

Mathematical modeling of nearshoring and reshoring in the reconfiguration of trade flows

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Abstract. *The paper develops an integrated mathematical framework for identifying and evaluating nearshoring and reshoring, two processes frequently associated in economic discourse, but analytically distinct. Nearshoring is treated as a redistribution of trade mass to partners located at shorter economic distances, and reshoring as an increase in the share of domestic supply in domestic absorption, conditional on the effective substitution of imports. Starting from a bilateral-sectoral-temporal tensor of flows, the analysis builds probability measures, proximity, concentration and entropy indicators, exact decompositions of the trade distance change, as well as a convex optimization problem of the supplier structure. Results are formulated and demonstrated regarding the monotony of trade transfer, the separation of geographical and sectoral effects, the necessary and sufficient conditions of reshoring and the uniqueness of the optimal solution. The framework is complemented by a PPML gravity specification, a dynamic adjustment model and a procedure for identifying structural breaks. The result is a quantitative architecture in which geographical proximity, production recovery, diversification and resilience are measured separately, but can be brought together in a formal decision rule.*

Keywords: nearshoring, reshoring, commercial distance, optimal transport, entropy, convex optimization, PPML, structural ruptures.

JEL Classification: F14, F15, F17, F23.

1. Introduction

The reconfiguration of international production chains has brought the decision of localization back to the center of economic analysis. In its simplified form, this decision is presented as a choice between production in an area with low unit costs and production in an area close to the final market. Such representation is insufficient, because the wage cost does not exhaust the total cost of supply, and the geographical distance does not fully capture time, uncertainty, infrastructure quality, geopolitical risk and operational flexibility. Moreover, the reduction of imports from a distant partner does not, in itself, demonstrate the existence of nearshoring: the flow can be replaced by an equally distant partner, it can disappear as a result of the contraction in demand or it can be absorbed by a sector with a different spatial structure. Similarly, the growth of domestic production is not automatically reshoring if it is export-oriented or if imports grow at a higher rate. The central problem is therefore one of mathematical identification: verifiable conditions must be established under which a change in the distribution of flows can be classified unambiguously (Anderson & Wincoop, 2003).

The objective of the paper is to build a model in which nearshoring and reshoring are defined by different mathematical objects. For nearshoring, bilateral and sectoral distribution of trade is necessary, as the phenomenon concerns the shift of the center of gravity of supply. For reshoring, the identity of domestic absorption is required, as the phenomenon concerns the ratio between net domestic supply and imports. The model does not aim to reduce the decision to a single index from the start; on the contrary, it separates volume, composition, proximity, concentration and risk, then shows under what conditions these dimensions can be aggregated (Tate, et al., 2014). This order is essential: a composite index constructed prior to identification may hide the fact that geographical proximity is achieved by excessive concentration or that the apparent internal return results only from decreased absorption.

The contribution is organised into five components.

The first normalizes trade flows in the form of a measure of probability and defines nearshoring by changing geographical moments.

The second demonstrates the effect of an elementary transfer of trade mass and provides an exact breakdown between the geographical reallocation within sectors and the change in sectoral shares.

The third defines reshoring by the logic of the domestic quota and derives a necessary and sufficient condition based on the difference between the growth rates of net domestic supply and imports.

The fourth formulates the choice of suppliers as a convex problem with cost, distance, risk and diversification.

The fifth transforms the theoretical construction into an estimable system through PPML, event study, and structural ruptures. Thus, mathematical analysis is not a succession of decorative formulas, but the mechanism by which concepts are defined, linked and tested.

2. Literature

The modern literature on the relocation of production starts from the revision of the classic decision of offshoring and from the extension of the notion of relevant cost. The choice of location cannot be based solely on the salary differential, as transport, safety stocks, variability of delivery times, coordination of activities, quality control and loss of flexibility can reverse the result of a simple accounting comparison. In this perspective, reshoring represents the return of a productive activity that had previously been transferred externally, and the relevant unit of analysis is not always the firm as a whole, but the product, operation, technological stage or economic function transferred. This delimitation separates the return of production from the mere expansion of domestic capacity. Consequently, a rigorous model must trace the origin of the flow, its destination, the nature of the activity and the economic function relocated, not just the aggregate variation in production (Bai & 3 Perron, 2003; Silva & Tenreiro, 2006).

Empirical studies have highlighted a set of factors that make the decision to relocate fundamentally multidimensional. Competitive pressures can favour the approximation of production to the consumer, design centres and innovation functions, especially when demand is volatile and product life cycles are short. In industries characterized by small series, customization and frequent changes in orders, the return of some activities can be explained by the need to reduce reaction time, not just by the convergence of salary costs (Martínez-Mora & Merino, 2014). However, backshoring remains selective, being more likely for products with high demands for quality, flexibility, automation and technology integration. Also, offshoring and backshoring are not perfectly symmetrical states, as the turnaround occurs after the accumulation of contractual costs, organizational routines, technological dependencies and supplier relationships that did not exist at the time of the initial move.

A second direction of the literature analyzes the organizational and institutional factors of relocation (McIvor & Bals, 2021). Determinants are typically grouped into costs, quality, time, risk, access to skills, proximity to customers, protection of knowledge and stability of the institutional framework. Reshoring can function as a mechanism to reduce risk and reconsider international engagement, without the firm abandoning internationalization; Rather, functions are redistributed between locations, depending on the strategic importance and vulnerability of each activity. A complete explanation must combine transaction costs, internal resources and capabilities, localization advantages, and production process characteristics. It follows that the optimal location depends simultaneously on the country, company, product and technological stage. From this perspective, reshoring should be modeled as a process with antecedents, contingent conditions, implementation, and outcomes, not as an isolated binary variable (Wiesmann, et al., 2017; Kinkel, 2014).

After recent global shocks, resilience has become a distinct motivation, but the literature warns against its automatic equivalence with localization. International diversification can

reduce exposure to a country-specific shock, while regional concentration can reduce transport time but amplify common risk. Therefore, proximity and resilience should be introduced separately in the objective function. This distinction is compatible with portfolio theory: the total risk depends not only on the risk of each provider, but also on the covariances between locations. A geographically close portfolio may have low logistical cost and high covariance of outages, and a dispersed one may have high cost and low covariance. It follows that nearshoring is not a universally dominant solution, but a movement on a border between cost, speed and risk (Di Mauro, et al., 2018).

Methodologically, the analysis of trade flows is based on the structural gravitational model, in which bilateral flows depend on the relative trade costs and the position of each economy vis-à-vis all partners. This structure justifies the use of large-scale fixed effects, capable of absorbing the unnoticed characteristics specific to the exporter, importer, sector and period. Flow logarithm and linear estimation can generate inconsistent coefficients in the presence of heteroscedasticity and eliminate zero-value observations; PPML estimation retains the multiplicative form, allows for the inclusion of null flows, and is suitable for dense commercial panels. Regime changes can be identified by multiple structural rupture procedures based on the overall minimization of the sum of square residues. In parallel, optimal transport theory provides a natural language for measuring the minimum cost needed to transform one geographic distribution into another, and entropic regularization makes the calculation applicable to large commercial networks (Correia, et al., 2020).

The main gap results from the incomplete separation of the phenomena. The managerial literature explains the reasons for relocation, and the econometric literature estimates flows, but less often is a system formulated in which nearshoring, reshoring, concentration and resilience are defined simultaneously and can produce divergent conclusions. The proposed framework addresses this gap with four requirements: the invariant of indicators to uniform scaling of trade; separation of geographical change from sectoral change; identification of reshoring by absorption, not by gross production; and the introduction of risk through a covariance matrix, not through an arbitrary penalty. These requirements transform analysis from a descriptive classification into a formal measurement and optimization problem (Ciabuschi, et al., 2019).

3. Mathematical analysis

3.1. Flow space and probabilistic normalisation

It is considered a home economy, partners, sectors and periods. The bilateral-sectoral trade flow is f_{ij} , and the total trade of the economy is T . For i , the elementary weight satisfies $w_i = f_{ij} / T$, so that the set of flows can be treated as a probability distribution on the Cartesian product $\mathcal{S} \times \mathcal{P}$. This normalization eliminates the effect of scale: if all flows are multiplied by the same constant c , then w_i remains the same. It follows that the indicators built from weights measure the structure, not

the level of trade. $i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}$ $X_{ijkt} \geq 0, X_{it} = \sum_{j,k} X_{ijkt}, X_{it} > 0, s_{ijkt} = X_{ijkt} / X_{it}, s_{ijkt} \in [0,1], \sum_{j,k} s_{ijkt} = 1, \mathcal{J} \times \mathcal{K} a > 0, s'_{ijkt} = a X_{ijkt} / (a X_{it}) = s_{ijkt}$

The share of the sector is s_{ijkt} , and the conditional share of the partner in the sector is p_{ijkt} . The relationship allows the separation of the sectoral composition from the geographical distribution within each sector. Be the economic distance between i and j , which can combine mileage, transportation time, logistics cost, and institutional risk. In order to limit the influence of extreme values, cost is used, and the average commercial distance becomes $q_{ikt} = \sum_j s_{ijkt} p_{ijkt} = s_{ijkt} / q_{ikt}, q_{ikt} > 0, s_{ijkt} = q_{ikt} p_{ijkt} d_{ij} > 0, j c_{ij} = \ln d_{ij}$

$$D_{it} = \sum_{j,k} s_{ijkt} c_{ij} = \sum_k q_{ikt} \mu_{ikt}, \quad \mu_{ikt} = \sum_j p_{ijkt} c_{ij}.$$

The first equality defines the aggregate geographic momentum, and the second shows that this is an average of sectoral distances. It follows from this identity that a decrease in it may come either from the proximity of the partners within the sectors or from the increase in the share of sectors that were already oriented towards proximity. Only the first mechanism represents actual geographical reallocation. $\mu_{ikt} D_{it}$

3.2. Definition of nearshoring and demonstration of monotony

Weak nearshoring between periods t and $t-1$ by δ and strictly nearshoring by δ , provided that the set of products analysed and the distance function are comparable. To demonstrate the meaning of this definition, it is considered an elementary mass transfer from the distant partner to the close partner, where $\delta > 0$. The weights also become $s'_{iakt} = s_{iakt} + \delta s'_{ibkt} = s_{ibkt} - \delta, 0 < \delta \leq s_{ibkt}$

$$D'_{it} - D_{it} = \delta c_{ia} - \delta c_{ib} = \delta (c_{ia} - c_{ib}) < 0.$$

H 1. Any positive transfer of trade mass from a partner with a higher distance cost to a lower cost partner strictly reduces the average trade distance, if the total volume and sector structure are kept constant. The proof is contained in the preceding relation: the sign of variation is the product of δ and the negative difference $c_{ia} - c_{ib}$. The result is linear for an elementary transfer and extends to a transfer matrix T , where it represents the mass moved from i to j . The overall variation is $\Delta D_i = \sum_{a,b} \tau_{ab} (c_{ia} - c_{ib}) c_{ia} < c_{ib}$; Therefore, a sufficient condition for nearshoring is that all active transfers satisfy $\tau_{ab} (c_{ia} - c_{ib}) c_{ia} < c_{ib}$. The condition is not necessary, as transfers to more distant partners can be offset by larger transfers to close partners. $\delta > 0, c_{ia} - c_{ib} T = (\tau_{ab}) \tau_{ab} \geq 0, \Delta D_i = \sum_{a,b} \tau_{ab} (c_{ia} - c_{ib}) c_{ia} < c_{ib}$

For comparisons between countries, the normalized proximity index is defined. If $c_i = \min_j c_{ij}$ and $c_i^{\max} = \max_j c_{ij}$, then $c_i^{\min} = \min_j c_{ij}, c_i^{\max} = \max_j c_{ij}$

$$N_{it} = \frac{c_i^{\max} - D_{it}}{c_i^{\max} - c_i^{\min}}, \quad 0 \leq N_{it} \leq 1.$$

The index is equal to one when the entire table is placed at the minimum distance and zero when it is placed at the maximum distance. Its derivative of a transference from is , which confirms the monotony. The advantage of normalization is comparability, and its limit is dependence on support: adding a very distant partner modifies even if that partner is not used. For this reason, robust analysis can use fixed percentiles of distance or a common support for all periods. $\partial N_{it} / \partial \delta = (c_{ib} - c_{ia}) / (c_i^{\max} - c_i^{\min}) > 0$

The average moment does not fully capture the cast. Two structures can have the same , but one can concentrate the flow at intermediate distances and the other at extremes. Enter the geographic variance and tail risk, where the weighted quantile is. Robust nearshoring requires not only , but also the avoidance of excessive growth . An economy can reduce the average by moving a large mass to the neighborhood, but it can maintain a critical dependence on a very distant supplier; The queue indicator detects this situation. $D_{it} V_{it} = \sum_{j,k} s_{ijkt} (c_{ij} - D_{it})^2 C_{it}(\alpha) = \sum_{j,k} s_{ijkt} c_{ij} \mathbf{1}(c_{ij} \geq Q_{it}(\alpha)) / (1 - \alpha) Q_{it}(\alpha) \Delta D_{it} < 0$

3.3. Exact breakdown of geographical change

For two periods, and , denote , and . Using the symmetric averages and , the exact identity is obtained $01 \Delta D_i = D_{i1} - D_{i0} \Delta q_{ik} = q_{ik1} - q_{ik0} \Delta \mu_{ik} = \mu_{ik1} - \mu_{ik0} \bar{q}_{ik} = (q_{ik1} + q_{ik0}) / 2 \bar{\mu}_{ik} = (\mu_{ik1} + \mu_{ik0}) / 2$

$$\Delta D_i = \sum_k \bar{q}_{ik} \Delta \mu_{ik} + \sum_k \bar{\mu}_{ik} \Delta q_{ik}.$$

The first term is the internal geographical effect of the sectors, and the second is the composition effect. The proof results from the algebraic identity applied to each sector. The development of the right side produces ; the cross-terms are canceled, remaining . By summing after the aggregate relation is obtained. $q_1 \mu_1 - q_0 \mu_0 = \bar{q}(\mu_1 - \mu_0) + \bar{\mu}(q_1 - q_0) \frac{1}{2}(q_1 \mu_1 - q_1 \mu_0 + q_0 \mu_1 - q_0 \mu_0) + \frac{1}{2}(q_1 \mu_1 - q_0 \mu_1 + q_1 \mu_0 - q_0 \mu_0) q_1 \mu_1 - q_0 \mu_0 k$

H 2. There is pure geographical nearshoring if, even if due to a positive sectoral effect. This sentence is important because it prevents the false rejection of nearshoring when the economy increases its share in a structural sector dependent on long distances. Conversely, it does not demonstrate geographical nearshoring if the first term is null or positive, and the decrease comes exclusively from the transfer of shares to sectors that are already close. In the application, the two components must be reported separately and, for comparability, can be divided by , obtaining relative contributions. $\sum_k \bar{q}_{ik} \Delta \mu_{ik} < 0 \Delta D_i \geq 0 \Delta D_i < 0 |D_{i0}|$

An additional breakdown separates the extensive margin from the intensive one. The indicator of the active link and the number of links are defined. The change in the total flow

can be written as the sum of the flows, appeared, disappeared, and persisted. For the set of persistent links, inputs, and outputs, the variation is . The same logic applies to distance-weighted mass. Thus, nearshoring can be produced by the intensification of existing close connections, the appearance of close connections or the disappearance of distant ones; The mechanisms have different implications for the productive capacity and stability of the network.

$$A_{ijkt} = \mathbf{1}(X_{ijkt} > 0)L_{it} = \sum_{j,k} A_{ijkt} \mathcal{P} \mathcal{E} \mathcal{X} \Delta X_{it} = \sum_{(j,k) \in \mathcal{P}} (X_{ijk1} - X_{ijk0}) + \sum_{(j,k) \in \mathcal{E}} X_{ijk1} - \sum_{(j,k) \in \mathcal{X}} X_{ijk0}$$

To measure the cost of complete distribution conversion, optimal transport is introduced. Let alone the geographical distributions over two periods, and the cost of moving a unit of mass from the location to . The set of feasible couplings is , and the discrete Wasserstein distance is $p^0 = (p_j^0)p^1 = (p_j^1)g_{ab}ba\Pi(p^0, p^1) = \{\Gamma \geq 0: \Gamma \mathbf{1} = p^1, \Gamma^\top \mathbf{1} = p^0\}$

$$W_g(p^0, p^1) = \min_{\Gamma \in \Pi(p^0, p^1)} \sum_{a,b} g_{ab} \Gamma_{ab}.$$

The value measures the intensity of the reconfiguration, but not the direction. The direction is given by . Therefore, a strong nearshoring type reconfiguration has simultaneously high and , while stability has small regardless of the sign of a minor variation. Entropic regularization adds term, ensuring a smooth solution and fast calculation by Sinkhorn scalings. $W_g D_{i1} - D_{i0} W_g \Delta D_i < 0 W_g \varepsilon \sum_{a,b} \Gamma_{ab} (\ln \Gamma_{ab} - 1)$

3.4. Formal definition of reshoring

Reshoring cannot be identified from the bilateral matrix alone, as it requires information about domestic production. For the sector, be it production, exports and imports. The net domestic supply destined for the national market is , and the internal absorption is . The internal supply share is defined by , with when the components are non-negative. Sector reshoring is identified by , but the logit transformation offers a stronger interpretation: $kQ_{ikt}E_{ikt}M_{ikt}S_{ikt}^D = Q_{ikt} - E_{ikt}A_{ikt} = S_{ikt}^D + M_{ikt}R_{ikt} = \frac{S_{ikt}^D}{A_{ikt0}} \leq R_{ikt} \leq 1 \Delta R_{ikt} > 0$

$$\text{logit}(R_{ikt}) = \ln \frac{R_{ikt}}{1 - R_{ikt}} = \ln S_{ikt}^D - \ln M_{ikt}.$$

Temporal differentiation results . $\Delta \text{logit}(R_{ikt}) = \Delta \ln S_{ikt}^D - \Delta \ln M_{ikt}$ **H 3.** The domestic share increases if and only if the net domestic supply grows proportionally faster than imports. The proof uses the strict monotony of the logit function on the interval : is equivalent to , and the last inequality is equivalent to . The condition also covers the situation in which both components decrease: there may be relative reshoring if imports contract faster than domestic supply, but this situation must be differentiated from

expansionary reshoring, further defined by $(0,1)\Delta R_{ikt} > 0 \Delta \logit(R_{ikt}) > 0 \Delta \ln S_{ikt}^D > \Delta \ln M_{ikt} \Delta S_{ikt}^D > 0$

Quota derivatives clarify the mechanism. For , we also get . The elasticities are also . The effect of a percentage increase in domestic supply is greater when the initial domestic share is small, as there is a large import base that can be substituted. For sector aggregation, the total share can be written , where . The aggregate variation is decomposed exactly into intra-sectoral effect and compositional effect, analogous to geographical decomposition: . Thus, the increase in the total share may come from reshoring in sectors or from an increase in the share of sectors that already had high domestic shares. $R = S/(S + M) \partial R / \partial S = M/(S + M)^2 > 0 \partial R / \partial M = -S/(S + M)^2 < 0 \partial \ln R / \partial \ln S = 1 - R \partial \ln R / \partial \ln M = -(1 - R)R_{it} = \sum_k S_{ikt}^D / \sum_k A_{ikt} R_{it} = \sum_k \omega_{ikt} R_{ikt} \omega_{ikt} = A_{ikt} / \sum_h A_{iht} \Delta R_i = \sum_k \bar{\omega}_{ik} \Delta R_{ik} + \sum_k \bar{R}_{ik} \Delta \omega_{ik}$

The separation between nearshoring and reshoring can be represented in a plane with the axes and . The dial indicates simultaneous external approach and internal return; indicates regionalisation without internal substitution; indicates an internal recovery without the approximation of residual imports; and indicates the expansion of external dependence. This representation prevents the terms from being used synonymously and allows for hybrid strategies, e.g. increasing domestic production for critical components and nearshoring for standardized components. $\Delta N_{it} \Delta R_{it} (+, +)(+, 0/-)(0/-, +)(-, -)$

3.5. Concentration, entropy and resilience

Closeness can be increased by moving flows to a single partner, which reduces distance but increases dependency. The geographical concentration is , where ; It satisfies when all partners are active. The Shannon entropy is , with , and the normalized diversification is . The relationship between the indicators is not linear, but both describe dispersion: it strongly penalizes large weights, while entropy is more sensitive to the entire support. $H_{it} = \sum_j u_{ijt}^2 u_{ijt} = \sum_k s_{ijkt} 1/J \leq H_{it} \leq 1 E_{it} = - \sum_j u_{ijt} \ln u_{ijt} 0 \leq E_{it} \leq \ln J E_{it}^* = E_{it} / \ln J H$

To measure structural change independent of distance, the Jensen–Shannon divergence is used. For distributions and , with , this is , where . The divergence is symmetrical, finite, and contained between and . A large value shows reconfiguration, but does not indicate approach; Combined with his sign, it separates the intense shift toward proximity from the mere volatility of partners.

$$\begin{aligned} u^0 u^1 m &= (u^0 + u^1) / 2 JS(u^0, u^1) = \frac{1}{2} KL(u^0 \parallel m) + \frac{1}{2} KL(u^1 \parallel m) KL(p \parallel q) \\ &= \sum_j p_j \ln(p_j / q_j) 0 \ln 2 \Delta D \end{aligned}$$

The risk of disruption is modelled by a semi-defined positive matrix, the elements of which represent the covariances of supplier shocks. For the vector of geographical weights , the

risk is . If the shocks are independent, the matrix is diagonal and diversification reduces risk by distributing the weights. If nearby suppliers are exposed to the same energy, climate or geopolitical shocks, non-diagonal elements may be large and regional concentration may increase. This provides an important mathematical conclusion: distance minimization and risk minimization coincide only under restrictive conditions regarding its structure .
 $\Sigma u \mathcal{R}(u) = u^\top \Sigma u \mathcal{R} \Sigma$

3.6. Optimisation of the supplier structure

The decision of the firm or the economy can be formulated as the choice of the vector on the simplex . Let it be the vector of total unit costs, the vector of economic distances, the risk matrix and the initial structure. Minimizes function $u = (u_1, \dots, u_J)^\top \Delta_J = \{u \geq 0: \mathbf{1}^\top u = 1\}$ $cd \Sigma u^0$

$$\min_{u \in \Delta_J} F(u) = c^\top u + \lambda d^\top u + \frac{\rho}{2} u^\top \Sigma u + \frac{\kappa}{2} \|u - u^0\|_2^2 - \eta \sum_j \ln(u_j + \epsilon),$$

where there is distance aversion, risk aversion, the cost of adjustment, and the last term favors diversification. The parameter prevents the singularity. The linear component would lead, on its own, to the allocation of the entire mass to the supplier with the lowest adjusted cost. The quadratic term of risk penalizes concentration in correlated locations, the cost of adjustment prevents unrealistic jumps, and the logarithmic barrier maintains an internal portfolio. $\lambda \rho \kappa \epsilon > 0$ $c^\top u + \lambda d^\top u$

Theorem 1. If, and , the function is strictly convex on the inside of the simplex, and the problem has a unique solution. The existence results from the continuity of function on a compact set, using the barrier extension to exclude the boundary when . Hessian is , which is positively defined because ; therefore, it is strictly convex and the minimizer is unique. $\kappa > 0, \rho \geq 0, \Sigma \succcurlyeq 0, \eta \geq 0, F\eta > 0, \nabla^2 F(u) = \rho \Sigma + \kappa I + \eta \text{diag}((u_j + \epsilon)^{-2})$ $\kappa I \succ 0$ F

The Karush–Kuhn–Tucker conditions make the connection between the parameters and the optimal weights. The Lagrangian is , with . The condition of stationarity for each is $\mathcal{L} = F(u) + \nu(\mathbf{1}^\top u - 1) - \zeta^\top u \zeta \geq 0$ j

$$c_j + \lambda d_j + \rho(\Sigma u)_j + \kappa(u_j - u_j^0) - \frac{\eta}{u_j + \epsilon} + \nu - \zeta_j = 0,$$

together with , , and . For an active supplier, , so the adjusted marginal cost must be equalized between locations. An increase of it penalizes distant suppliers; However, the exact effect on it depends on and on the cost of the adjustment. Differentiating the first-order system inside we get , where . After projecting onto the hyperplane , it results with . The formula shows that the reaction does not depend only on the order of distances, but also on the curvature induced by risk, diversification and inertia. $u_j \geq 0, \zeta_j \geq 0, \zeta_j u_j = 0$ $\sum_j$

$$u_j = 1\zeta_j = 0\lambda u\Sigma H du/d\lambda + d + \mathbf{1} dv/d\lambda = 0H = \nabla^2 F(u)\mathbf{1}^\top u = 1du/d\lambda = -P_H dP_H = H^{-1} - H^{-1}\mathbf{1}(\mathbf{1}^\top H^{-1}\mathbf{1})^{-1}\mathbf{1}^\top H^{-1}$$

In the dynamic version, the structure evolves according to , with and . The value function satisfies the Bellman equation, where it contains costs and risks. The quadratic cost explains the gradual adjustment: even if the new static optimum suddenly moves towards proximity, the dynamic solution may involve partial nearshoring over several periods. This feature is compatible with the rigidities of suppliers' capacity, contracts and certification. $u_{t+1} = u_t + a_t \mathbf{1}^\top a_t = 0u_t + a_t \geq 0V(u_t, z_t) = \min_{a_t} \{f(u_t + a_t, z_t) + \frac{\kappa}{2} \|a_t\|^2 + \beta \mathbb{E}[V(u_t + a_t, z_{t+1}) | z_t]\} z_t$

3.7. Econometric model and temporal identification

For bilateral flows, the conditional average is specified

$$\begin{aligned} \mu_{ijkt} &= \mathbb{E}(X_{ijkt} | \mathcal{F}_{ijkt}) \\ &= \exp(\alpha_{it} + \gamma_{jt} + \delta_{kt} + \phi_{ij} + \beta_1 \ln d_{ij} + \beta_2 \text{Near}_{ij} \times \text{Post}_t \\ &\quad + \beta_3 \text{Adv}_{ikt} \times \text{Near}_{ij} \times \text{Post}_t). \end{aligned}$$

The fixed export-year, importer-year and sector-year effects absorb supply, demand and technology shocks, and the pair effect controls the invariable bilateral characteristics. The variable identifies partners below a distance threshold or in the same region, marks the reconfiguration regime, and can represent the comparative advantage. The coefficient shows a relative increase in near-term flows after the change, and tests whether it is stronger in sectors where the economy has a competitive capacity. $\text{Near}_{ij} \text{Post}_t \text{Adv}_{ikt} \beta_2 > 0 \beta_3$

Poisson's pseudo-log-likelihood is , where it indexes observations. The score is , and Hessian is negatively semi-defined. The estimator satisfies the momentary condition. For a continuous variable, , and for a binary variable the exact percentage effect is . The use of PPML does not assume equality between mean and variance for the consistency of the coefficients; It is sufficient to correctly specify the conditional mean, and the inference is made with a sandwich matrix and adequate clustering. $\ell(\theta) = \sum_n [X_n \ln \mu_n - \mu_n - \ln(X_n!)]$ $nS(\theta) = \sum_n x_n (X_n - \mu_n) H(\theta) = -\sum_n \mu_n x_n x_n^\top \sum_n x_n (X_n - \hat{\mu}_n) = 0$ $\partial \ln \mu / \partial x_r = \beta_r 100(e^{\beta_r} - 1)$

To avoid arbitrary choice of period, the model can be extended to an event study. If it is time for change for the economy, it is estimated. Pre-event coefficients test pre-trends, and subsequent coefficients describe the speed and persistence of reallocation. Empirical nearshoring is sustained when the pre-event coefficients are close to zero, and the post-event coefficients become positive and persistent for close partners, at the same time as the indicator decreases. $\text{Post} T_i i \sum_{r \neq -1} \beta_r \text{Near}_{ij} \mathbf{1}(t - T_i = r) D_{it}$

Structural breaks are estimated for the indicator series or for the distance elasticity. For breaks to the data, the model is on the segment. The estimator minimizes , below a

minimum segment length. Dynamic programming identifies the global minimum, and the number of breaks can be chosen through sequential tests and informational criteria. A negative break in its level or an increase in the coefficient for close partners provides evidence of regime change, but the interpretation requires simultaneous verification of the sectoral composition and the internal share.

$$mT_1 < \dots < T_m y_t = x_t^\top \beta_r + \varepsilon_t T_{r-1} < t \leq T_r SSR(T_1, \dots, T_m) \\ = \sum_{r=1}^{m+1} \min_{\beta_r} \sum_{t=T_{r-1}+1}^{T_r} (y_t - x_t^\top \beta_r)^2 D_t$$

3.8. Integrated classification rule

Based on previous constructions, a period is classified as robust nearshoring if three conditions are met: the pure geographical effect is negative, ; the change is not explained solely by the disappearance of trade, which requires extensive and intensive margins to be checked; and the queue risk does not increase above an accepted threshold. Expansive reshoring also imposes . Diversification is measured separately by and $.G_i = \sum_k \bar{q}_{ik} \Delta \mu_{ik} < 0 \Delta \logit(R_{ik}) > 0 \Delta S_{ik}^D > 0 \Delta H \Delta E^*$

A synthetic score can be defined only after standardization of the components: , with non-negative weights that add up to one. However, the score does not replace the identification conditions. Two economies can have the same, one through reshoring and the other through nearshoring; Component reporting is mandatory. In a policy issue, weights can be determined by minimizing a losing function or by analyzing the Pareto boundary, not by arbitrary choice. $Z_{it} = w_N z(\Delta N_{it}) + w_R z(\Delta R_{it}) - w_H z(\Delta H_{it}) - w_C z(\Delta C_{it}) Z$

4. Conclusions

The model shows that nearshoring and reshoring are different responses to the same general localization problem. Nearshoring changes the distribution of imports, while reshoring changes the ratio between domestic production destined for the domestic market and imports. The two may be complementary, but there is no identity between them. A firm can shift supply from Asia to Central Europe without increasing production in the country of origin; This is regionalization, not reshoring. Conversely, an increase in domestic production can substitute imports from a nearby country, producing reshoring without a relevant decrease in the average distance of the remaining imports.

Sectoral breakdown is key to interpretation. An aggregate decrease in distance can be generated by the decline of a globalised sector and the growth of a regional sector, even if no sector changes its partners. In the absence of decomposition, that amendment would be erroneously classified as nearshoring. Symmetrically, the rapprochement of partners from several sectors can be masked by the increase in the share of a sector that requires distant

raw materials. The proposed framework avoids both errors and allows variation to be attributed to a geographical and a structural component.

The optimization problem highlights the fact that proximity should not be unconditionally maximized. If nearby suppliers have high costs, limited capacity or strongly correlated shocks, a fully regionalized solution may be dominated by a mixed portfolio. The parameters λ , ρ , and κ provide a direct interpretation of the trade-off: increasing distance aversion moves mass towards proximity, increasing risk aversion favors locations with low covariance, the cost of adjustment produces gradualism, and the preference for diversification limits dependence on a single supplier. In practice, these parameters can be calibrated by the costs of delays, probabilities of interruption and contractual costs. $\lambda\rho\kappa\eta$

Econometric results must be interpreted together with structural indicators. A positive interaction coefficient shows a relative increase in close flows, but does not prove that the average distance decreased if flows to distant partners increased further in other sectors. Similarly, a break in the aggregate series may reflect a price shock, not the relocation of quantities. Therefore, the empirical application must use deflated trade values or quantities, fixed sector-year effects, composition indicators, and robustness analyses at the distance function. $Near \times Post$

The framework also has limits. The economic distance is difficult to observe in full, and the kilometers are only an approximation. Commercial data identifies the place of shipment, not always the origin of the added value. Domestic production and exports can be measured in different classifications, which affects the reshoring share. The risk matrix requires series of interruptions or credible proxies. In addition, the causal relationship between shocks and relocation can be affected by anticipation and selection. These limits do not cancel the model, but establish the conditions under which the results must be interpreted as structural evidence, not as mere correlations. Σ

The paper formulated a unitary mathematical system for the analysis of nearshoring and reshoring. Nearshoring has been defined as a reduction in the geographical timing of commercial distribution and its monotony towards mass transfers to closer partners has been demonstrated. Symmetric decomposition separated the internal geographic effect of sectors from the effect of changing sectoral composition, and extensive and intensive margins identified the mechanism by which reallocation occurs. Reshoring was defined by the share of domestic supply in domestic absorption and logit transformation produced the necessary and sufficient condition that net domestic supply must grow proportionately faster than imports.

The integration of concentration, entropy and risk matrix has demonstrated that closeness is not equivalent to resilience. The convex problem of the supplier structure formalized the compromise between cost, distance, risk, diversification and inertia, and the KKT conditions offered the marginal interpretation of the optimal weights. The PPML specification, the event study and the structural breaks transform the model into an

empirical protocol applicable to bilateral-sectoral flows. The central conclusion is that a reconfiguration can be called nearshoring or reshoring only after separating the volume, composition, geography and origin of the supply. This mathematical discipline allows the comparison of countries and sectors without confusing regionalization, the return of production, diversification and trade contraction.

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