Theoretical and Applied Economics
Volume XXXII (2025), No. 4(645), Winter, pp. 27-42

Co-movement and causality among exchange rates: a wavelet analysis

Tejesh H R
SUBN Theosophical Women's College, India
hrtejesh@gmail.com

Abstract. *This paper investigates the dynamic co-movement and causality among the daily returns of four currency pairs: USD/INR, EUR/INR, GBP/INR, and JPY/INR, covering the period from 2000 to 2024. Using Wavelet Power Spectrum (WPS) and Wavelet Coherence Analysis (WCA), we explore the relationships and causal dynamics between these currency pairs across various time scales. Our findings reveal significant coherence among the currency pairs from 2008 to 2019, likely influenced by global events such as the 2008 financial crisis. The findings also reveal that the majority of the pairs exhibit positive and strong co-movement across different scales. Finally, bidirectional causality is observed between USD and EUR, while the other pairs exhibit either unidirectional causality or independent movements.*

Keywords: co-movement, causality, exchange rates, returns, wavelet analysis.

JEL Classification: C10, C58, F31, G11, G15.

1. Introduction

Foreign exchange rate co-movement is a crucial concept in international finance. It has sparked growing interest among investors and financial analysts due to its significant implications for global trade and investment. In currency markets, understanding co-movement is essential for optimizing portfolio diversification strategies in both domestic and international settings. Much like stock markets, foreign exchange markets exhibit interconnected dynamics that influence asset allocation and risk management. The effective co-movement between currency pairs increases the ability to predict exchange rate trends, playing a critical role in risk management and expected returns. Currency markets have become highly integrated, forming complex relationships across regions and time frames. This interconnectedness allows investors to adjust their strategies based on currency fluctuations and global economic conditions. Technological advancements have accelerated this integration, providing real-time data flows that drive periods of both synchronized and independent movement in currency markets. However, the inherent volatility of exchange rates and the scarcity of reliable information make it difficult for investors to anticipate market movements with certainty. Despite increasing focus on financial markets, co-movement in exchange rates has received relatively limited attention, with only a handful of studies addressing this topic (Kuşkaya et al., 2022; Kumar et al., 2016; Yang et al., 2016; Nikkinen et al., 2011). This presents an exciting opportunity to explore exchange rate co-movement, particularly in the time-frequency domain, given the limited existing literature in this area.

On the econometric side, most previous studies have relied on traditional methods, such as correlation coefficients (Negro et al., 2004), co-integration techniques, and error correction models (Arouri et al., 2013). Other approaches, like the copula model (Nguyen et al., 2013; Samarakoon, 2011) and ARCH/GARCH frameworks (Hammoudeh et al., 2015; Ito et al., 1994; Mensi et al., 2016; Theodossiou et al., 1993), have been applied to investigate stock market co-movement. While these models have proven useful, they often produce mixed results and are not well-suited to capturing co-movement across multiple time frequencies, which is an important concern for portfolio managers (Hkiri et al., 2014). Additionally, long-term co-integration may not be enough to understand the complexities of exchange rate dynamics, especially when short-term fluctuations and broader time-frequency interactions are considered. As a result, predicting the dynamic co-movement and causality in foreign exchange markets remains a complex yet vital task for financial practitioners and researchers. Unlike traditional approaches, co-integration examines long-term equilibrium relationships between exchange rates, which are influenced by a wide range of complex and interconnected economic and political factors. Exchange rates are also categorized by noise and non-stationarity, making them difficult to forecast using classical statistical methods. As Daubechies (1990) suggested, wavelet transform or wavelet analysis is an effective tool for analysing time series with non-stationary properties across different frequencies. One of the important strengths of wavelet analysis is its ability to examine relationships between variables from both time and frequency perspectives simultaneously. This allows the separation of currency movements into various frequency components, enabling a more detailed study of their dynamics. Building on this approach, in this paper, we examine the dynamic co-movement and causality in daily returns of USD/INR,

EUR/INR, GBP/INR, and JPY/INR currency pairs using the wavelet coherence approach. This study is expected to contribute to the existing literature by exploring the relatively under-researched area of co-movement and causality in exchange rates. Applying the wavelet coherence approach, to analyse the time-frequency dynamics of exchange rate co-movement.

The structure of this study is outlined as follows: Section 2 describes the methodology employed and details the dataset used for wavelet analysis. Section 3 presents the empirical results and discussion of the key findings. Lastly, Section 4 provides a summary of the main findings and concludes the study.

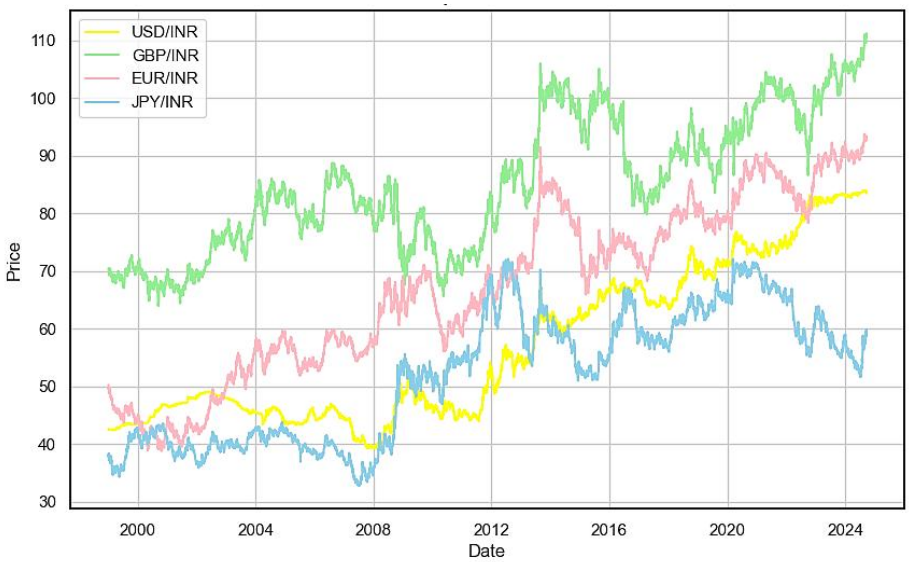
2. Research Methodology

This section presents the theoretical framework, empirical methodology, and the data used in the study, followed by an explanation of the wavelet analysis process. We start by describing the data.

2.1. Data

This study investigates the dynamic co-movement and causality in the daily returns of selected currency pairs using the wavelet coherence approach. The dataset includes daily exchange rates of four major currency pairs, namely USD/INR, EUR/INR, GBP/INR, and JPY/INR, covering the period from 1 April 1999 to 20 September 2024. This time frame spans 6222 daily observations and was selected based on data availability from the Reserve Bank of India (RBI). The exchange rate data are sourced entirely from the official website of the RBI (rbi.org.in).

Figure 1. *Daily time series of exchange rates*



Source: Developed by the author(s).

The dataset consists of daily exchange rates between the Indian Rupee (INR) and the US Dollar (USD), Euro (EUR), British Pound (GBP), and Japanese Yen (JPY). These currency pairs are chosen due to their significance in India's foreign trade, investment flows, and overall economic interactions. They represent India's primary economic partners and have a major influence on its foreign exchange market and the value of the INR. Figure 1 shows the daily exchange rate trends of all the selected currency pairs over the study period. Consequently, this study is expected to provide insights into how global economic trends and bilateral relationships impact India's exchange rate dynamics.

2.2. Data Transformation

To analyse the currency returns, we computed the daily returns using the first differences of the natural logarithms of exchange rates. The returns formula is expressed as:

$$R_t = log\left(\frac{P_t}{P_{t-1}}\right) \times 100$$

where R_t represents the currency returns on day t, while P_t and P_{t-1} represent the exchange rate levels at time t and $t-1$, respectively. The returns are expressed as percentages by multiplying the logarithmic differences by 100. This method allows us to normalize the data and ensure comparability across the different currency pairs.

2.3. Wavelet Analysis

The Wavelet methodology is a mathematical approach increasingly used in various fields, including economics and finance, for modeling time series data. Unlike the traditional Fourier Analysis, which cannot capture time-dependent changes in a signal, the wavelet transform offers the advantage of representing a signal in both time and frequency domains (Caraiani 2012). Wavelets are oscillating functions that decay over time, making them effective filters for decomposing signals into different resolution levels. This process, known as multi-resolution decomposition, allows the analysis of both large and small features in a time series, with each level corresponding to a specific time scale. The application of wavelets is particularly useful for non-stationary phenomena – those that exhibit time-varying characteristics and do not maintain a constant mean or variance. In addition to its frequency and time domain features, this emerging method addresses econometric synergy by overcoming issues related to error terms and structural breaks (Antonakakis et al., 2017; Yang et al., 2017). This versatility has led to their widespread use in disciplines such as geophysics, medicine, statistics, economics, and finance. Ftiti et al. (2016) emphasized some benefits of wavelets, including their ability to decompose data into different frequency components, which made it easier to identify seasonality, structural changes, volatility clusters, and local and global dynamics Nini et al., (2023). However, Dibal et al. (2018) pointed out drawbacks such as redundancy, high computational demand, and imperfect signal reconstruction.

This study attempts to examines the co-movement among USD/INR, EUR/INR, GBP/INR, and JPY/INR currency pairs to understand their linkages across different time horizons (short, medium, and long-term). The wavelet coherency method, as proposed by Grinsted et al. (2004), allows for decomposing these relationships into time-frequency components.

It can be applied in both bivariate and multivariate analyses to explore covariation and causal relationships across different scales over time. Similar to the Pearson Bivariate Correlation Coefficient, wavelet coherency measures the degree of co-movement in the time-frequency domain between two time series (Singh et al. 2022).

2.4. The Continuous Wavelet Transform (CWT)

The CWT is a multiresolution analysis technique that focuses on the dilation and contraction of wavelet functions (Sahabuddin et al., 2022). The wavelet technique decomposes return series into time-scale components while capturing the variability and structure of stochastic processes at each scale. A wavelet is a small wave that can be stretched or compressed to extract frequency components from a complex signal (Bouri et al. 2020).

The mother wavelet, which generates these smaller waves, is defined by time and scale *s* as:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

Hear, *τ, s,* and $1/\sqrt{s}$ represent the translation parameter, dilation parameter, and normalization factor, respectively. $\psi_{\tau,s}(t)$ are elementary functions derived from the wavelet transform of a time series, obtained from a time-localized mother wavelet $\psi(t)$.

To analyse wavelet coherence among USD/INR, EUR/INR, GBP/INR, and JPY/INR currency pairs, the Morlet Wavelet is applied (Morlet et al. 1982). This wavelet comprises both smooth (father) and detailed (mother) functions within the wavelet family and provides an optimal balance between time and frequency localization. Grinsted et al. (2004) noted that the Fourier period for the Morlet wavelet closely matches the scale used:

$$\psi^{M}(t) = \frac{1}{\pi^{1/4}} e^{i\omega_o t} e^{-t^2/2}$$

In this context, $\pi^{1/4}$ preserves wavelet energy within a band while maintaining the central frequency. The term ω_o refers to the localizations that balance time and frequency. Also, $e^{-t^2/2}$ represents the Gaussian envelope, while $e^{i\omega_o t}$ denotes a complex analytic wavelet.

2.5. Wavelet Power Spectrum

Wavelet analysis can use either continuous (CWT) or discrete wavelet transforms (DWT). The CWT has advantage of flexibility in wavelet selection based on data length and its redundancy. The CWT of a discrete-time series *x(t)*, where *t = 0, 1,….., n,* with respect to $\psi(t)$, is given by:

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\, \psi * \left(\frac{t-\tau}{s}\right) dt$$

Here, * represents the complex conjugate. The wavelet power spectrum (WPS) measures the relative contribution of each time and scale to a time series' variance, represented as $|W_x(\tau, s)|^2$. The WPS was used in this paper to analyse the behavior of the time series.

Wavelet transform preserves the energy of a time series, making it useful for power spectrum analysis. The total variance is calculated by integrating across both time *(τ)* and scale *(s)*, and is expressed as:

$$\sigma_x^2 = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|W_x(\tau,s)|^2\,\frac{d\tau ds}{s^2}$$

As stated above, the cross-wavelet spectrum is notable for revealing co-movement between two series in both frequency and time. Through analysing the contour plot of the wavelet cross-spectrum, one can identify the time-frequency regions where the series co-move and assess the variation in co-movement over time and frequency.

2.6. Wavelet Coherence

The cross-wavelet power identifies areas of strong shared power between two time series in the time-frequency domain. The wavelet squared coherence between the two series is defined as:

$$R_{xy}^2(\tau,s) = \frac{\left|S(s^{-1}W_{xy}(\tau,s))\right|^2}{S\left(s^{-1}|W_x(\tau,s)|^2.S(s^{-1}\left|W_y(\tau,s)\right|^2\right)}$$

The term $R_{xy}^2(\tau,s)$ is a measure of contemporaneous correlations between *x(t)* and *y(t)* at each time point and frequency. The function S(.) is a smoothing parameter for scale and time. The value of $R_{xy}^2(\tau,s)$ ranges from zero (indicating no co-movement) to one (indicating high co-movement), reflecting a scale-specific squared correlation between the two series. The wavelet coherence framework helps the analysis of lead-lag relationships between series, overcoming the limitation of squared coherence in distinguishing between positive and negative correlations. Phase differences, which span the entire time frame for frequency band functions, provide clear idea into these lead-lag relationships. They are represented by arrows that indicate positive or negative correlations in both frequency and time domains. According to Webster et al., (1999) and Bloomfield (2013), the phase relationship between *x(t)* and *y(t)* is expressed as:

$$\phi_{xy}(\tau,s) = tan^{-1}\frac{\Im\{W_{xy}(\tau,s)\}}{\Re\{W_{xy}(\tau,s)\}}, \phi_{x,y}\epsilon[-\pi,\pi]$$

In this context, the parameters $\Im$ and $\Re$ represent the imaginary and real parts of the smoothed power spectrum, respectively. A zero-degree phase difference indicates synchronization between *x(t)* with *y(t)* at a specific time-frequency. In wavelet coherence plots, $\phi_{xy}(\tau,s)$ is demonstrated with arrows – rightward, leftward, upward, and downward – within statistically significant areas. A rightward arrow suggests that *x(t)* and *y(t)* are in phase and positively associated, while a leftward arrow indicates they are out of phase and negatively associated, with minimal time lag. An upward arrow signifies that the first series leads the second by $\pi/2$, and a downward arrow indicates the opposite.

The results of wavelet coherence are generally presented in a chart where time and frequency (or scale) are plotted on the respective axes. The degree of association (co-

movement) between the two analysed time series is represented by a color gradient. Warmer colors, such as red, indicate strong co-movements, while cooler colors, like blue, signify weak co-movements. Areas located outside the black line, known as the cone of influence, indicate that the estimates of wavelet coefficients are statistically insignificant at the 5% significance level and are therefore not considered in the analysis.

3. Empirical results and Discussion

In this section, we begin with preliminary statistical analysis, which includes descriptive statistics for each series, normality tests, serial correlation tests, and graphical representations, as detailed in Subsection 3.1.

3.1. Preliminary Analysis

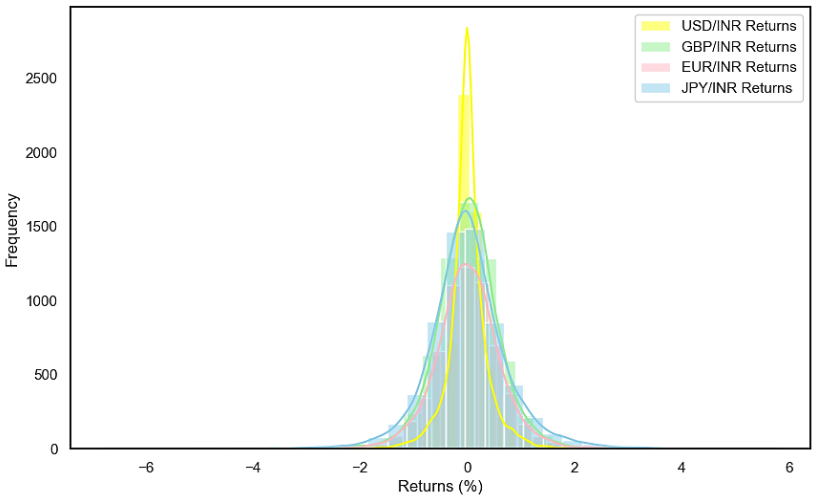
Table 1. *Summary statistics for the return series of currency pairs*

	USD/INR	GBP/INR	EUR/INR	JPY/INR
Descriptive statistics:				
Mean	0.0109	0.0074	0.0099	0.0073
Standard Deviation	0.3858	0.6104	0.6146	0.7557
Minimum	-0.1409	-0.3325	-0.3446	-0.3978
Maximum	0.1562	0.3471	0.3547	0.3776
Skewness	4.0200	3.6789	4.1545	5.7640
Kurtosis	0.2094	-0.5519	0.0529	0.3035
Normality tests:				
Jarque-Bera test	18174.13 (0.000)	12629.12 (0.000)	1754.12 (0.000)	4228.04 (0.000)
Shapiro-Wilk test	0.8986 (0.000)	0.9552 (0.000)	0.9779 (0.000)	0.9564 (0.000)
Kolmogorov-Smirnov test	0.2579 (0.000)	0.1521 (0.000)	0.1439 (0.000)	0.1147 (0.000)
Serial correlation tests:				
Ljung-Box test	45.75 (0.000)	23.70 (0.0084)	19.71 (0.0321)	17.24 (0.0691)
Breusch-Godfrey test	44.40 (0.000)	23.22 (0.00995)	19.98 (0.02945)	17.24 (0.06925)

Note: The values in the parentheses are p-values.
Source: Author(s) calculation.

We start by presenting summary statistics for the return series of each currency pair. Table 1 summarizes the daily return statistics for four currencies, namely USD, EUR, GBP, and JPY, relative to the INR. Among these, the USD/INR pair stands out with the highest average return and the lowest volatility, indicating both favorable and stable performance compared to the others. The JPY shows the lowest average daily returns among the currencies. Additionally, JPY returns are the most volatile within the group, with a standard deviation of 0.76%. All currency pairs are positively skewed, representing a longer tail on the right side of the distribution. On the other hand, only GBP/INR has a negative kurtosis value, indicating a flatter distribution. The hypothesis of normality of the unconditional distribution for all currency pairs are rejected, confirming the presence of outliers and non-standard behaviors in each series. This can also be seen in Figure 2. With the exception of JPY/INR, all the other pairs show strong autocorrelation in both the return series and the squared return series.

Figure 2. *Distribution of returns series of currency pairs*

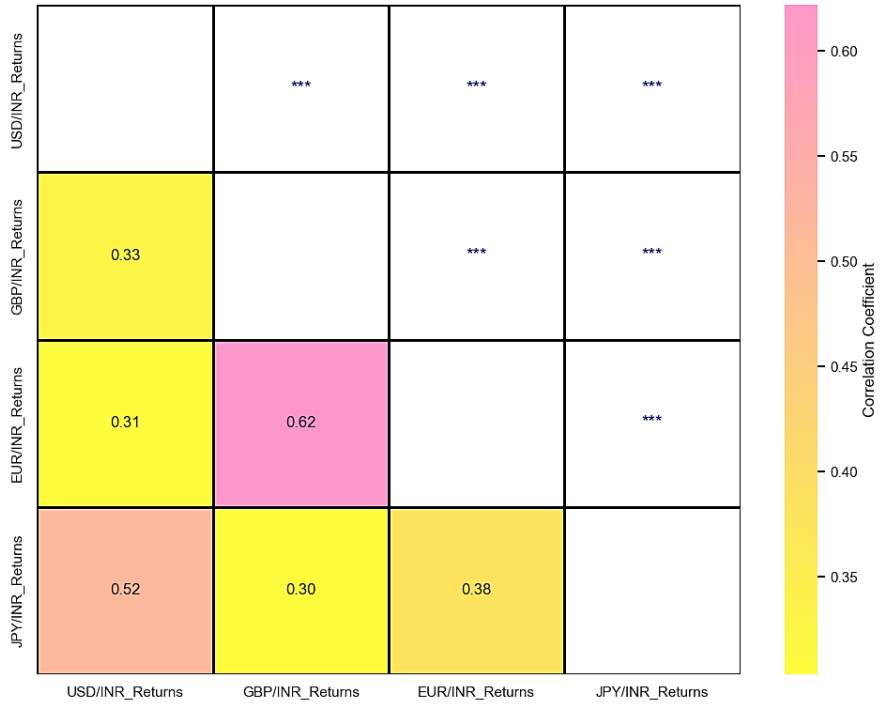


Source: Developed by the author(s).

3.2. Unconditional Correlation Analysis

Figure 3 presents the pairwise correlations across the returns of the currencies considered.

Figure 3. *Unconditional correlation matrix*



Source: Developed by the author(s).

All currency pairs show positive, significant, and moderate correlations with one another. The highest association is observed between EUR/INR and GBP/INR at 62%, followed by JPY/INR and USD/INR at 52%, among others.

3.3. Non-Stationarity Tests

The results of the unit root tests for the return series of the currency pairs, using the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests, are presented in Table 2. For all the currency pairs, highly significant negative statistics with p-values below the significance level are shown by the ADF test results. In contrast, relatively low KPSS test statistics with p-values greater than 5% are observed. Together, these results confirm that the return series for all currency pairs are stationary, as the absence of a unit root is suggested by both tests.

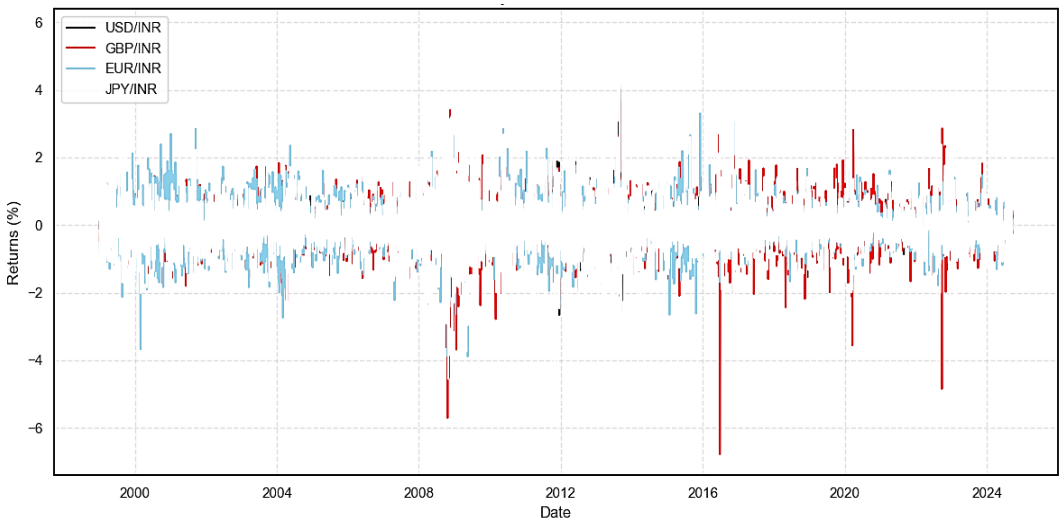
Furthermore, the time-series plot of the returns in Figure 4 provides additional visual confirmation of the absence of a unit root, reinforcing the stationarity of the data.

Table 2. *Unit root test for the return series of currency pairs*

Currency	Augmented Dickey-Fuller (ADF) test		Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test	
	Statistic	p-value	Statistic	p-value
USD/INR	-14.2430	0.0000	0.1258	0.1000
GBP/INR	-17.6173	0.0000	0.0361	0.1000
EUR/INR	-26.5274	0.0000	0.0419	0.1000
JPY/INR	-81.4603	0.0000	0.0706	0.1000

Source: Author(s) calculation.

Figure 4. *Daily returns series of currency pairs*



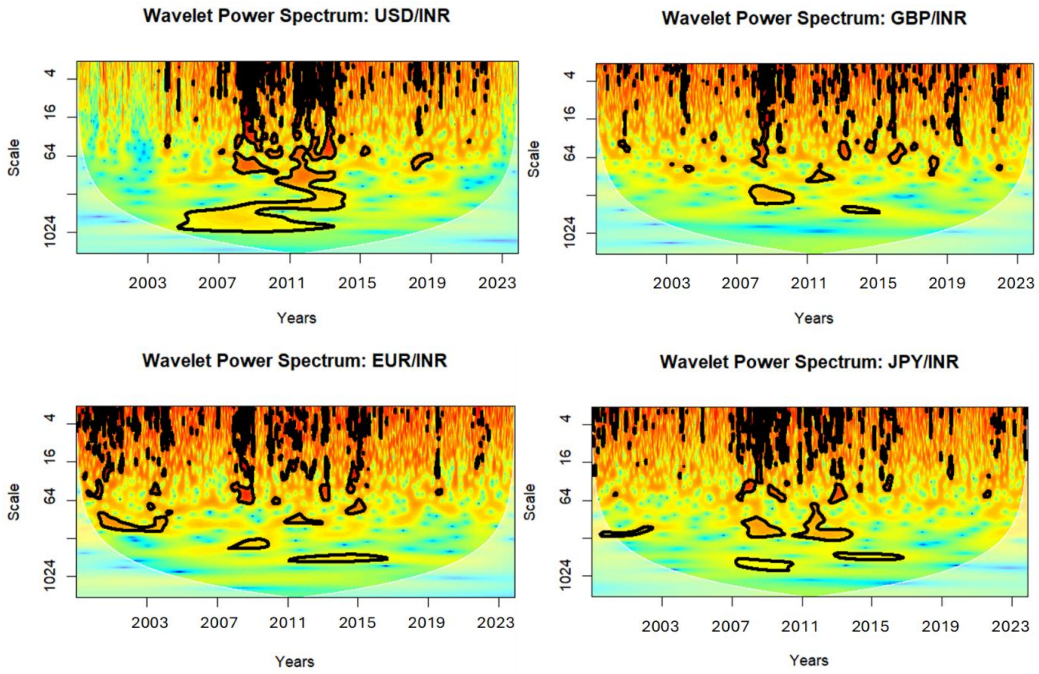
Source: Developed by the author(s).

3.4. Wavelet Power Spectrum

Figure 5 demonstrations the wavelet power spectrum for USD/INR, EUR/INR, GBP/INR, and JPY/INR returns from 2000 to 2024. The scale represents different frequencies, with lower values (4-16) representing short-term fluctuations and higher values (1024) corresponding to long-term variations. Orange areas indicate high power and strong volatility, while light blue/green areas signify low power, reflecting periods of stability.

For the USD/INR series, the early period from 2000 to 2004 displays a relatively stable market, with mostly blue/green regions reflecting minimal volatility. However, volatility begins to rise around 2005, peaking between 2009 and 2013 (where noticeable short-term fluctuations dominate the spectrum). A more turbulent period follows from 2019 to 2021, with significant medium- and long-term volatility, especially between 64-16 scales. By 2022 and 2023, volatility diminishes, though some medium-term instability remains.

Figure 5. *Wavelet power spectrum*



Source: Developed by the author(s).

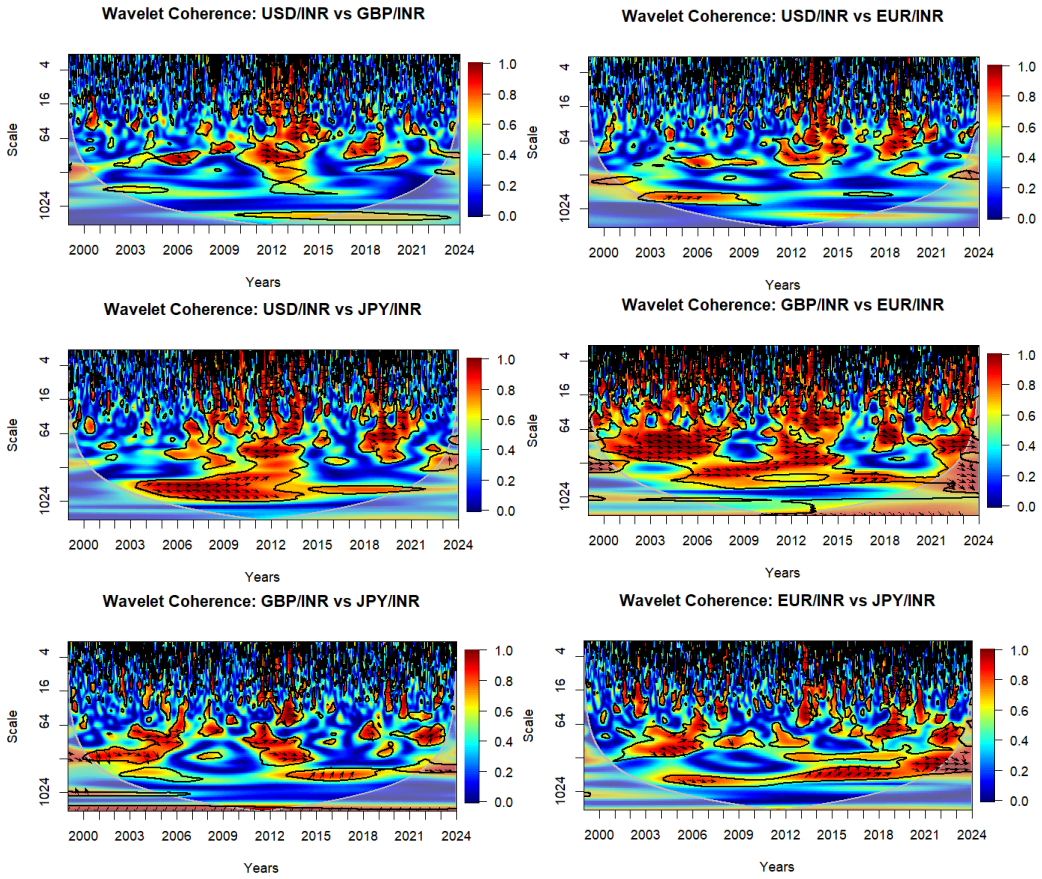
Turning to GBP/INR, long-term fluctuations become more significant during 2008-2015. The shorter and medium-term volatility, on the other hand, is a consistent feature across the entire time frame. The EUR/INR series demonstrates clear instability from 2000 to 2005, were volatility peaks at low and medium frequencies. Between 2008 and 2017, volatility becomes more widespread, influencing the entire frequency spectrum, indicating continuous market disturbances during this time. Lastly, the JPY/INR series reveals significant market fluctuations from 2007 to 2017, with high volatility across all

frequencies. Notable short-term fluctuations appear in 2019 and 2023, while the early period (2000-2002) experiences both short- and medium-term volatility.

3.5. Wavelet Coherence Approach

The wavelet coherence approach is applied to capture the causal relationship between the return series of selected currency pairs. Figure 6 present the results from the wavelet coherence and phase difference between each time series pair. This shows the co-movement of these four currency pairs in the time-frequency space. The x-axis represents time (in days), and the y-axis represents the frequency domain. The analysis considers five frequency bands, namely 1–4 days (short-term relationship), 4–16 days, 16–64 days, 64–256 days (medium-term relationship), and 256–1024 days (long-term relationship). The color gradient on the right is coherence, with dark blue indicating low power and dark red indicating high power (close to one). The arrows represent the phase relationship, show the direction of influence or lead-lag dynamics between two currencies.

Figure 6. *Wavelet coherence dynamic.*



Source: Developed by the author(s).

From 2008 to 2019, high coherence across all scales is observed for most pairs, likely due to global events like the 2008 financial crisis. Lions' share arrows point right or up-right, indicating positive relationships during those periods. The wavelet coherence shows up arrows in 2002 and 2004, suggesting that USD/INR positively influenced GBP/INR in the medium term. Between 2000-2005, the return series did not move consistently together, except for occasional coherence above 16 days, indicating a weak long-term association. USD/INR and EUR/INR had a strong positive relationship from 2009 to 2016 and again from 2019 to 2021, mainly across short and medium scales. A similar pattern is noted from 2004 to 2006, but only over the long term. However, overall long-term coherence is weak, except during 2004-2006. A strong connection between USD/INR and JPY/INR is observed from 2003 to 2014, with multi-scale coherence from 2010 to 2014. The return series of GBP/INR and EUR/INR show positive and stronger co-movement across all scales from the beginning to the end of the period. In contrast, GBP/INR and JPY/INR exhibit medium to long-term negative association during 2000-2005 and 2012-2014, but a positive association from 2015-2017 and 2021-2023. Similarly, EUR/INR and JPY/INR display negative association in the medium to long term between 2003-2008 and 2020-2024.

3.6. Wavelet-Based Decomposition Granger Causality Analysis

In this study, wavelet-based decomposed Granger Causality tests are used to verify the robustness of the Continuous Wavelet Transform (CWT) and Wavelet Coherence (WC) analysis results. The tests are applied to twelve currency return pairs, and the results are presented in Figure 7.

The results show a combination of bidirectional and unidirectional causality across the

Figure 7. *Granger causality test results*



Source: Developed by the author(s).

currency pairs, with some pairs moving independently. Only USD and EUR have bidirectional causality between each other, indicating that changes in both currency pairs can influence one another. There is no significant causality in either direction between USD-JPY and GBP-EUR, suggesting that these currency pairs move independently. USD-GBP, GBP-JPY, and JPY-EUR show unidirectional causality, implying that the past movements in USD, GBP, and JPY can predict GBP, JPY, and EUR, respectively, but not vice versa.

4. Summary and conclusions

In this study, an attempt has been made to analyse the co-movement of major currency pairs (USD, EUR, GBP, JPY) against the INR from 2000 to 2024 using wavelet power spectrum and coherence across short, medium, and long-term frequencies. The findings reveal dynamic changes in volatility over time and across scales. The USD experienced stable periods in the early 2000s, with sensitive volatility during 2009-2013 and 2019-2021. The GBP showed consistent medium-term volatility, while the EUR faced market instability from 2000-2005, with extensive volatility from 2008-2017. On the other hand, the JPY displayed significant fluctuations from 2007-2017, with notable short-term volatility in 2019 and 2023.

The results of the coherence analysis indicate strong relationships between several pairs, particularly during global events like the 2008 financial crisis. The USD showed positive coherence with the GBP in the medium term, while USD and EUR exhibited strong connections during 2009-2016 and 2019-2021. GBP and EUR maintained positive co-movements throughout the period, while GBP and JPY, as well as EUR and JPY, demonstrated negative associations at times. Bidirectional causality was found only between USD and EUR, while other pairs exhibited unidirectional or independent movements. Overall, the analysis emphasizes evolving co-movement dynamics influenced by both global and currency-specific factors.

Our results reveal dynamic co-movements among major currencies, offering key insights for portfolio managers and international investors with exposure to INR. These findings are also significant for Indian banks and import-export firms seeking to hedge against fluctuations in USD, EUR, GBP, and JPY.

Given India's large trade potential with major economies, the identified co-movement patterns suggest limited benefits from currency diversification over monthly, quarterly, and semi-annual horizons. However, international investors may still find opportunities for gains at shorter intervals, such as intraweek, weekly, and fortnightly periods, where lower correlations among currencies are observed. This highlights the need for a nuanced approach to currency diversification and risk management in the Indian market. There is significant potential for further research on exploring the relationships among currency returns using advanced techniques, such as artificial intelligence and machine learning models, which have been applied in market prediction.

In particular, deep learning strategies employing neural networks could be instrumental in measuring co-movements among the variables considered. Additionally, future studies could explore different currencies, especially where data scarcity has made this type of research limited.

References

Aloui, C., and Hkiri, B. (2014). Co-movements of GCC emerging stock markets: New evidence from wavelet coherence analysis. *Economic Modelling*, 36, 421-431. <https://doi.org/10.1016/j.econmod.2013.09.043>

Aloui, R., Hammoudeh, S., and Nguyen, D.K. (2013). A time-varying copula approach to oil and stock market dependence: The case of transition economies. *Energy Economics*, 39, 208-221. <https://doi.org/10.1016/j.eneco.2013.04.012>

Antonakakis, N., Chang, T., Cunado, J., and Gupta, R. (2017). The relationship between commodity markets and commodity mutual funds: A wavelet-based analysis. *Finance Research Letters*, 24(1), 1-9. <https://doi.org/10.1016/j.frl.2017.03.005>

Arouri, M.E.H., Hammoudeh, S., Lahiani, A., and Nguyen, D.K. (2013). On the short- and long-run efficiency of energy and precious metal markets. *Energy Economics*, 40, 832-844. <https://doi.org/10.1016/j.eneco.2013.10.004>

Balcilar, M., Demirer, R., and Hammoudeh, S. (2015). Global risk exposures and industry diversification with Shariah-compliant equity sectors. *Pacific-Basin Finance Journal*, 35(Part B), 499-520. <https://doi.org/10.1016/j.pacfin.2015.09.002>

Bloomfield, P. (2013). Fourier analysis of time series: An introduction (2nd ed.). *John Wiley & Sons*.

Bouri, E., Shahzad, S.J.H., Roubaud, D., Kristoufek, L., and Lucey, B. (2020). Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis. *The Quarterly Review of Economics and Finance*, 77, 156-164. <https://doi.org/10.1016/j.qref.2020.03.004>

Brooks, R., and Del Negro, M. (2004). The rise in comovement across national stock markets: Market integration or IT bubble? *Journal of Empirical Finance*, 11(5), 659-680. <https://doi.org/10.1016/j.jempfin.2003.08.001>

Caraiani, P. (2012). Stylized facts of business cycles in a transition economy in time and frequency. *Economic Modelling*, 29(6), 2163-2173. <https://doi.org/10.1016/j.econmod.2012.06.014>

Daubechies, I. (1990). The wavelet transform, time-frequency localization, and signal analysis. *IEEE Transactions on Information Theory*, 36(5), 961-1005. <https://doi.org/10.1109/18.57199>

Dibal, P.Y., Onwuka, E., Agajo, J., and Alenoghena, C. (2018). Application of wavelet transform in spectrum sensing for cognitive radio: A survey. *Physical Communication*, 28, 45-57. <https://doi.org/10.1016/j.phycom.2018.03.004>

Ftiti, Z., Guesmi, K., and Abid, I. (2016). Oil price and stock market co-movement: What can we

learn from time-scale approaches? *International Review of Financial Analysis*, 46, 266-280. <https://doi.org/10.1016/j.irfa.2015.08.011>

Grinsted, A., Moore, J., and Jevrejeva, S. (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics*, 11(5/6), 561-566. <https://doi.org/10.5194/npg-11-561-2004>

Kumar, S., Pathak, R., Tiwari, A.K., and Yoon, S.M. (2016). Are exchange rates interdependent? Evidence using wavelet analysis. *Applied Economics*, 49(33), 3231-3245. <https://doi.org/10.1080/00036846.2016.1257108>

Kuşkaya, S., Toguc, N., and Bilgili, F. (2022). Wavelet coherence analysis and exchange rate movements. *Quality & Quantity*, 56(3-4), 4675-4692. <https://doi.org/10.1007/s11135-022-01327-7>

Lin, W.L., Engle, R.F., and Ito, T. (1994). Do bulls and bears move across borders? International transmission of stock returns and volatility. *Review of Financial Studies*, 7(3), 507-538. <https://doi.org/10.1093/rfs/7.3.507>

Marín-Rodríguez, N.J., González-Ruiz, J.D., and Botero, S. (2023). A wavelet analysis of the dynamic connectedness among oil prices, green bonds, and CO_2 emissions. *Risks*, 11(1), 15. <https://doi.org/10.3390/risks11010015>

Mensi, W., Hammoudeh, S., Nguyen, D.K., and Kang, S.H. (2016). Global financial crisis and spillover effects among the U.S. and BRICS stock markets. *International Review of Economics & Financ*e, 42, 257-276. <https://doi.org/10.1016/j.iref.2015.11.005>

Morlet, J., Arens, G., Fourgeau, E., and Glard, D. (1982). Wave propagation and sampling theory-Part I: Complex signal and scattering in multilayered media. *Geophysics*, 47(2), 203-221. <https://doi.org/10.1190/1.1441328>

Nikkinen, J., Pynnönen, S., Ranta, M., and Vähämaa, S. (2011). Cross-dynamics of exchange rate expectations: A wavelet analysis. *International Journal of Finance & Economics*, 16(3), 205-217. <https://doi.org/10.1002/ijfe.v16.3>

Sahabuddin, M., Hassan, M.F., Tabash, M.I., Al-Omari, M.A., Alam, M.K., and Islam, F.T. (2022). Co-movement and causality dynamics linkages between conventional and Islamic stock indexes in Bangladesh: A wavelet analysis. *Cogent Business & Management*, 9(1). <https://doi.org/10.1080/23311975.2022.2034233>

Samarakoon, L.P. (2011). Stock market interdependence, contagion, and the U.S. financial crisis: The case of emerging and frontier markets. *Journal of International Financial Markets*, Institutions and Money, 21(5), 724-742. <https://doi.org/10.1016/j.intfin.2011.05.001>

Singh, S., Bansal, P., and Bhardwaj, N. (2022). Correlation between geopolitical risk, economic policy uncertainty, and Bitcoin using partial and multiple wavelet coherence in P5 + 1 nations. *Research in International Business and Finance*, 63, 101756. <https://doi.org/10.1016/j.ribaf.2022.101756>

Theodossiou, P., and Lee, U. (1993). Mean and volatility spillovers across major national stock markets: Further empirical evidence. *Journal of Financial Research*, 16(4), 337-350. <https://doi.org/10.1111/jfir.1993.16.issue-4>

Torrence, C., and Webster, P.J. (1999). Interdecadal changes in the ENSO-monsoon system. *Journal of Climate*, 12(8), 2679–2690. <https://doi.org/10.1175/1520-0442>

Yang, L., Cai, X.J., Zhang, H., and Hamori, S. (2016). Interdependence of foreign exchange markets: A wavelet coherence analysis. *Economic Modelling*, 55, 6-14. <https://doi.org/10.1016/j.econmod.2016.01.022>

Yang, Z., Ce, L., and Lian, L. (2017). Electricity price forecasting by a hybrid model, combining wavelet transform, ARMA, and kernel-based extreme learning machine methods. *Applied Energy*, 190, 291-305. <https://doi.org/10.1016/j.apenergy.2016.12.130>