Models of Macroeconomic Systems using Genetic Algorithms

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Abstract. *AG represents only one, it's true that it is very fashionable today, of the new instruments which began to be used lately in the macroeconomic modelling. Here, it may also be mentioned the classification systems, the genetic programming, the models based on agents, the evolutionist games etc. The characteristics of all these methods consist in the effort to detect the internal dynamic processes of the modelled macroeconomic systems and not only the answer of these to external chocks and perturbations. Among these internal dynamic processes maybe the most interesting is that one of continuous adapting of the modelled macroeconomic system to the environment.*

In the macroeconomic environment are included different systems made of subsystems and heterogene agents and the decisions adopted by these affect both the systems as such and other subsystems and processes from the environment. This general independency, neglected several times in the macroeconomic modelling due to the lack of instruments and methods able to detect it, may be however approached with enough rigour resorting to new modelling methods as the one described in this chapter.

In order to illustrate such a tendency, we shall present further on some of the most recent macroeconomic models which have been reformulated and studied using AG.

Key words: genetic algorithm; fitness function; decision rule; string; multiple equilibrium points.

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1. Cobweb model (Arifovic, 1994)

One of the first models elaborated in this sense was *the cobweb model,* very famous and studied in the economic dynamics. Arifovic uses this model in order to study the adapting process of the decisions of production of the firms to the demand and offer on the market.

Due to the fact that the model, in its classical form, was studied for a long time, it appears the immediate advantage of comparing the results obtained, by applying AG, with the ones resulted previously.

Within the model, there are considered *n* active firms on a competitive market, dealing with a single perishable good (non-stackable). Due to the delay in production, the quantities from the respective good offered by firms on the market depend on the expected level of the price.

The cost of the production intended for sale within the firm *i* at the moment *t* is given by the relation:

$$C_{i,t} = x \times q_{i,t} + \frac{1}{2} y \times q_{i,t}^2, \ x > 0, \ y > 0, \qquad (1)$$

where:

$q_{i,t}$ is the quantity produced for sale at the moment *t*, whereas *x* and *y* are parameters.

The profit of the firm i at the la moment t is then:

$$\Pi_{i,t} = p_t \times q_{i,t} - C_{i,t} \quad (2)$$

where:

p_t represents the price of the respective good at the moment t.

The optimum quantity which may be produced by the firm I at the moment t is obtained of the condition of first order:

$$\frac{\partial \Pi_{i,t}}{\partial q_{i,t}} = 0$$

of which we have:

$$q_{i,t} = \frac{1}{y}\left(p^e_{i,t} - x\right)$$

where:

$p^e_{i,t}$ represents the price expected by the firm I at the moment t.

The product demand on the respective market is given by an inverse function of the demand:

$$p_t = A - B\sum_{i=1}^{n} q_{i,t} \quad (5)$$

where:

A and B are given positive parameters.

When the market arrives in balance, therefore $p^e_{i,t} = p^e_t = p_t$, the demanded and offered quantity on the market, $q_{i,t} = q^*$ and the price $p_t = p^*$ are in this case constant.

If the firms have perfect (naïve) expectations concerning the price (therefore the price of the next period is expected to be equal with the one of the current period), the model has a solution that converges towards a stable and unique price and quantity, then $B/y < 1$ and diverges when $B/y > 1$.

For the AG application to this model, it was considered that the decisions rules of the firms are represented through binary strings. A firm i, $i = 1, n$ takes a decision concerning its production at the moment t using a binary string of finished length l, written in the alphabet {0,1}. A binary string is first of all decoded and transformed in a whole positive number and then nominalised to obtain a quantity $q_{i,t}$ ε $[0, q_{max}]$, where q_{max} is the maximum quantity that a firm may produce. The quantity $q_{i,t}$ represents the production decision of the firm i at the moment t.

When the quantities $q_{i,t}$ that the firms shall produce are determined, it may also be determined the clearing price of the market p_t using the relation (5). This price is used then to determine the profit of the firms at the moment t, using the relation (2). A certain value of the i firm's profit represents the fitness $\mu_{i,t}$ of the firm i following the application of the respective decision.

The population of decision rules is then updated in order to create a new population that shall be used at the moment t+1. There are two varieties of this model. The first one uses only the selection, crossing and mutation operations. The second variety uses only a new genetic operation, besides those three mentioned previously and namely the selection operation.

This operation is testing new individuals (chromosomes) before they become members of the new population. It is being calculated a potential fitness beginning with the price of the last period for each individual. Then two parents and two individuals newly appeared are ordered according to the calculated value of the fitness function, from the highest value to the lowest one, the first two individuals being accepted as members of the new population of decisional rules.

In case of an equality of the fitness value between a parent and a successor, it is selected a successor to become member of the new population.

By applying the genetic operations on the members of a population of decisional rules at the moment t, it results a new population of rules that shall be used at the moment t+1. The population from the initial moment, t = 0 is generated randomly and the genetic operations are applied iteratively until the moment T.

The above iterative process may be interpreted economically as it follows. The reproduction works as an imitation of the successful individuals. The binary strings of these individuals (firms) have high fitness values and they are copied by other firms. The strings with lower fitness values, which represent decisions to produce less and with a low profit, find few imitators (or none) in the following generation.

The crossing and the mutation are used to generate new ideas concerning the manner to produce more and to offer for sale on the market the product, recombining the existing decisional experiences and generating others new. If it is included the selection operation as well, the above interpretation is being modified in the sense that, during each period, the firms generate new production decisions using genetic operators. The fitness values of these new potential members of the population of decisional rules are compared with the ancient population, within the market conditions observed in the previous period. If the new ideas occurred are better than the previous ones, then they are implemented within the firms. The AG usage under these conditions determines the individual firms to learn in time to adopt decisions that lead to the increase of the profits. These are eventually maximised within all the firms, but they adopt production decisions that direct then gradually towards a higher profit.

The simulations performed showed that for different values of the parameters (different number of firms, different values of the coefficients *a, b, A* and *B* etc.) both stable and instable solutions are obtained.

Arifovic developed an AG with a multiple population, where each firm is endowed with an entire population of strings. We may consider that this population represents admissible rules of decision within which each firm selects, at a certain moment, a decision concerning the best conduit in a given environment. In each moment of time, only a string is being selected, this determining further on the conduit of the agent (firm).

The probability to select a certain type of string is proportional with its performances in predefined conditions. Although a firm selects only a string from an entire amount, all the alternative decisions are evaluated ex post. Therefore, in the context of the model presented previously, at each moment of time *t*, a firm selects a binary string out of an amount and then uses this string in order to update its decision of production. Once the price of clearing the market is calculated, the firm uses that price in order to determine the profits that each string from its amount of decisional rules shall bring for the level of the respective price. These profits determine the fitness values of the respective binary string. Once the fitness values are calculated, the genetic operators are applied within each population of binary strings, associated to the individual firms.

It is noticed that, through this modification of the model, it is obtained a more varied general frame which leads to more diversified ideas concerning the decisions that refer to the quality of product that shall be offered on the market. Although this frame becomes more complex, the calculation effort is identical with the previous one, where AG is applied to a single population of decision rules.

The application of the selection operation ensures the convergence towards balance of the model solution. When the convergence appears, all the binary strings, of all the populations of decision rules associated to the firms, are decoded in quantities. Without this operation of selection, the simulation shall lead to high fluctuations which are not reduced in time. Therefore, it was shown that, in the model of individual learning of the firms, these must use more sophisticated operations (for instance the operation of selection), in order to realise the convergence towards a balance. But this balance may be stable or instable, depending on the values that the given constants of the model have.

2. Economies with superposed generations and the selection of the economic balance

Another interesting application of AG is the one of determining the balance in different types of economies which has multiple points of balance: for instance, the economies with superposed generations, the economies with increases and the monetary economies.

Further on, we shall present a model of an economy with superposed generations which, under certain conditions, possesses multiple stationary balances. These stationary points of balance have different properties of stability, depending on the nature of the expectations incorporated in the model: the rational or the adaptive expectations.

First of all, we shall consider an economy which consists of two superposed generations of agents, therefore in which are living two generations: the young generation and the old generation. Each generation has an equal number of agents, N. We shall note the two generations with *t* and *t*+1. Each agent from the *t* generation lives only two consecutive periods, *t* and *t*+1, whereas the agents from the generation *t*+1 live only a single period, *t*+1.

An agent from the generation *t* consumes $c_t(t)$ during the first period (during youth) and c_{t+1} in the second period (when old). The agents have identical preferences and endowments with different goods. The young agent is endowed with w^1 units from a perishable consumable good, and the old agent w^2 units from the same consumable good ($w^1 > w^2$).

The money quantity offered in economy at the moment t is M(t).

Each agent from the young generation has to solve the following problem of optimum:

$$\max c_t(t) \times c_{t+1}(t)$$

under the conditions:

$$c_t(t) \le w^1 - \frac{m(t)}{p(t)}$$

$$c_{t+1}(t) \le w^2 + \frac{m(t)}{p(t+1)}$$

where:

m(t) represents the nominal monetary balances that an agent saves in the first period and spends in the second period of his life, and p(t) is the nominal level of the consumable good price in the period *t*.

The nominal monetary balances are obtained by dividing the money offer at the number of agents from economy at the moment *t*.

The dynamic of the nominal price p(t) under the conditions of the hypothesis concerning the perfect (naïve) anticipations is described by a relation of the type:

$$p(t) = \frac{S(t)}{S(t-1)} \times p(t-1)$$

where:

S(t) represents the total saving of the agents of *t* generation. p(0) and S(0) are considered given. It is noticed that the prices increase with a rate equal with the one of increase of the saving from a period to another.

If in economy is applied a monetary policy with a constant offer of money, therefore:

$$M(t) = M, \forall t$$

Then the equation of prices dynamics has an unique stationary *paretian* equilibrium point, given by

$$p(t) = p^*$$

where:

$$p^* = \frac{2M}{(w^1 - w^2)N}.$$

This equilibrium point is instable under the conditions of perfect anticipations and is reached every time the price p(t) becomes equal with p^*.

There is also a continuum of *indexed* monetary balances after the initial level of the prices $p(0) = p_0$ in the interval (p^*, ∞). In this continuum, all the monetary balances with an initial price $p_0 > p^*$ converge towards a stationary equilibrium point where the money loose their value.

Another possible monetary policy is the one with a constant monetary deficit of G dimension, financed through money print. The value of G is given by

$$G = \frac{M(t) - M(t-1)}{p(t)}$$

supposing that the existing quantity of money in economy in the initial period, M(0). Under the conditions of such a policy, the solution of the model contains two points of stationary equilibrium: a point π_1^* corresponding to a reduced inflation and a point π_2^* corresponding to a high inflation.

The point π_1^* is of the type superior Pareto. The point π_2^* is stable equilibrium, being a drawer for the equilibrium trajectory under the conditions of the hypothesis of the rational expectations, trajectory which leaves from an initial point $\pi_0 \in \left(\pi_1^*, \frac{w^1}{w^2}\right) \times \pi_0$ is equal even with π_1^*, the economy arrives at a stationary equilibrium with reduced inflation. The stability condition involves, also, that an increase of the budgetary deficit G determines a decrease of the inflation rate in a stable stationary equilibrium.

In order to apply AG to the above model, there are considered two populations of binary strings at each moment of time *t*. One represents the set of rules for the young members of the generation *t* and the other the set of rules for the old members of the generation *t*+1. Each population is updated in alternative periods of time after its members passed through a cycle of life of two periods.

The binary strings refer to the values of consume of the agents during the first period. A member *i*, $i \in \{1,\ldots,N\}$ of the generation *t* takes a decision concerning the consume in the first period (young) at the moment *t*, noted $c^1_{i,t}$, using a binary string. The economies of the agent *i* from the generation *t* are given by

$$s_{i,t} = w^1 - c^1_{i,t}$$

The sequence of events which takes place at the moment *t* is the following:

The values of the consume from the first period (young) are obtained through decodification and normalisation of the binary strings associated to the individuals from the population and then determine, for each agent *i,* the individual economies $s_{i,t}$.

Then, it is determined the value of the aggregated economy S(t) amounting the individual economies of the agents from the young generations:

$$S(t) = \sum_{i=1}^{N} s_i(t)$$

The price of the consumable good at the moment *t*, p(t) is obtained then from the relation:

$$p(t) = \frac{M}{S(t)}$$

in case of the monetary policy with constant offer of money, or

$$p(t) = \frac{S(t-1) \times p(t-1)}{S(t) - G}$$

for $S(t - 1) > G$, in case of the monetary policies with constant budgetary deficit, G.

It is then determined the consume in the second period (old) of the agent *i,* $i \in \{1,\ldots,N\}$, of the generation *t*-1:

$$c_{i,t}(t-1) = \frac{s_i(t-1) \times p(t-1)}{p(t)} + w^2$$

Eventually, there are calculated the values of the fitness function of the members of the generation t-1. The fitness function for a string *i* from the generation t-1 is given by the value of the utility consume of the agent *i* at the moment t+1(the second period of life):

$$\mu_{i,t-1} = U[c_{i,t-1}(t-1), c_{i,t-1}(t)] = c_{i,t-1}(t-1) \times c_{i,t-1}(t).$$

The population of rules of the generationt+1 is then obtained from the population of rules of generation *t* using genetic operators of reproduction, crossing, mutation and selection. Once the new generation t+1 of population is created, the entire cycle repeats. The population of rules of the generation t+1 represents the younger agents, whereas the members of the generation *t* become now old agents.

The populations of generations 0 and 1 year are generated aleatory. The system has at the beginning $N \times h$ monetary units distributed initially to the generation 1 (old).

The simulations in case of the economy with constant monetary offer converge towards a stationary equilibrium where the money has value. This equilibrium is also a convergence point in case of the economies with adaptive expectations which use an average of the levels of the passed prices for the price prognosis.

In the economies with constant positive value of the budgetary deficit, AG converges towards the stationary equilibrium point corresponding to a low inflation rate.

3. The economies with superposed generations and economic increase

Another application of AG is in the models of economic increase obtained leaving from the models with superposed generations.

Therefore, we consider an economy with a constant number N of agents, born in each period *t*. The agents are living two periods, a young and an old one, and each of them are endowed with a time unit at each moment *t*. All the agents from economy have the same function of utility:

$$U = \ln c_{i,t}(t) + \ln c_{i,t}(t+1)$$

There is only one perishable good which is being used both for consume and as an input for production. The output for labour unit is given by a function of neoclassic production:

$$f(k(t)) = k(t)^{\alpha}, \alpha \in (0,1)$$

k(t) being the technical endowment of work.

The rate of the physical capital productivity and the rate of the salary are given by:

$$r(t) = \alpha k\, t^{\alpha-1}$$

and respectively

$$w(t) = (1-\alpha)\times k\,(t)^{\alpha}$$

A young agent *i* from the generation *t* takes a decision to spend a fraction of time $\tau_{i,t} \in [0,1]$ for instruction. Each young agent inherits a efficiency level x(t) available in economy at the moment *t*. The level x(t) is obtained as an average of the efficiency units (human capital accumulated) of the agents of generation t-1:

$$x(t) = \frac{1}{N}\sum_{j=1}^{N} x_{j,t-1}(t),$$

where $x_{j,t-1}(t)$ represents the number of efficiency units of the agent *j* of generation t-1 at the moment *t*.

The young agents may combine this inherited endowment x(t) with the instruction decision $\tau_{i,t}(t)$ in order to obtain $x_{i,t}(t+1)$ effective units of work when they become old, using for this a method of instruction, noted $h(\tau_{i,t}(t), x(t))$.

An essential trait of the model is that the income obtained through training depends positively of the level x(t). Therefore $x_{i,t}(t+1)$ is given by the relation:

$$x_{i,t}(t+1) = h(\tau_{i,t}(t), x(t))x(t) = 1 + \gamma(x(t))\tau_{i,t}(t),$$

where $\gamma(\cdot)$ represents the productivity of the human capital and is given by a sigmoid function of the type:

$$\gamma(x(t)) = \frac{\lambda}{1+e^{-x(t)}} - \frac{\lambda}{2}$$

The function $\gamma(x(t))$ strictly increasing in relation with x(t), $\gamma(0) = 0$ and

$$\lim_{x(t)\to\infty} \gamma(x(t)) = \frac{\lambda}{2} = \hat{\gamma}$$

The parameter λ controls therefore the income obtained by training the agents.

The accumulation equation of the efficiency x(t) following the training process may be written:

$$x(t+1) = x(t)[1 + \gamma(x(t))\times\overline{\tau}(t)]$$

Where $\overline{\tau}(t) = \frac{1}{N}\sum_{i=1}^{N}\tau_{i,t}(t)$ represents the average of the training time of the agents of *t* generation.

Besides the decision concerning the allocation of the training time, the agents also take a decision concerning the fraction $\Phi_{i,t}$ from the available time that they save (leisure). This saving of time is then equal with:

$$s_{i,t}(t) = \Phi_{i,t}(t)\times w(t)[1-\tau_{i,t}(t)]\times x(t)$$

The decision concerning the leisure influences the accumulation of physical capital in economy in time.

The model has two state variables, one corresponding to the low income (the poverty course) and the second one corresponding to the high income (maximum increase). The first variable is equivalent with the state of stationary increase from the model of neoclassic increase without accumulation of human capital and with no technical progress. In the stationary state, $\tau = 0$ for all the agents *i*, $i \in \{1,...,N\}$ and all *t*, which makes that the human capital as well to stay at its initial level, therefore the efficiency x(t) stays constant for all *t*.

The other variable corresponds to the state of stationary increase where $\tau > 0$ for all *i* and all *t*. Therefore, x(t) shall increase with a constant rate such as for *t* enough high, $\gamma(t) \to \hat{\gamma}$.

The stationary state corresponding to the reduced income (the poverty course) is locally stable under the conditions of dynamics with rational expectations, whereas the stationary state corresponding to the high income (maximum increase) is a stable saddle point.

For the AG application, the decision of the agent *i*, $i \in \{1,...,N\}$ concerning the fraction of time that he spends for training, $\tau_{i,t} \in [0,1]$ in the decision concerning the fraction of time that he is saving, $\Phi_{i,t} \in [0,1]$, are represented by the same binary string of length *l*, where l/2 bits are used to encode the first decision, and the other bits to encode the second decision.

The fitness values of the decision rules are equal with the values of the function of utility registered at the end of the second period of life. The population of decision rules is updated using genetic operators of reproduction, crossing and mutation.

At each moment of time *t* there are two populations of such rules, one associated to the young agents and the other associated to the old agents.

No matter the initial conditions given, such an economy shall develop, following the simulation, towards a stationary state corresponding to the maximum increase which represents the global equilibrium of such an economy. Once such a state is reached (which happens in the case of the probability equal with 1), the economy stays in this state forever.

The initial level selected for x(t) represents the essential variable with respect to the time necessary to attain such a state. The more the initial level of x(t) is reduced, the more time is necessary for the economy to come out of the poverty course.

Initially, AG attains enough fast the stationary state corresponding to a low income. In this point, many of the decisional rules prescribe not to invest time in training because the investment in the human capital brings a low income and determines a reduced value depending on the fitness.

However, due to the effect of mutation, there is always a small fraction of rules which lead to positive values of the time dedicated to training, $\tau_{i,t}$. These rules may disappear from the population due to the pressure of selection. However, gradually, they contribute to the increase of x(t). In time, more x(t) increases, it reaches a sill beginning with which the income from the human capital becomes high and the fitness values of the decisional rules which invest in instruction begin to increase. Once this thing happens, the pressure of selection diminishes because the rules of decision which determine the positive decisions during the time dedicated to training bring now fitness values higher than those that determine the agents to invest zero time in training.

In this point, AG leads the economy fast towards a stationary state corresponding to a higher income, where $\tau_{i,t}$ take positive values for all the agents *i* and all the moments of time *t*. The transition phase is relatively short and, once the economy arrived in this stationary state, as we have shown, it stays here forever.

Since the accurate date of commutation depends on the specific sequence of mutations which leads to the accumulation of capital, the economies that have identical initial conditions may have different periods of development. In general, higher rates of the rules of decision of the economies which have invested in training determine average periods of realisation of the lower commutation.

Such models, although they show that there are different stationary states, which explains the different developments between the economies, do not show how it may pass from the economy in course of poverty to the economy with maximum increase or how much of the economy remains in the first state. However, these models capture two important aspects of the development process. The first one is for the low initial levels of the human capital per capita, which characterises the underdeveloped economies, the population of agents crosses several generations in the neighbourhood of the stationary state corresponding to the poverty course before that, eventually, to begin to develop on a trajectory that leads towards a stationary state corresponding to a high income. This thing explains why countries which at present are developed used to have a stationary level of initial development for hundreds of years.

The second aspect is that the economies with identical initial conditions may register transition periods between the two states of different duration. This thing is important because different dates of commutation involve different levels of income per capita, in the stationary state corresponding to the high income. This thing would explain as well the great differences that appear between the levels of the income per capita in the developed countries.

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