. The falls were particularly detected during the 2007-2016 period of the great recession. This finding limited national stabilization policies. After the great recession, business cycles became more synchronized between these Euro zone countries. These business cycle externalities are beneficial for the creation of a common monetary policy in Europe. A different approach is that suggested by Compos et al. (2019) who show that synchronization increased by 50% between European countries pronounced particularly after the introduction of the euro in 1999 and that this increase was more noticed specifically in Euro zone countries.

Benhida (2015) studied the synchronization of business cycles in Morocco with the major Euro zone countries, using the correlation index on the one hand, and Bry-Boshan's (1971) parametric method on the other. These methods were adopted by Benhida (2015) to detect both turning points in cycle activity and to determine the degree of cyclical synchronization between Morocco and Euro zone countries. These approaches attracted the attention of several researchers after the 1970s, notably (Elachheb, 2010; Hassad and EL Ghak, 2010).

In this sense, the question of cyclical synchronization between European and North African countries occupies a central place in the empirical literature. That's why several researchers, such as Elachheb (2010), Medhioub (2011) and Knani (2012), have focused on this debate in the Tunisian economy and that of Euro zone countries. They studied the degree of cyclical correlation between Tunisia and certain Northern Mediterranean countries on a monthly basis from January 1994 to December 2007. In their analysis, these authors used several methods, including Harding and Pagan's (2006) concordance index method and the non-linear approach based on regime change models. The results show a degree of convergence between the Tunisian economy and those of Mediterranean countries, implying that Tunisian industrial cycles are perfectly synchronized with those of Euro-Mediterranean countries.

Elachhab (2010) also attempted to assess and determine the causes of cyclical synchronization/de-synchronization between Tunisia and its main European partners. The results of this study showed that there is a positive and significant impact of the Tunisian cycle with France, essentially linked to the greater intensity of their bilateral trade and the similarity of their economic policies. On the other hand, the result is not verified for the case of Tunisian cycles with Italy and Germany cycles. Some studies have reported the fragility of bilateral trade, which could be explained by dissimilar supply structures and the decline in its trade with these two countries. (Knani, 2012).

Similar to Elachheb (2010), Knani (2012) examined the degree of cyclical correlation between Tunisia and its European partners, such as France, Italy and Germany. He used Harding and Pagan's (2006) approach and (ARDL) model to estimate the long-term effects of financial, trade and common factors on the synchronization of economic activities. The results of this study showed an average synchronization of around 50%, and demonstrated the existence of long-term negative effects of bilateral trade and positive effects of common factors on the correlation between Tunisia's economic fluctuations and those of its main partners.

As suggested by De Haan et al., (2008), there is no single method of analysis, and no consensus on how to proceed. In this context, we use two approaches to address our research problem: the classical approach (Bry-Boschan, (1971) and the non-linear approach Hamilton, (1989), which changes the parametric Markovian regimes, with the aim of identifying the characteristics and synchronization of business cycles between the Euro zone countries and Tunisia.

3. Methodology and database presentation

3.1. Methodology

In this research, we adopted a non-parametric procedure from Bry-Boschan (1971)[(1)] and a parametric method based on a non-linear model (MSVAR) from Hamilton (1989). In particular, the Markov regime-switching processes developed recently in economics by Hamilton (1989) have proved to be well suited to detecting the characteristics of international business cycles. Indeed, in empirical research, a number of authors have taken up the Markov regime-switching model in the analysis of business cycles, including Phillips (1991), who treats the multivariate model on two countries and with two regimes. Following the same approach, Filardo and Gordon (1994) enriched their studies by introducing leading indicators for two-regime trivariate model to predict turning points.

The innovation of this model lies in the presence of unobservable common factors that possess their own dynamics governed by a k-regime Markov chain of a discrete, stochastic and unobservable nature, dealing with the state of the economy of the countries in question (Tunisia, Germany, Italy, France, Spain and the euro zone). To this end, we use a multivariate three-regime model with a regime-dependent intercept and covariance[(2)] to find out whether there are conditions under which a common business cycle can be asserted between these countries. The importance of the markov chain model specifications that enable us to generate statistically and economically significant results.

Based on Hamilton's (1989) model, the general framework of the model is written as follows:

$$Y_t = \mu(S_t) + \sum_{j=1}^{p} \phi_j(S_t)(Y_{t-j} - \mu(S_{t-j})) + \varepsilon_t \qquad (1)$$

where $(S_t)_t$ follows a first-order Markov chain with values in the countable set $[1,..., K]$. where ε_t is a centered white Gaussian noise process of variance-covariance matrix, of dimension(N,N) and where $\phi_j(S_t),...,\phi_p(S_t)$ are (N,N) matrices describing the model's dependence on the regime (S_t).

As noted by (Medhioub, 2015; Moradi, 2016; Dua, and Tuteja, 2023), the basic idea of the MSVAR model is that the variable (S_t) is specified to the K-regime Markov chain process. This means that the current regime (S_t) depends only on the regime of the previous state (S_{t-1}), for i, j = 1,..., K:

$$P(S_t = j / S_{t-1} = i, S_{t-2} = k, ...) = P(S_t = j / S_{t-1} = i) = P_{ij} \qquad (2)$$

$(P_{ij})i, j = 1,.... k$ is the probability of moving from state i at date t-1 to state j at date t. Probabilities (P_{ij}) are also called transition probabilities; they measure the probability of remaining in an identical regime or, on the contrary, of changing regime.

The transition probabilities that can be indicated in the transition matrix (N, N) are denoted by P. Indeed, as pointed out by (Hamilton, 1989; Medhioub and Eleuch, 2013; Dua, and Tuteja 2023), the transition from state i to state j in the Markov regime-switching model is probabilistic. The transition probability matrix is written as follows:

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{21} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix} \qquad (3)$$

The properties of probabilities are:

- $(P_{ij}) \geq 0 \; \forall i, j \in [1,.....k]$ i i.e. probabilities are not negative

- $\sum_{j=1}^{k} P_{ij} = 1 \forall j = 1,....k$ is the probability of the economy passing through one of the K states. The sum of the probabilities is equal to unity.

Given that there are three states or regimes. To model the process we choose a specification of this model based on Markov Switching Intercept Autoregressive Heteroscedasticity (MSIAH) or a Markov mean-switching heteroscedasticity (MSMH) specification. Other process types, such as MSI and MSIA, are adopted to verify the absence of regime shifts in autoregressive parameters (A) and heteroscedasticity (H). Other specifications are MSM and MSMH, introduced by Hamilton (1990). The former contains regime-dependent means while MSMH, while the latter, includes regime-dependent means and variances.

Once or regimes are recognized. It is also possible to establish a multivariate vector autoregression (VAR) model for the industrial production growth rates of Tunisia, France; Italy, Spain, Germany and the euro zone. To do this, we relied on a univariate Hamilton model, focusing on which of the MSIAH/ MSIH/ MSI/ MSIA/ MSMH/ MSM specifications devoted to this model. Usually, referring to this univariate model encountered by Hamilton (1989), the various alternative specifications discussed in (Guidolin, 2011; Dua and Tuteja, 2023) under the name of Markov intercept autoregressive heteroscedasticity model (MSIAH) is written in the following form:

$$Y_t = \mu_{St} + \sum_{j=1}^{k} \phi_{Stj} Y_{t-j} + \sigma_{St} \varepsilon_t \tag{4}$$

The Writing of the specification of Markov Switching Intercept Autoregressive (MSIA) Model:

$$Y_t = \mu_{St} + \sum_{j=1}^{k} \phi_{Stj} Y_{t-j} + \sigma \ \varepsilon_t \tag{5}$$

The Mean Markov Switching Heteroscedasticity (MSMH) specification for the k-regime MSVAR model is as follows:

$$Y_t - \mu_{St} = \sum_{j=1}^{k} \phi_j (Y_{t-j} - \mu_{St-j}) + \sigma_{St} \varepsilon_t \tag{6}$$

The expression for the Markov Switching Mean (MSM) model specification at K regime is written as follows:

$$Y_t - \mu_{St} = \sum_{j=1}^{k} \phi_j (Y_{t-j} - \mu_{St-j}) + \sigma \varepsilon_t \tag{7}$$

The Markov Switching Intercept (MSI) Model:

$$Y_t = \mu_{St} = \sum_{j=1}^{k} \phi_j Y_{t-j} + \sigma \varepsilon_t \tag{8}$$

The Markov Switching Intercept Heteroscedasticity (MSIH) Model:

$$Y_t = \mu_{St} = \sum_{j=1}^{k} \phi_j Y_{t-j} + \sigma_{St} \varepsilon_t \tag{9}$$

The Estimation of MSVAR (1) model parameters by the maximum likelihood (ML) method, which is based on a version of the Expectation Maximisation (EM) algorithm developed by Hamilton (1990) and Krolzig (1997). The basis of the Expectation Maximisation (EM) algorithm is a 2-step process used by Dempster et al. (1977): In the first step, “Expectation (E)”, we assume that there are three states or regimes in the world and the probabilities of the hidden states of nature are estimated. Whereas, in the second step (M) the probabilities estimated by step (E) are used in the estimation of the model parameters and the likelihood function is maximized. Once the parameters have been estimated from the data, we obtain the filtered probabilities and the smoothed probabilities[3] noted $P[S_t = j/(y_t, y_{t-1},..., y_1)]$, for j = 1, 2, 3. These smoothed probabilities are used to classify observations between regimes and to date the business cycle. Typically, the rule applied in Hamilton's (1989) model is to assign the observation to time t, according to the highest smoothed probability.

$$m^* = \arg\max P[S_t = j/(Y_t)] \quad (10)$$

Following the articles by (Hamilton, 1989; Damette and Rabah, 2010; Medhioub and El Euch, 2013), given the existence of two regimes, the dating and identification of business cycle turning points for j = 1, 2 at date t is based on the following classification rules[4]. |For Peaks, both filtered and smoothed probabilities verify this relationship $P[S_t = 1/(y_t, y_{t-1},..., y_1)] > 0.5$ *and* $P[S_{t+1} = 2/(y_t, y_{t-1},..., y_1)] < 0.5$, and vice versa for Troughs $P[S_t = 1/(y_t, y_{t-1},..., y_1)] < 0.5$ *et* $P[S_{t+1} = 2/(y_t, y_{t-1},..., y_1)] > 0.5$.

3.2. Database

To examine the business cycle synchronization between Tunisia and the Euro zone countries, we use the Industrial Production Index (IPI) series[5], which is considered to be a fundamental statistical indicator that measures variations in the quantities produced in industries over a short period. Indeed, the series of variables in first difference extends over a monthly period from January 1, 1990 to May 31, 2017 (base 100 in 2010), (Figure 1). Data were collected from the National Institute of Statistics (INS) and the OECD, covering six countries (Tunisia, Germany, Italy, France, Spain and the Euro zone).

In this study, the statistical description of the variables presented in Table 1 leads to several major comments. The analysis of standard deviations shows that the IPI of the majority of countries fluctuates enormously from one month to the next. The coefficient of asymmetry or skewness, with negative values (Skewness<0) for each of the series, allows us to conclude that the distributions of the series are asymmetrically spread to the left, with the exception of the distribution of the IPI series for Spain and Tunisia, which are asymmetrically spread to the right, with a positive skewness coefficient greater than 0. At the same time, the same findings are observed for the analysis of kurtosis coefficients, which are low, below 3, with a (Kurtosis<3) for the majority of series (the IPI of Tunisia, Spain, Italy, France and the Euro zone) are Platykurtic. This low Kurtosis allows us to note that the seasonally-adjusted IPI series present Platykurtic distributions. Furthermore, the probability analysis for the Jarque-Bera statistics, which are all below 5%, leads us to confirm that the data do not follow a normal distribution. Consequently, this distribution rejects the null hypothesis of normality.

Figure 1. *Variables in first difference*

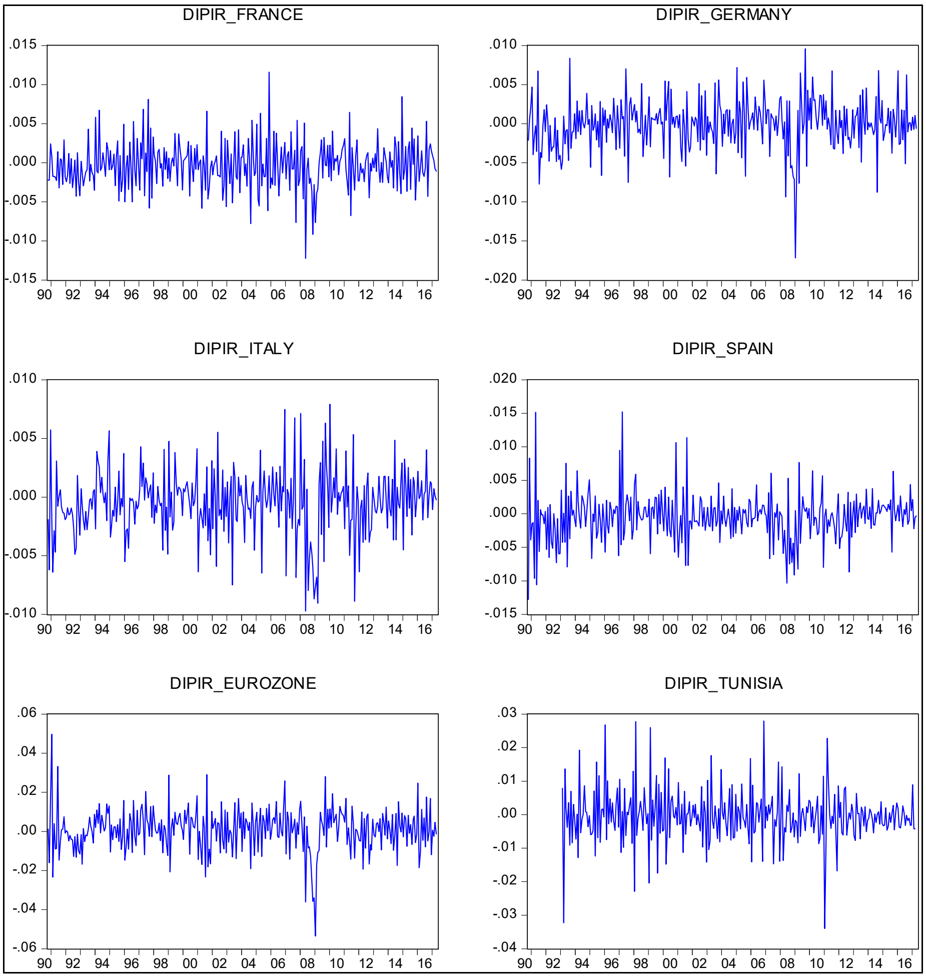


Table 1. *Descriptive statistics of variables*

Designations	IPI^{FRA}	IPI^{GER}	IPI^{ITA}	IPI^{SPA}	IPI^{EUR}	IPI^{TUN}
Mean	1,72e-06	1,57e-05	-1,88e-05	6,78e-06	1,32e-05	1,50e-50
Median	1,047	1,005	1,059	1,074	1,030	1,009
Maximum	1,110	1,051	1,157	1,160	1,054	1,045
Minimum	0,983	0,969	0,963	0,961	0,982	0,922
Standard error	0,00574	0,00893	0,00844	0,00775	0,00851	0,00500
Skewness	-0,54339	-0,65154	-0,48536	0,11031	-0,51253	0,27136
Excess Kurtosis	1,66513	2,61958	1,78756	0,37138	1,38221	-0,52917
Jarque-Bera	40,69077	88,09860	42,58337	1,92041	22,70284	5,24301
Probability	0	0	0	0	0	0
SumSq, Dev,	0,00222	0,00291	0,00210	0,003024	0,02671	0,01106
Observations	320	320	320	320	320	320

Source: author's calculations.

4. Empirical results

4.1. Dating of turning points estimated by the Bry-Boschan method

We applied Bry Boschan's (1971) non-parametric method to the Industrial Production Index (IPI), with the aim of obtaining business cycle turning points. Indeed, the results shown in Table 2 shows that the turning points are different for all countries. To this end, there are clearly defined peaks and troughs for the classic cycle. Indeed, it appears that over the period 1990 to 2017, there are 7 periods of cyclical movement marked by 3 troughs and 4 peaks that are easily identifiable. The first period has four common peaks (1994:12; 1995:1, 2000:05 and 2006:12) relative to the countries (France, Spain, Italy and Germany). But the three common trough dates (1994:1; 1996:12 and 2009:3) are periods of roughly the same duration for Tunisia as for the other Euro zone countries except France.

Table 2. *Turning points in the Classic cycle of Tunisia and Euro zone countries*

	France	Germany	Italy	Spain	Euro zone	Tunisia
Trough	1993:12 1997:1 1999:3 2009:4 2014:11 2016 :7	1994:1 1996:2 2002:5 2003:8 2009:4 2013:1	1994:01 1996:12 1999:05 2005:05 2009:03 2014:05	1993:08 1996:12 1999:02 2006:04 2009:03 2013:02	1996:7 1998:12 2003:9 2005:5 2009:4 2013:2	1995:5 1998:1 2003:11 2005:11 2009:3 -
Peak	- 1995:1 1998:5 2000:5 2011:1 2015:10	1991:1 1994:12 2001:2 2002:11 2008:2 2011:7 2016:1	- 1994:12 1997:12 2000:05 2006:12 2011:04	- 1995:01 1998:02 2000:02 2007:03 2009:12	- 1998:1 2000:12 2004:7 2006:12 2011:2 2016:1	- 1997:1 2001:1 2004:9 2007:6 2010:1
(P to T)	- 24 10 100 40 9	36 14 15 09 14 18	- 24 17 60 27 37	- 11 33 10 28 24	- 11 33 10 28 24	- 12 34 14 21 -
(T to P)	- 13 16 21 21 17	- 11 60 06 54 27 36	- 11 12 12 19 25	- 17 14 12 11 9	- 18 24 10 19 22 45	- 20 36 10 19 10
(P to P)	- - 40 31 121 57	- 47 74 21 63 41 54	- - 36 29 79 52	- - 37 24 85 33	- - 35 43 29 50 69	- - 48 44 33 31
(T to T)	- 37 26 121 61 26	- 25 17 15 68 45	- 35 29 72 46 62	- 40 26 86 35 47	- 29 57 20 47 46	- 32 70 24 40 -

Source: author's calculations.

From an economic point of view, there seems to be little relationship between the duration of a recession or an expansion. As Burns and Mitchell (1947) point out, the duration of cycle phases is at least a few months, without specifying how long. For Bry and Boschan (1971), in their seminal article on cycle dating for the NBER, the identification of turning points follows very clear rules: the minimum duration of a phase is five months, and the minimum duration of a complete cycle, from peak to peak or trough to trough, is fifteen months. By definition, if we look at the empirical literature, we find that these empirical values are found in certain research works or articles which consider a country to be in recession if its GDP growth rate remains negative for two consecutive quarters. Among the works we can cite (Ferrera, 2009; compos et al. 2019; Oman 2019; Arčabić and Škrinjarić, 2021).

Another point for reflection drawn from Table 2 is that there are 05 major periods of recession for Euro zone countries that are easily identifiable: the first recession took place from early June 1995 to late 1996, followed by a slowdown between 1997 and 1999 as a result of the Asian crisis. Then came the third shock, the American recession, which worsened after September 11, 2001 and affected the European economy between January 2002 and March 2003. Finally, the last episode was triggered by the subprime crisis between 2007 and 2009. Focusing on the severity of recession phases, Table 2 shows that the average length of recession is characterized by classic medium-long cycle amplitude for the majority of Euro zone countries, at around 2.5 years for France, Italy and Spain, and 1 year and 2 months for Germany. In the case of Tunisia, on the other hand, the duration of recession phases is short and does not exceed 13.5 months. At this point, following the seminal article by De Grauwe and Ji (2018), most Euro zone countries qualified a period of expansion between the periods 2000 and 2007. But after this date, a crisis known as the "*subprime crisis*" hit the majority of economies worldwide and, in particular, the Euro zone countries. This common shock had a negative impact on these group countries (France, Italy, Germany and Spain), leading to deep and prolonged recessions during this crisis episode. As a result, the movement of the Euro zone cycle is asymmetrical, characterized by stronger cyclical amplitude.

However, for France, the Industrial Production Index (IPI) appears to have a long (peak-peak) cycle with 45.17 months. Whereas for other countries, there is little evidence that average (Peak-Peak) cycle lengths are different, such as in Germany, where the average cycle length is probably lower at 24.29 months. On the other hand, in Italy and Spain, cycles are of roughly the same length, at around 3 years. For Tunisia, cycle lengths (Trough- Trough) are shorter, averaging 27.67 months. As a result, the Tunisian economy appears to be relatively less cyclical than other Euro zone economies, notably those of France, Germany, Italy and Spain.

Table 3. *Average duration of expansion and recession in the Classic cycle*

	France	Germany	Italy	Spain	Euro zone	Tunisia
Recession duration	30.5	15.14	27.5	28.5	15.14	13.5
Expansion duration	14.67	27.71	13.17	10.5	19.71	15.83
Cycle (P to P) Average duration	41.5	42.86	32.67	29.83	32.29	26
Cycle (T to T) Average duration	45.17	24.29	40.67	39	28.43	27.67

Source: author's calculations.

Table 3 on the classic cycle shows that, after the great depression, the economic cycles of the Euro zone countries are relatively well synchronized. This high degree of synchronization was revived by the launch of a single currency in 1999. This led to the creation of the Economic and Monetary Union (EMU), which fostered the emergence of a common economic cycle in the Euro zone. Indeed, the implementation of accommodating monetary policies increased synchronization between European countries by 50%, particularly after the introduction of the euro in 1999, Compos et al. (2019). On the other hand, there was little synchronization between Tunisia and the Euro zone.

Table 4. *Turning points in the Growth cycle of Tunisia and Euro zone countries*

	France	Germany	Italy	Spain	Euro zone	Tunisia
Trough	1996:9	1996:2	1996:10	1996:07	1996:07	1997:09
	1999:2	1999:4	1999:02	1999:03	1998:12	1999:11
	2003:7	2003:7	2002:01	2002:04	2003:09	2003:06
	2005:5	2005:4	2003:07	2005:06	2005:05	2006:03
	2009:6	2009:6	2005:05	2009:04	2009:04	2009:05
	2012:11	2013:3	2009:06	2013:01	2013:02	2011:04
	-		2013:02		-	-
Peak	1995:3	1995:1	1995:04	1995:02	-	1996:09
	1998:2	1998:5	1997:12	1998:06	1998:01	1998:12
	2000:11	2000:11	2000:09	2000:04	2000:12	2001:01
	2004:7	2004:5	2002:09	2004:02	2004:07	2004:04
	2007:12	2008:2	2004:03	2007:10	2006:12	2007:06
	2011:7	2011:9	2008:01	2011:06	2011:02	2010:03
	2013:7		2011:05		2016:11	2013:04
(P to T)	18	13	18	17	-	12
	12	11	14	9	11	11
	32	32	16	24	33	29
	10	11	10	16	10	23
	18	16	14	18	28	23
	16	18	17	19	24	13
	-		21		-	-
(T to P)	-	-	-	-	-	-
	17	27	14	23	18	15
	21	19	19	13	24	14
	12	10	8	22	10	10
	31	34	8	28	19	15
	25	27	32	26	22	10
	8		23		45	24
(P to P)	-	-	-	-	-	-
	35	40	32	40	-	27
	33	30	33	22	35	25
	44	42	24	46	43	39
	41	45	18	44	29	38
	43	43	46	44	50	33
	24		40		69	37
(T to T)	-	-	-	-	-	26
	29	38	28	32	29	43
	53	51	35	37	57	33
	22	21	18	38	20	38
	49	50	22	46	47	23
	41	45	49	45	46	-
	-		44		-	

Source: author's calculations.

A simple comparison of the growth cycle and the classic cycle between Tunisia and some Euro zone countries brings out some important facts that can be observed with a visual examination of Table 4. Indeed, it appears to identify 6 periods of cyclical movement marked by 4 troughs and 2 peaks that are easily identifiable. Starting with the growth cycle troughs that are similar for the majority of Euro zone countries, we can find the following trough points (1999:2; 2003:7; 2005:5 and 2009:6). While, in the case of the Peak points, the hardnesses (2000:11 and 2004:7) are very close for this group of countries (France, Germany and Tunisia).

The identification turning points of growth cycle obtained by the BBQ algorithm highlights several phases and cycles. Indeed, Table 5 clearly shows that the average length of the growth cycle in Tunisia (Peak -Peak) is very close to that identified in the Euro zone (23.29 for Tunisia versus 28.43 for the Euro zone). As Elachhab (2009) points out, over the last thirty years, the financial integration of the Tunisian economy with the Euro zone has been weak (the share of foreign direct investment in GDP was around 1.9% during the period 1970-2000), and it could be argued that it is trade links (import and export) that could explain the bulk of the transmission of European contractions to the Tunisian economy.

Table 5. *Average duration of expansion and recession in the Growth cycle*

	France	Germany	Italy	Spain	Euro zone	Tunisia
Recession duration	15.14	16.83	15.71	17.17	15.14	15.86
Expansion duration	16.29	19.5	14.86	18.67	19.71	12.57
Cycle (P to P) Average duration	31.43	33.33	27.57	32.67	32.29	28.43
Cycle (T to T) Average duration	27.71	34.17	28	33	28.43	23.29

Source: author's calculations.

Similarly, there appears to be a fair degree of symmetry in the duration and scale of recession phases in the Tunisian cycle and the Euro zone. The hardness of the slowdown phase for Tunisia is around 15.86, compared with 15.14 for recession phases in this group of Euro zone countries.) Nevertheless, the average duration of expansion phases evolved over the period 1990-2017 at a low rate, not exceeding 12.57 months for Tunisia versus 19.71 months for the Euro zone). These results show that the behavior of the Tunisian growth cycle is similar to that of other countries, with a slight delay compared to the recession phases of the Euro zone cycle. Overall, it also appears that the irregularities of the Tunisian growth cycle are convergent and similar to those observed in the Euro zone countries.

4.2. Dating of turning points estimated by the Markov-Switching model

The identification of classical cycle and growth characteristics for the industrial production index (IPI) of the majority of countries in our sample estimated by the MSVAR model (Table 6), that there are five periods of recession that characterize the state of the Tunisian economy. Indeed, these periods of decline are essentially marked during the periods 1992-1998, 1999-2000 and 2001-2004, as well as for the great financial crisis from late 2007 to early 2009 and from 2009 to mid-2011. All these episodes have occurred in recent years, the most serious of which have been marked by financial crises, social protests (the Gafsa mining basin revolt against unemployment) and corruption (the nepotism of the Ben Ali clan). Thus, the start of the Tunisian revolution in 2010 and the fall of the Ben Ali regime in January 2011.

According to the results obtained in Table 6, the transition probability matrix estimated by the MS(2) VAR(1) model shows that the "*high-growth regime*" can only be reached through the "*low-growth regime*". Indeed, the asymmetry of the business cycle in terms of the average duration of the expansion (low and high growth) and recession phases is observed by the transition matrix. For the countries in our sample (France, Germany, Italy, Spain, Tunisia and the Euro zone), the coefficients associated with the transition probabilities are highly significant and fairly differentiated, ranging from 0.989 to 0.739. In the case of France and Spain, the regime-dependent values are not in ascending order (i.e., recession is longer than growth). This is why the persistence of the three regimes is different. In the case of regime 1, where the state is in recession, the coefficients of the transition probability are of duration 0.989 and 0.954, with an average duration of 5.985 and 4.204 months for France and Spain respectively. On the other hand, in the second state characterized by low growth, the probability of transition is significant for the cases of Italy, Tunisia and the Euro zone, at 0.902, 0.984 and 0.920 respectively, with an average expansion duration almost doubling from 2.735 months to 5.151 months. Finally, the last state, which is characterized by high growth, shows a much higher growth rate, 0.939 in the case of Germany, with an average duration of 4.231 months. As we have mentioned, the asymmetry in the economic cycle that characterizes the Tunisian economy and the Euro zone is observed specifically in regime 3. This advantage makes the MSVAR model particularly interesting for capturing asymmetry in the country cycle. Ferrera (2009) argues that Markov change models are particularly useful for determining the asymmetry of Euro zone country cycles. He argues that, since 1970, the duration of expansionary and recessionary phases has been asymmetric, lasting between eight and eleven years in euro zone countries. Similarly, Medhioub and Mraihi (2011) confirm this result, proving that regime-switching models can capture the business cycle asymmetries of Tunisian economic activity.

Table 6. *Parameter Estimates for Markov-Switching VAR models*

Parameter	MSIH(3)-VAR(2) Specifications																	
	$DLIPI^{FRA}$			$DLIPI^{Ger}$			$DLIPI^{ITA}$			$DLIPI^{SPA}$			$DLIPI^{EUR}$			$DLIPI^{TUN}$		
	Coef.	S-Er	T-Stat	Coef.	S-Er	T-Stat	Coef.	S-Er	T-Stat	Coef.	S-Er	T-Stat	Coef.	S-Er	T-Stat	Coef.	S-Er	T-Stat
μ_1	0.989	0.015	63.824	0.838	9.733	0.000	0.893	0.066	13.487	0.9538	0.030	31.354	0.760	0.207	3.676	0.329	0.358	0.919
μ_2	0.006	0.007	0.835	0.073	0.853	0.394	0.049	0.024	2.010	0.001	0.000	0.000	0.001	0.000	0.000	0.278	0.209	1.330
μ_3	0.001	0.000	0.000	0.013	0.877	0.381	0.023	0.051	0.450	0.069	0.050	1.375	0.060	0.064	0.936	0.071	0.116	0.611
ϕ_{Fra}	0.010	0.015	0.680	0.100	1.199	0.232	0.106	0.066	1.609	0.046	0.030	1.518	0.239	0.207	1.157	-0.087	0.153	-0.569
ϕ_{Ger}	0.954	0.044	21.793	0.739	6.850	0.000	0.902	0.048	18.642	0.949	0.029	33.117	0.984	0.012	82.058	-0.157	0.091	-1.720
ϕ_{ITA}	0.144	0.141	1.019	0.048	1.154	0.250	0.124	0.102	1.217	0.041	0.051	0.795	0.084	0.087	0.970	0.480	0.222	2.162
ϕ_{SPA}	-0.453	0.116	-3.914	-1.358	5.720	0.000	-0.815	0.182	-4.472	-0.494	0.175	-2.822	-3.054	0.436	-7.005	-0.000	0.073	-0.002
ϕ_{EUR}	0.106	0.114	0.933	0.263	3.095	0.002	0.032	0.123	0.260	0.101	0.072	1.404	0.156	0.056	2.777	-0.332	0.192	-1.724
ϕ_{TUN}	0.162	0.172	0.943	0.211	1.920	0.056	1.101	0.245	4.500	0.587	0.287	2.044	-0.700	0.331	-2.118	0.069	0.077	0.9087
σ_1	0.418	0.103	4.036	0.589	2.427	0.016	0.612	0.165	3.699	1.177	0.258	4.568	0.387	0.302	1.281	0.140	0.162	0.866
σ_2	1.258	0.228	5.519	0.084	1.927	0.055	0.458	0.105	4.358	0.316	0.060	5.224	0.417	0.052	8.008	0.345	0.126	2.742
σ_3	0.389	0.227	1.711	0.793	5.601	0.000	0.427	0.129	3.312	1.383	0.484	2.855	0.843	0.312	2.697	1.179	0.248	4.748
Log L	-259.318			-238.777			-232.017			-246.466			-205.466			-243.714		
AIC HQ SBC	-0.084 -0.077 -0.067			-0.554 -0.540 -0.520			-0.995 -0.981 -0.961			-0.292 -0.285 -0.274			-0.863 -0.857 -0.846			-0.416 -0.395 -0.364		
Matrix of Transition Probabilities																		
	P_{1i}	P_{2i}	P_{3i}	P_{1i}	P_{2i}	P_{3i}	P_{1i}	P_{2i}	P_{3i}	P_{1i}	P_{2i}	P_{3i}	P_{1i}	P_{2i}	P_{3i}	P_{1i}	P_{2i}	P_{3i}
P_{1j}	0.989*	0.006	0.001	0.838	0.072	0.013	0.893	0.049	0.023	0.954*	0.001	0.069	0.760**	0.001	0.060	0.506	0.017	0.525
P_{2j}	0.010	0.954	0.144	0.100	0.739**	0.048	0.106	0.902*	0.124	0.046	0.949	0.0406	0.239	0.984*	0.084	0.494	0.920*	0.001
P_{3j}	0.000	0.0393	0.855*	0.061	0.187	0.939*	0.000	0.049	0.852*	0.000	0.050	0.891**	0.000	0.014	0.856	0.000	0.063	0.474**
Duration	5.985			4.231			5.151			4.204			5.036			2.735		

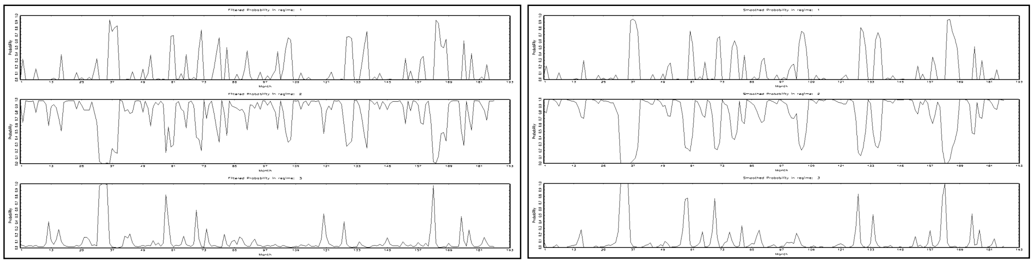
Note: $DLIPI^{FRA}$ denotes the industrial production index of France, $DLIPI^{GER}$ denotes the industrial production index of Germany, $DLIPI^{ITA}$ denotes the industrial production index of Italy, $DLIPI^{SPA}$ denotes the industrial production index of Spain, $DLIPI^{EUR}$ denotes the industrial production index of Euro zone, and $DLIPI^{TUN}$ denotes the industrial production index of Tunisian, respectively

Regime 1 = Recession State.

Regime 2 = Expansion State (with Low growth rate).

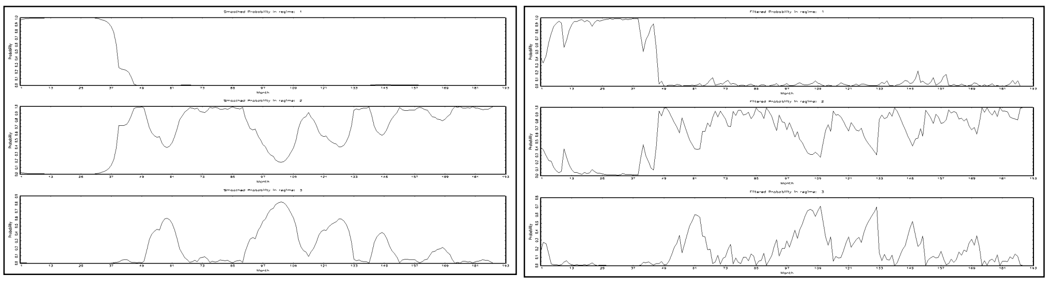
Regime 3 = Expansion State (with High growth rate).

Figure 2. *Filtered and Smoothed probabilities (MSVAR) with 3 regimes for Tunisia*



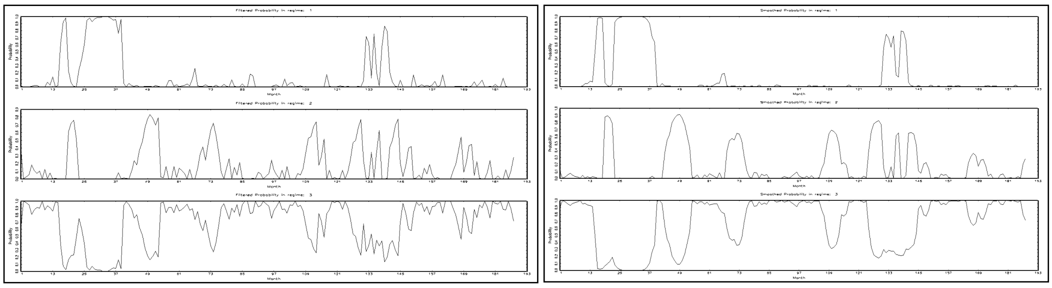
The figure 2 represents the filtered and smoothed probabilities of 3 regimes for the case of Tunisia. These probabilities illustrate the clear state of the Tunisian economy, with regimes varying between values close to 0 and 1. Indeed, Figure 2 suggests major, abrupt and much shorter bearish periods. The periods of decline that characterize the state of the Tunisian economy according to figure 2 are essentially marked for the following months: September 1992 to April 1993; October 1994 to February 1995; October 1995 to January 1996; September 1998 to January 1999; September 1999 to May 2000; February 2001 to August 2001 and April 2003 to January 2004. In line with the results in Table 5, the coefficient of the probability of recession for regimes 1 and 3 reaches 95%, specifically during the period 1992-1998; 1999-2000 and 2001-2004, and the great financial crisis of late 2007 to early 2009 and 2009 to mid-2011. All these events have taken place over the past few years, and it is clear that this period of stagnation is justified by the decline in real economic activity, which has deteriorated particularly with the succession of crises, most recently the outbreak of the Tunisian revolution in December 2010.

Figure 3. *Filtered and Smoothed probabilities (MSVAR) with 3 regimes for France*



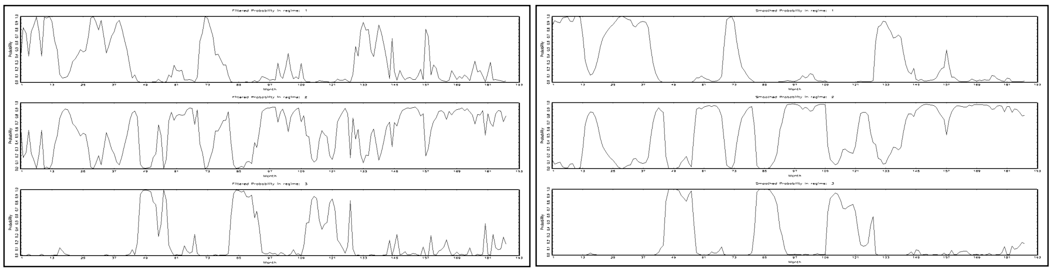
Looking at figure 3, we can see from the analysis of filtered and smoothed probabilities in three regimes for the case of France three major recessions. All these recessions are preceded by an abnormal increase in filtered probabilities: the first recession is observed between 1994 and 1996, the second from early 1998 to 2000. The two oil shocks (smoothed probabilities for the case of 3 regimes worth 0.90 and 0.70 respectively on Figure 3). Finally, the last recession, following the subprime crisis, spans the years 2008 and 2009. This result confirms the work of Damette and Rabah (2010), for the case of France where the model result of Hamilton (1989), identifies three recession dates (1994, 1998 and 2008 to 2009)[(6)].

Figure 4. *Filtered and Smoothed probabilities (MSVAR) with 3 regimes for Germany*



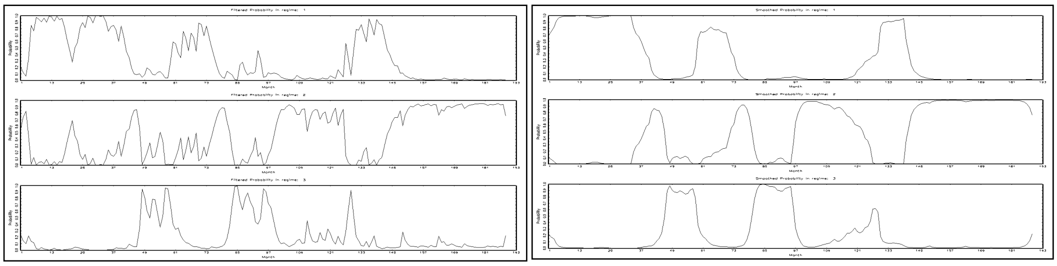
As shown in figure 4, the filtered probability of MS(2)-VAR(1) model describes very precisely 3 recession phases for the Germany case with an average duration of 6.3 months. These short periods of recession that characterize the German economy are marked by this sharp decline specifically between April 1991 to November 1991, December 1993 to May 1994 and July 2001 to December 2001. After these periods of economic instability from 1991 to 2001 there was a substantial recovery period (probability smoothed regime 2). This result is consistent with the main argument of de Castro (2015) for the case of the Portuguese economy.

Figure 5. *Filtered and Smoothed probabilities (MSVAR) with 3 regimes for Italy*



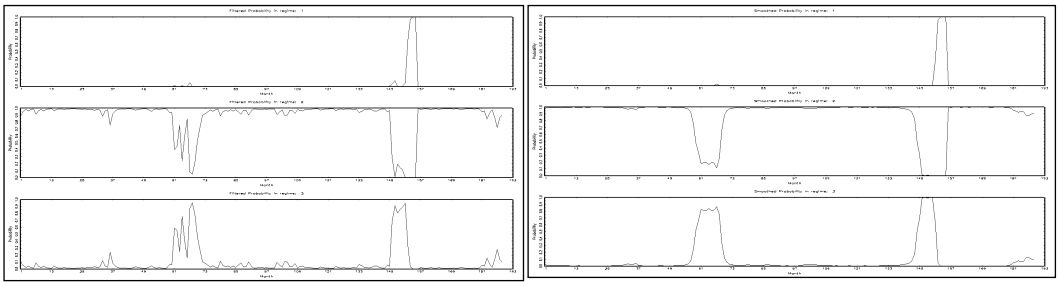
The observation in figure 5 of the identification of expansion and recession phases estimated by the MS (2) VAR (1) model, (filtered and smoothed probabilities) for the case of Italy confirms the hypothesis that the conditions for the crisis emerged during this period in the sense that the economy was still operating in one of the other two regimes associated with expansions. In fact, the slowdowns in growth that are signaled by the model. For example, in September 1994, May 1995 and January 1998, the filtered probability did not exceed the 5% thresholds.

Figure 6. *Filtered and Smoothed probabilities (MSVAR) with 3 regimes for Spain*



In the case of Spain, as can be seen from figure 6 and in line with the result in table 5, the filtered and smoothed probabilities increase progressively from the first three years of the '90s, reaching 97% when the recession actually begins in October 1993 in the case of regime 1. Secondly, we should note the significant predictive content of Spain's Industrial Products, since by using this variable exclusively, we obtain a probability of recession already equal to 95% at the end of 1995 in the same regime.

Figure 7. *Filtered and Smoothed probabilities (MSVAR) with 3 regimes for Euro zone*



Based on the estimation result of the MSVAR model presented in figure 7, which describes the recession and expansion phases for Euro zone countries. The shape of this graph shows that the Euro zone IPI can identify 5 major recessions corresponding to significant movements with a probability close to one. The first recession occurred from early June 1995 to late 1996, followed by a downturn between 1997 and 1999 as a result of the Asian crisis. This was followed by the third shock, the American recession, which worsened after September 11, 2001 and affected the European economy between January 2002 and March 2003. Finally, the last episode was triggered by the "*subprime crisis*" between 2007 and 2009. The result of our study confirms some empirical works such as (Burak et al 2003; Ferrera 2009 and Oman 2019), which proves five recession periods. However, it does not confirm other works such as Castro (2015), which proves 4 periods of recession (Medhioub 2015; Damette and Rabah, 2010) for three periods of recession.

5. Discussion and conclusion

The beginning of the history of business cycle research has been observed by economists since the 20th century in the work of Hayek (1933) and Von Mises (1933). Indeed, since the 1970s, the NBER has been researching the chronology of the business cycle for the US economy. However, for European countries, the Centre for Economic Policy Research (CEPR) and the Economic Cycle Research Institute (ECRI) has been extending this task to the Euro zone since the 1990s. In developing countries, renewed interest in business cycle analysis, has been developed since the 2000s (Elachheb, 2010; Medhioub and Mraihi, 2011; Benhida, 2015; Medhioub, 2015). Indeed, the economic crises and changes that have hit the global economy have given particular attention to the search for points of reflection on the origins of economic fluctuations and the identification of the characteristics of economic cycles, with an emphasis on the explanatory factors of the cyclical transition mechanism between countries. With this in mind, we have chosen as the main objective of this article to study the economic cycle in the Tunisian context on the one hand, and to compare it with that of the Euro zone countries on the other, using Bry Boschan's non-parametric method with particular attention paid to Hamilton's (1989) regime-switching models, with a view to producing results that would help policy-makers to make better decisions and adopt appropriate, sound and effective economic policies to cushion falls and avoid periods of depression.

The empirical results of this article show the fragility of the Tunisian economy to international conditions, in particular those of Euro zone countries, by focusing on the explanatory factors of the cyclical transition mechanism between countries Berkmen et al. (2012). Indeed, the growth cycle and classic analysis obtained by applying the BBQ algorithm clearly shows that the average length of the growth cycle in Tunisia (Peak-Peak) is very close to that identified in the euro zone. Thus, the average length of the recession is characterized by classical cycle amplitude that is moderately long for most Euro zone countries, i.e. around 2.5 years. By contrast, the duration of recessionary phases in Tunisia is short, not exceeding 13.5 months. Furthermore, on the basis of the transition probability matrix obtained with regime-switching models (MSVAR). We note that the coefficients associated with transition probabilities are highly significant and quite differentiated, ranging from 0.739 to 0.989. This means that these probabilities illustrate the state of the Tunisian economy, which experiences major downturns, characterized by more stable recessionary situations throughout the study period. However, it should be noted that for the Euro zone economy, the smoothed and filtered probabilities can identify 5 major recessions corresponding to significant movements with a probability close to one. These periods of decline were most marked between the beginning of June 1995 and the end of 1996, followed by a slowdown between 1997 and 1999 in the wake of the Asian crisis. This was followed by a third shock, the US recession, which worsened after September 11, 2001 and affected the European economy between January 2002 and March 2003. Finally, the most serious incident to affect the global economy, and European countries in particular, was the so-called "*subprime crisis*" between 2007 and 2009.

Notes

(1) Bry Boschan's (1971) methods were modified by Harding and Pagan (2003) and adapted for the quarterly time series (Elachhab, 2007). The assumptions of the (BB) algorithm procedure for dating a series (monthly period) can be summarized in the following 06 points: (1) A peak (end of an expansion) must be followed by a trough (end of a recession); (2) In the presence of consecutive double peaks, respectively troughs, the higher, respectively lower value is chosen; (03) A phase is the set of monthly periods, that separate a consecutive peak and trough; (04) A cycle is the set of two consecutive phases, i.e. the set of periods from a peak to a peak or from a trough to a trough; (05) The duration of a cycle (Peak-Peak or Trough-Trough) must be greater than or equal to 15 months for monthly data; (06) The duration of a phase (Peak-Trough or Trough-Peak) must be at least five periods for monthly data". Concerning the determination of dating for monthly data, we find that the series achieves a local maximum at date t if it satisfies the condition: $\{Y_t > Y_{t\pm k}\}$ and a minimum if $\{Y_t < Y_{t\pm k}\}$ with k = 5. The amplitude for a recession phase and for an expansion phases with Y_P and Y_C is the values of the series at the peak and trough of the cycle considered. For more details, see Harding and Pagan (2003).

(2) The MSVAR model was chosen on the basis of the AIC, the HQ criterion and the LR test.

(3) These two types of probabilities are calculated on the basis of all the information available at date T.

(4) These rules are; Peak and trough must alternate; Phase must last at least six months; Cycle must have a minimum duration of fifteen months Medhioub and El Euch (2013).

(5) The industrial production index (IPI) is seasonally adjusted in Eviews 10.0 using the Census X11 method, and standardized by the consumer price index (CPI).

(6) cf. fig. 1 pp 146 Damette and Rabah (2010).

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