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Optimal Licensing Contracts with Adverse Selection and Informational Rents

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Abstract. *In the paper we analyse a model for determining the optimal licensing contract in both situations of symmetric and asymmetric information between the license's owner and the potential buyer. Next we present another way of solving the corresponding adverse selection model, using the informational rents as variables. This approach is different from that of Macho-Stadler and Perez-Castrillo.*

Keywords: optimal contract; innovation; adverse selection; informational rents.

JEL Codes: C61, D82, D86.
REL Codes: 7J, 7L.

Introduction

In the actual economic context, competition plays a very important role for the firm's existence on different markets. One of the main goals of any productive firm is finding new production technologies generating lower production costs, but high qualitative goods or services and yielding efficient results. The firms pay a special attention to development and research activity carried out by specialized departments. This activity results in new production possibilities, new patents or innovations and yields to a higher competitive position on the market for the respective firm.

In the paper we will analyse the effects of asymmetric information between the two contractual partners on the form of optimal licensing contract. We will use an adverse selection model, proposed for the first time by Macho-Stadler and Perez-Castrillo (1991).

Usually, a licensing contract is based on a fixed fee (and this fee is independent of the output produced or the activity) and a variable fee (royalty), dependent on the volume of output. Although the optimality of licensing contracts has been considerable studied in the last twenty years, there is no a general conclusion about the form of these contracts. The terms of licensing contracts depend on many factors such as: if the innovator (the licensor) competes with the innovation in the same market as the potential buyer, the number of firms competing for the innovation (potential buyers of the innovation), the seller's possibility to verify and to monitor the buyer after selling the patent or risk sharing between the two contractual partners (the seller and the buyer).

Recent papers tried to determine, on theoretical reasons, the optimal form of licensing contracts and to explain the necessity of including some variable fees a license contract. With the rapid development of the incentive theory in the last twenty years, many theoretical studies pointed out that the presence of asymmetric information between the licensor and the licensee influenced the form of licensing contracts: Gallini and Wright (1990), Macho-Stadler and Perez-Castrillo (1991), Begg (1992), Erutku et al. (2008), Antelo (2009).

In their paper, Gallini and Wright (1990) analyse the consequences of seller's private information on the optimality of contracts and they show that the high quality innovations are signalled through two-part contracts (a fixed fee and a variable fee). Macho-Stadler and Perez-Castrillo (1991) derive the features of optimal licensing contracts in two different situations of asymmetric information between the partners: signalling (the Principal-the seller has some private information and can signal the high quality on the patent through contracts based on royalties) and screening (the Agent-the buyer has some private information about the true value of the patent and the contracts must be based only on fixed fees). Begg (1992) shows that a contract based on royalties is more efficient than a licence contract based on fixed fee, due to the existence of a separating equilibrium of the game he analysed. In another theoretical study on the design of optimal contracts in the situation of incomplete information, Erutku et al. (2008) consider that the contracts based only on royalties are suboptimal from the buyer's perspective since there is always a two part contract dominating the contract based on royalties. Antelo (2009) analyses in a signalling game the effects of asymmetric information (the contract duration and the choice of the royalties) on the design of royalties-only contracts. Crama et al (2008) study the form of optimal licensing contracts when the buyer's evaluation of innovation represents incomplete information; they show that the presence of moral hazard and adverse selection can yields to a higher loss.

There are not so many empirical studies about the optimal terms of licensing contracts and the results don't confirm all the features generated in theoretical studies. Some interesting analysis belong to Rostocker (1983), Taylor and Silberstone (1973), Macho-Stadler et al. (1996), Jensen and Thursby (2001), Mukherjee et al. (2008).

The paper is organized as follows. In Section 1 we briefly present the main assumptions and the model from Macho-Stadler and Perez-Castrillo's paper (1991, pp. 189-208). In Section 2 we characterise the optimal licensing contract in the situation of symmetric information (complete information) and we comment the results. Section 3 presents another approach for solving the Principal's optimization program in the situation of asymmetric information; we use here the variables "informational rents" which permit us to interpret the features of the second best optimal contracts in terms of the problems with asymmetry of information between the Principal and the Agent. A section with conclusions closes the paper.

1. The model

In their paper, Macho-Stadler and Perez-Castrillo analzed two types of problems with informational asymmetry between the seller (the owner of a patent) and the buyer (a firm who's intention is to use the patent) of one innovation (license). The first problem corresponds to a classical signalling problem where the seller knows better than the buyer the quality of innovation. The second problem, where the buyer has an informational advantage (private information), corresponds to an adverse selection problem. This last type of problem is also described in our paper.

We assume that a research institute owns a patent (innovation) and it can't obtain profit directly, but only through a licensing contract. We will refer further at this institute as being the Principal.

On the other hand, the Agent is a monopolist (a firm) having a constant average production cost c^0, so that the total cost of production is given by $c^0 \times Q$.

Definition 1. A licensing contract in the situation of symmetric information is a pair (F, ε), where:

F represents the amount paid by the monopolist when signing the contract (this is usually referred as a *fixed fee*);

ε is the amount paid by the Agent to the Principal for each unit produced by the firm using the patent (this is usually referred as a *variable fees* or *royalties*).

In the new situation – after buying the license, the average production cost becomes lower than c^0, $c < c^0$.

We assume that the market demand function for the good produced by the monopolist is given by $D(P)$ and the monopoly price is denoted by $P^m(x)$. Then, for each level of the average cost *x*, the firm's profit function is denoted by $\pi^m(x)$.

Proposition 1. The profit function satisfies:

$$\frac{d\pi^m(x)}{dx} = -D\left(P^m(x)\right)$$

Proof

The profit function for each value of x is:

$$\pi^m(x) = \left[P^m(x) - x\right]D\left(P^m(x)\right) \quad (1)$$

where

$$P^m(x) \in \arg\max_P (P - x)D(P)$$

Then:

$$\frac{d}{dP}(P - x)D(P) = 0 \text{ for } P = P^m(x)$$

or

$$D\left(P^m(x)\right) + \left[P^m(x) - x\right]D'\left(P^m(x)\right) = 0 \quad (2)$$

Differentiating the profit function from (1) with respect to x yields:

$$\frac{d\pi^m(x)}{dx} = \left[\frac{dP^m(x)}{dx} - 1\right] D\big(P^m(x)\big) + \left[P^m(x) - x\right] D'\big(P^m(x)\big)\frac{dP^m(x)}{dx} =$$

$$= \frac{dP^m(x)}{dx}\left[D\big(P^m(x)\big) + \left[P^m(x) - x\right] D'\big(P^m(x)\big)\right] - D\big(P^m(x)\big) =$$

$$= -D\big(P^m(x)\big)$$

Remarks:

1. In the last equality we used the relation (2).

2. The above result could be alternatively obtained differentiating the function $\pi^m(x)$ with respect to x as an explicit variable and using the envelope theorem (Details are presented in *Appendix 1*).

2. The optimal contract in the situation of symmetric information

First suppose that there is no asymmetry of information between the Principal and the Agent. This corresponds to the situation where both the seller and the buyer (of the patent) know the true value of the innovation.

The Principal's problem is to maximize the total profit subject to Agent's participation constraint (he is willing to pay no more than the profit excess).

We use the following notation: $D^m(x) = D\big(P^m(x)\big)$.

Then, the optimization problem is written as:

$$\underset{F,\varepsilon}{(\max)}\left[F + \varepsilon \times D^m(c+\varepsilon)\right]$$

$s.t.$

(I) $F \le \pi^m(c+e) - \pi^m(c^0)$

$F \ge 0$

$\varepsilon \ge 0$

Solving the problem

In the above program, we associate the Kuhn-Tucker multipliers to each constraint, including the sign constraints as inequalities. We present this variant of solving the problem in Appendix 2. The Lagrangian is given by:

$$L(F,\varepsilon;\lambda,\alpha,\beta) = F + \varepsilon \times D^m(c+\varepsilon) +$$

$$+ \lambda\left[\pi^m(c+\varepsilon) - \pi^m(c^0) - F\right] + \alpha \times F + \beta \times \varepsilon$$

Optimizing with respect to the variables, the first order conditions are:

$$\frac{\partial L}{\partial F} = 0 \tag{3}$$

$$\frac{\partial \mathrm{L}}{\partial \varepsilon} = 0 \tag{4}$$

$$\frac{\partial L}{\partial \lambda} \geq 0,\ \ \lambda \geq 0 \text{ and } \lambda \frac{\partial L}{\partial \lambda} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \alpha} \geq 0,\ \alpha \geq 0 \text{ and } \alpha \frac{\partial L}{\partial \alpha} = 0 \tag{6}$$

$$\frac{\partial L}{\partial \beta} \geq 0,\ \beta \geq 0 \text{ and } \beta \frac{\partial L}{\partial \beta} = 0 \tag{7}$$

From (3) we get:

$1 - \lambda + \alpha = 0$ or $\lambda = 1 + \alpha > 0$.

From (5) it follows that $\frac{\partial L}{\partial \lambda} = 0$; therefore, we have:

$$F = \pi^m(c+\varepsilon) - \pi^m(c^0) \tag{*}$$

This result has a well known interpretation: the participation constraint is binding at the optimum.

The first order condition (4) is equivalent to:

$$\frac{\partial L}{\partial \varepsilon} = D^m(c+\varepsilon) + \varepsilon \times D'^m(c+\varepsilon) + \lambda \times \pi'^m(c+\varepsilon) + \beta = 0$$

We use in the above relation the result from *Proposition 1* and $\lambda = 1 + \alpha$. This yields to:

$$D^m(c+\varepsilon) + \varepsilon \times D'^m(c+\varepsilon) - (1+\alpha)D^m\left(c+\varepsilon\right) + \beta = 0$$

or:

$$\varepsilon \times D'^m(c+\varepsilon) - \alpha \times D^m(c+\varepsilon) + \beta = 0$$

We will show that, for any value of β, at the optimum it must be that $\varepsilon = 0$.

Suppose that β is equal to 0. Then, we obtain:

$$\varepsilon \times D'^m(c+\varepsilon) - \alpha \times D^m(c+\varepsilon) = 0,$$

But, in this last equation there is a sum of two negative terms. The only conclusion is that $\varepsilon = 0$ and $\alpha = 0$.

On the other hand, if $\beta > 0$, from the first order condition (7), it must be that $\frac{\partial L}{\partial \beta} = 0$; therefore $\tilde{\varepsilon} = 0$.

Using the result $\tilde{\varepsilon} = 0$, the condition (*) becomes:
$\tilde{F} = \pi^m(c+\varepsilon) - \pi^m(c^0) = \pi^m(c) - \pi^m(c^0) > 0$

We note that, if $\tilde{F} > 0$, then it follows immediately from (6) that $\alpha = 0$.

Hence, we got the optimal licensing contract $\left(\tilde{F},0\right) = \left(\pi^m(c) - \pi^m(c^0),0\right)$, which is a Pareto-optimal contract.

We can now formulate the following theorem:

Theorem 1. In the case of symmetric information, the optimal contract (the first best solution) in the sense of *Definition 1* is given by $(\tilde{F},0)$, with $\tilde{F} = \pi^m(c) - \pi^m(c^0)$.

In such a situation, the optimal licensing contract is based only on a fixed fee, which is exactly equal to the buyer's profit excess and the Agent's production is efficient.

3. The optimal contract in the situation of asymmetric information

With asymmetry of information, the Agent has some private information (hidden information) and knows better than the Principal how important the innovation is. In some situations, the asymmetry of information consists in a better knowledge of the buyer related to the demand function for the output he produces or related to the production technology impact and the real value of the innovation. These situations are more probable when the patent's owner doesn't act on the industry or on the respective product market.

The Principal (the owner of the license) considers that the Agent's evaluation is one of two types – the type G (good) or the type B (bad), and having the corresponding average cost $c^G < c^0$ and $c^B < c^0$ respectively, with

$c^G < c^B$. The probability the Principal believes that the innovation has the type G is denoted by γ.

Without knowing the innovation type, it is optimal for the Principal to propose a menu of contracts, one contract for each type of Agent.

Definition 2. A menu of contracts $\left((F^G,\varepsilon^G),(F^B,\varepsilon^B)\right)$ is *feasible* if it satisfies the following participation constraints:

$$\pi^m(c^G+\varepsilon^G)-\pi^m(c^0)-F^G \geq 0 \qquad (8)$$

$$\pi^m(c^B+\varepsilon^B)-\pi^m(c^0)-F^B \geq 0 \qquad (9)$$

This definition means that the contracts (F^G,ε^G), $\left(F^B,\varepsilon^B\right)$ assigned to each type of Agent yield a profit greater then the profit obtained by the Agent without using the patent.

Furthermore, for a menu to be accepted it must satisfies the *incentive compatibility constraints* – each type of Agent will select the contract the Principal designs for him.

Definition 3. A menu of feasible contracts $\left((F^G,\varepsilon^G),(F^B,\varepsilon^B)\right)$ is incentive feasible if it satisfies also the following *incentive compatibility constraints*:

$$\pi^m(c^G+\varepsilon^G)-F^G \geq \pi^m(c^G+\varepsilon^B)-F^B \qquad (10)$$

$$\pi^m(c^B+\varepsilon^B)-F^B \geq \pi^m(c^B+\varepsilon^G)-F^G \qquad (11)$$

This means that when the Agent chooses the contract $\left(F^G,\varepsilon^G\right)$ his profit evaluated at the average cost c^G is greater than the profit obtained if he would choose the other contract $\left(F^B,\varepsilon^B\right)$. The same interpretation corresponds to the second constraint: when the Agent chooses the contract $\left(F^B,\varepsilon^B\right)$ having the average cost c^B, his profit is greater than the profit obtained when he chooses the other contract $\left(F^G,\varepsilon^G\right)$.

The inequalities (8)-(11) fully characterise the set of incentive feasible contracts.

We can now write the Principal’s program:

$$\text{(II)} \quad \underset{F^G,\varepsilon^G,F^B,\varepsilon^B}{(\max)} \left\{\gamma\left[F^G+\varepsilon^G\times D^m(c^G+\varepsilon^{G)}\right]+(1-\gamma)\left[F^B+\varepsilon^B\times D^m(c^B+\varepsilon^B)\right]\right\}$$

s.t.

$$\pi^{\mathrm{m}}(\mathrm{c}^{\mathrm{G}}+\varepsilon^G)-\mathrm{F}^{\mathrm{G}} \geq \pi^{\mathrm{m}}(\mathrm{c}^{\mathrm{G}}+\varepsilon^B)-\mathrm{F}^{\mathrm{B}} \qquad (10)$$

$$\pi^{m}(c^{B}+\varepsilon^{B}) - F^{B} \geq \pi^{m}(c^{B}+\varepsilon^{G}) - F^{G} \qquad (11)$$

$$\pi^{m}(c^{G}+\varepsilon^{G}) - \pi^{m}(c^{0}) - F^{G} \geq 0 \qquad (12)$$

$$\pi^{m}(c^{B}+\varepsilon^{B}) - \pi^{m}(c^{0}) - F^{B} \geq 0 \qquad (13)$$

$$F^{G} \geq 0,\ \varepsilon^{G} \geq 0,\ F^{B} \geq 0, \varepsilon^{B} \geq 0$$

Proposition 2. If the set of incentive feasible contracts is nonempty, the constraint (12) is implied by the constraints given by (10) and (13).

Therefore, if the innovation of type *B* is accepted, it is also accepted the other type of innovation.

Proof

We use the monotonicity property of the profit function. From *Proposition 1* it follows that the profit function $\pi^{m}(\cdot)$ is strictly decreasing and using the constraint (10) we get:

$$\pi^{m}(c^{G}+\varepsilon^{G}) - F^{G} \geq \pi^{m}(c^{G}+\varepsilon^{B}) - F^{B} \geq \pi^{m}(c^{B}+\varepsilon^{B}) - F^{B}$$

or

$$\pi^{m}(c^{G}+\varepsilon^{G}) - F^{G} - \pi^{m}(c^{0}) \geq \pi^{m}(c^{B}+\varepsilon^{B}) - F^{B} - \pi^{m}(c^{0})$$

So, the constraint (12) is not a relevant one and can be ignored when solving the Program (II).

Theorem 2. The optimal contract in the situation of *adverse selection* (the second best solution) $\left((\overline{F}^{G}, \overline{\varepsilon}^{G}), (\overline{F}^{B}, \overline{\varepsilon}^{B})\right)$ satisfies the following:

$$\overline{F}^{G} < \tilde{F}^{G},\ \overline{\varepsilon}^{G} = \tilde{\varepsilon}^{G} = 0$$

and

$$\overline{F}^{B} < \overline{F}^{G},\ \overline{\varepsilon}^{B} > 0\,.$$

Next, we will give another proof of this theorem, different from that of Macho-Stadler and Perez-Castrillo. For this, we will use the informational rents as variables.

The model transformed using the variables informational rents – variable fees

Definition 3. The expressions

$$U^{G} = \pi^{m}(c^{G}+\varepsilon^{G}) - F^{G} - \pi^{m}(c^{0})$$

and

$U^B = \pi^m(c^B + \varepsilon^B) - F^B - \pi^m(c^0)$

are the *informational rents* for each type of innovation G and B, respectively.

We transform the objective function and all the constraints of the Program (II) using these new variables.

If we substitute F^G and F^B we obtain:

$$F^G = \pi^m(c^G + \varepsilon^G) - \pi^m(c^0) - U^G \tag{14}$$

$$F^B = \pi^m(c^B + \varepsilon^B) - \pi^m(c^0) - U^B \tag{15}$$

Then, the objective function of the Principal becomes:

$$H\left(\varepsilon^G, \varepsilon^B, U^G, U^B\right) = \gamma\left[\pi^m(c^G + \varepsilon^G) - \pi^m(c^0) + \varepsilon^G \times D^m(c^G + \varepsilon^G)\right] + \\ + (1-\gamma)\left[\pi^m(c^B + \varepsilon^B) - \pi^m(c^0) + \varepsilon^B \times D^m(c^B + \varepsilon^B)\right] - \\ - \left[\gamma \times U^G + (1-\gamma)U^B\right]$$

This expression shows that the Principal objective is maximizing the expected social value of trade minus the expected informational rent of the Agent.

Before we go further, we need to introduce the following function:

Definition 4. The function $\phi : [0, \infty) \to R$, $\phi(\varepsilon) = \pi^m(c^G + \varepsilon) - \pi^m(c^B + \varepsilon)$ is the *excess function* with respect to the patent's type.

The function $\phi(\varepsilon)$ has the following properties:

i) $\phi(\varepsilon) > 0, \ \forall \varepsilon \geq 0$

ii) $\phi'(\varepsilon), \ \forall \varepsilon > 0$

While the first property is induced by the monotonicity property of the profit function with respect to the average cost, the second property of the excess function shows that the marginal excess is strictly decreasing with respect to the marginal cost.

The higher the average cost is, the lower the excess function is. At such high average cost, there is almost any difference between the two types of innovation.

With the above change of variables, the constraints (10)-(13) will be written in terms of ε^G, ε^B, U^G and U^B.

The incentive constraint (10) is transformed as follows:

$$\pi^m(c^G+\varepsilon^G)-F^G-\pi^m(c^0)\geq \pi^m(c^B+\varepsilon^B)-F^B-\pi^m(c^0)+$$
$$+\pi^m(c^G+\varepsilon^B)-\pi^m(c^B+\varepsilon^B)$$

or

$$U^G\geq U^B+\phi(\varepsilon^B) \qquad \text{(upward constraint)} \tag{16}$$

Similarly, the incentive constraint (11) becomes:

$$U^B\geq U^G-\phi(\varepsilon^G) \qquad \text{(downward constraint)} \tag{17}$$

The participation constraints (12) and (13) are now equivalent to the following sign constraints $U^G\geq 0$ and $U^B\geq 0$.

The optimization problem of the Principal (II) is rewritten as:

$$\underset{\varepsilon^B,\varepsilon^G,U^G,U^B}{(\max)}\ G\left(\varepsilon^G,\varepsilon^B,U^G,U^B\right)$$

(III) s.t.

$$U^G\geq U^B+\phi(\varepsilon^B) \tag{16a}$$
$$U^B\geq U^G-\phi(\varepsilon^G) \tag{17a}$$
$$U^G\geq 0 \tag{18}$$
$$U^B\geq 0 \tag{19}$$

Proposition 3. In the optimization Program (III), at optimum we have:
i) the constraints $U^B\geq 0$ and $U^G\geq U^B+\phi(\varepsilon^B)$ are binding;
ii) the constraints $U^B\geq U^G-\phi(\varepsilon^G)$ and $U^G\geq 0$ are not relevant.

Proof

i) Suppose that $U^B>0$ and let $u>0$ be a small real value such that $U^B-u\geq 0$.

If $(\varepsilon^G,\varepsilon^B,U^G,U^B)$ represents the optimal solution of the problem (III), then $(\varepsilon^G,\varepsilon^B,U^G-u,U^B-u)$ is at least a feasible solution for this problem. The objective function evaluated in this point is:

$$G(\varepsilon^G,\varepsilon^B,U^G-u,U^B-u)=G(\varepsilon^G,\varepsilon^B,U^G,U^B)+u>G\left(\varepsilon^G,\varepsilon^B,U^G,U^B\right)$$

This is a contradiction with the assumption that $(\varepsilon^G,\varepsilon^B,U^G,U^B)$ represents the optimal solution. Therefore, $\overline{U}^B=0$ is optimal.

Suppose now that $U^G > \phi(\varepsilon^B)$. For a small real value $u > 0$, the solution $(\varepsilon^G, \varepsilon^B, U^G - u, 0)$ is at least a feasible solution and the corresponding value of the objective function is:

$$G(\varepsilon^G,\ \varepsilon^B,\ U^G - u, 0) = G(\varepsilon^G,\ \varepsilon^B,\ 0,\ U^B) + \gamma \times u > G\left(\varepsilon^G,\ \varepsilon^B,\ 0,\ U^B\right)$$

This contradicts the above assumption.

Therefore $U^G = \phi(\varepsilon^B) > 0$ is optimal.

ii) With the results proven above, the constraint $U^B \geq U^G - \phi(\varepsilon^G)$ becomes:

$$0 \geq \phi(\varepsilon^B) - \phi(\varepsilon^G) \text{ or } \phi(\varepsilon^G) \geq \phi(\varepsilon^B)$$

(but this is true from the *implementability condition*).

The constraint $U^G \geq 0$ is a simple consequence of the conditions: $U^B \geq 0$ and $U^G \geq U^B + \phi(\varepsilon^B)$.

With all the results, the optimization problem is simplified and becomes a simple optimization problem with respect to only two variables:

$$\begin{aligned}\max_{\varepsilon^G,\varepsilon^B} H(\cdot) = \{ & \gamma[\pi^m(c^G + \varepsilon^G) + \varepsilon^G \times D^m(c^G + \varepsilon^G)] + \\ & + (1-\gamma)\left[\pi^m(c^B + \varepsilon^B) + \varepsilon^B \times D^m(c^B + \varepsilon^B)\right] \\ & - \gamma \times \phi(\varepsilon^B)\}\end{aligned}$$

For this new optimization problem, the necessary conditions are:

$$\frac{\partial(\cdot)}{\partial \varepsilon^G} = 0 \Rightarrow$$
$$\Rightarrow \gamma\left[\frac{d\pi^m(c^G + \varepsilon^G)}{d\varepsilon^G} + D^m(c^G + \varepsilon^G) + \varepsilon^G \times D'^m(c^G + \varepsilon^G)\right] = 0 \tag{20}$$

and

$$\frac{\partial(\cdot)}{\partial \varepsilon^B} = 0 \Rightarrow$$
$$\Rightarrow (1-\gamma)\left[\frac{d\pi^m(c^B + \varepsilon^B)}{d\varepsilon^B} + D^m(c^B + \varepsilon^B) + \varepsilon^B \times D'^m(c^B + \varepsilon^B)\right] - \tag{21}$$
$$- \gamma \times \phi'\left(\varepsilon^B\right) = 0$$

Using the result of *Proposition 1*, these first order conditions (20) and (21) can be written as:

$$\varepsilon^G \times D'^m(c^G + \varepsilon^G) = 0 \tag{22}$$

and

$$(1-\gamma)\varepsilon^B \times D'^m(c^B + \varepsilon^B) - \gamma \times \phi'(\varepsilon^B) = 0 \tag{23}$$

The condition (22) yields to $\bar{\varepsilon}^G = 0$.

Solving for ε^B into the condition (23) it follows immediately that $\bar{\varepsilon}^B > 0$.

Suppose that $\bar{\varepsilon}^B > 0$ is not optimal. Then if $\bar{\varepsilon}^B = 0$, we would have:
$\phi'(0) = 0 = -D^m(c^G) + D^m(c^B) < 0$ and this is a contradiction.

Therefore, the optimal solution for the program (III) is given by:
$\bar{\varepsilon}^G = 0$, $\bar{\varepsilon}^B > 0$, $\bar{U}^B = 0$ and $\bar{U}^G = \phi(\bar{\varepsilon}^B)$

We can easily derive now the optimal fixed fees (the old variables).

From $\bar{U}^B = 0$, we get:

$$\bar{F}^B = \pi^m(c^B + \bar{\varepsilon}^B) - \pi^m(c^0) < \pi^m(c^B) - \pi^m(c^0) = \tilde{F}^B$$

Similarly, from $\bar{U}^G = \phi(\bar{\varepsilon}^B)$ we get:

$$\bar{F}^G = \pi^m(c^G + \bar{\varepsilon}^G) - \pi^m(c^0) - \bar{U}^G =$$
$$= \pi^m(c^G) - \pi^m(c^0) - \phi(\bar{\varepsilon}^B) < \pi^m(c^G) - \pi^m(c^0) = \tilde{F}^G$$

If we evaluate the difference $\bar{F}^G$ - $\bar{F}^B$ we obtain:

$$\bar{F}^G - \bar{F}^B = \pi^m(c^G) - \pi^m(c^0) - \phi(\bar{\varepsilon}^B) - \pi^m(c^B + \bar{\varepsilon}^B) + \pi^m(c^0) =$$
$$= \pi^m(c^G) - \pi^m(c^G + \bar{\varepsilon}^B) + \pi^m(c^B + \bar{\varepsilon}^B) - \pi^m(c^B + \bar{\varepsilon}^B) =$$
$$= \pi^m(c^G) - \pi^m(c^G + \bar{\varepsilon}^B) > 0$$

Hence $\bar{F}^G > \bar{F}^B$.

We found the results given in Theorem 2. The main advantage of our approach is the direct way of deriving the Agents payoffs (named informational rents).

Remarks

We can now summarize the features of the optimal licensing contracts in the situation of asymmetric information:

A. The contract designed for the firm with high evaluation (type G) is completely based on a fixed fee $\overline{F}^G$ lower than the fixed fee corresponding to the situation of symmetric information (the first best solution). We have therefore, $\overline{\varepsilon}^G = 0 = \tilde{\varepsilon}^G$.

This contract gives to the Agent of type G a positive informational rent $\overline{U}^G = \phi(\overline{\varepsilon}^B)$.

B. The contract designed for the low evaluation (type B) generates the same profit as the profit obtained without using the new technology. This second best contract is based on two parts: a fixed fee $\overline{F}^B$ and a variable fee, $\overline{\varepsilon}^B > \tilde{\varepsilon}^B = 0$. With this contract the Agent gets no informational rent.

An example

Suppose that the demand function is $D(p) = p^{-\alpha}$, with $\alpha > 1$.

We first find the price such that $p^m(x) \in \arg\max[p - x]D(p)$.

We denote by $f(p) = (p - x)p^{-\alpha} = p^{1-\alpha} - x \times p^{-\alpha}$ the objective function from the above optimization problem. This function is a concave function.

And its derivative with respect to p is:

$$f'(p) = (1-\alpha)\, p^{-\alpha} - x(-\alpha)\, p^{-\alpha-1}$$

The first order condition for the maximization problem is given by:

$$f'(p) = 0$$

or

$$p^{-\alpha}\left[1 - \alpha + \alpha \times x \times p^{-1}\right] = 0$$

Therefore:

$$p^m(x) = \frac{\alpha \times x}{\alpha - 1} = \left(1 + \frac{1}{\alpha - 1}\right)x \qquad (24)$$

The above expression shows that the price is a linear positive transformation with respect to the average production cost and a hyperbolic decreasing transformation with respect to the parameter α.

From (24) we can find the optimal demand:

$$D^m(x) = \frac{(\alpha-1)^\alpha}{\alpha^\alpha} \times \frac{1}{x^\alpha} \tag{25}$$

and the profit function

$$\pi^m(x) = \frac{x^{1-\alpha}}{\alpha-1} \times \frac{(\alpha-1)^\alpha}{\alpha^\alpha} \tag{26}$$

It is easy to see the link between the profit function $\pi^m(x)$ and the demand $D^m(x)$:

$$\frac{d\pi^m(x)}{dx} = \frac{(1-\alpha)x^{-\alpha}}{\alpha-1} \times \frac{(\alpha-1)^\alpha}{\alpha^\alpha} = -\frac{(\alpha-1)^\alpha}{\alpha^\alpha} \times \frac{1}{x^\alpha} = -D^m(x)$$

This result confirms the result of *Proposition 1*.

The optimization problem with respect to the contract variables (F, ε) in this particular case is written as:

$$\underset{F,\varepsilon}{(\max)} \left\{ F + \varepsilon \frac{(\alpha-1)}{\alpha^\alpha} \times \frac{1}{(c+\varepsilon)^\alpha} \right\}$$

s.t.

$$\frac{(c+\varepsilon)^{1-\alpha}}{(\alpha-1)} \times \frac{(\alpha-1)^\alpha}{\alpha^\alpha} - \frac{(c^0)^{1-\alpha}}{(\alpha-1)} \times \frac{(\alpha-1)^\alpha}{\alpha^\alpha} \geq F$$

$$\varepsilon \geq 0$$

$$F \geq 0$$

Let λ be the multiplier assigned to the first constraint in the above program, and let a and b be the Kuhn-Tucker multipliers of the sign constraints.

As in Appendix 2, the Lagrange function is:

$$\begin{aligned} L(F, \varepsilon; \lambda, a, b) = F + \varepsilon \frac{(\alpha-1)^\alpha}{\alpha^\alpha} \times \frac{1}{(c+\varepsilon)^\alpha} + \\ + \lambda \left[\frac{(c+\varepsilon)^{1-\alpha}}{\alpha-1} \times \frac{(\alpha-1)^\alpha}{\alpha^\alpha} - \frac{(c^0)^{1-\alpha}}{\alpha-1} \times \frac{(\alpha-1)^\alpha}{\alpha^\alpha} - F \right] + \\ + a \times \varepsilon + b \times F \end{aligned}$$

And the first order conditions are:

$$\frac{\partial L}{\partial F}=0 \text{ or } 1-\lambda+b=0.$$

It follows that $\lambda>0$ and then, at the optimum we have:

$$F=\frac{(c+\varepsilon)^{1-\alpha}}{\alpha-1}\times\frac{(\alpha-1)}{\alpha^{\alpha}}-\frac{\left(c^{0}\right)^{1-\alpha}}{\alpha-1}\times\frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}} \tag{27}$$

$$\frac{\partial L}{\partial \varepsilon}=0 \text{ or,}$$

$$a+\frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}}\times(c+\varepsilon)^{-\alpha}+\varepsilon\times\frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}}\times\frac{-\alpha}{(c+\varepsilon)^{\alpha+1}}+$$

$$+\lambda\times\frac{-1}{(c+\varepsilon)^{\alpha}}\times\frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}}=0$$

In this last equation, using $\lambda=1+b$ we get:

$$-\varepsilon\times\frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}}\times\frac{\alpha}{(c+\varepsilon)^{\alpha+1}}-b\times\frac{1}{(c+\varepsilon)^{\alpha}}\times\frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}}+a=0 \tag{28}$$

Analysing the relation (28), we note that if $a>0$, then $\varepsilon=0$.
Otherwise, if $a=0$ then $\varepsilon=0$ and $b=0$.
Therefore, the optimal contract is given by:

$$\tilde{\varepsilon}=0 \text{ and } \tilde{F}=\frac{(\alpha-1)^{\alpha-1}}{\alpha^{\alpha}}\left[c^{1-\alpha}-\left(c^{0}\right)^{1-\alpha}\right]>0$$

Conclusions

We analyzed in this paper the effects of asymmetric information between the owner of an innovation and the potential buyer (in a monopoly position) on the design of optimal licensing contracts. If it is the buyer who has private information, then the contracts based only on fixed fees are designed for the firm with a high evaluation of innovation and this type of firm gets a positive informational rent. On the other hand, if the firm has a low evaluation about the innovation, then the contract designed by the Principal (the seller) is based also on variable fees.

However, in practice, the contractual terms or the particular form of a license contract don't depend only on incentive aspects or on the presence of asymmetric information between the Principal and the Agent. These terms depend also on some other factors such as financial firm's situation on the market uncertainty or business risk. But these informational problems could explain some contractual forms that are used in different economic activities.

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References

Antelo, M., „On contract duration of royalty licensing contracts", *Spanish Economic Review*, 11, 2009, pp. 277-299

Begg, A.W., „The licensing of patents under asymmetric information", *International Journal of Industrial Organization*, 10, 2002, pp. 171-194

Crama, P., Peyck, B., Degraeve, Z., „Milestone Payments or royalties? Contract design for R&D licensing", *Operations Research*, 56, 2008, pp. 1539-1552

Erutku, C., Freire, A.P., Richelle, Y., „Licensing innovations with exclusive contracts", *Review of Industrial Organization*, 31, 2007, pp. 261-273

Gallini, N.T., Wright, B.D., „Technology transfer under asymmetric information", *Rand Journal of Economics*, 21, 1990, pp. 147-160

Jensen, R., Thursby, M., „Proofs and prototypes for sale: the licensing of university inventions", *American Economic Review*, 91, 2001, pp. 240-259

Katz, M.L., Shapiro, C., „On the licensing of innovations", *Rand Journal of Economics*, 16, 1985, pp. 504-520

Laffont, J.J., Martimort, D. (2002). *The Theory of Incentives. The Principal-Agent Model*. Princeton University Press

Macho-Stadler, I., Perez-Castrillo, D.J., „Contrats de license et asymetrie d'information", *Annales d'Economie et de Statistique*, 24, 1991, pp. 189-208

Macho-Stadler, I., Martinez-Giralt, X., Perez-Castrillo, D.J., „The role of information in licensing contract design", *Research Policy*, 25, 1996, pp. 43-57

Mukherjee, A., Broll, U., Mukherjee, S., „Unionized labor market and licensing by a monopolist", *Journal of Economics*, 93, 2008, pp. 59-79

Appendix 1

The Envelope theorem

Consider the following parameterized optimization problem:

$(\max) f(x,a)$

(P) s.t.

$g(x,a)=0$

where

$f: R^n \times R \to R$ and $g: R^n \times R \to R$.

Let $L(x,t,a) = f(x,a) - tg(x,a)$ be the Lagrange function associated to the above problem and let $x(a)$ be a maximizing choice – an optimal solution of the problem (P).

We denote by $M(a) = f(x(a),a)$ the optimal value for the objective function. We are often concerned with examining how this optimal value changes when the parameter *a* is changed. These derivatives are easily evaluated using the first order conditions and the chain rule. We have:

$$\frac{dM(a)}{da} = \frac{\partial L(x(a),a)}{\partial a} = \frac{\partial f(x(a),a)}{\partial a} - t\frac{\partial g(x(a),a)}{\partial a}.$$

Appendix 2

The Kuhn-Tucker conditions

The problem

Consider $f, g, g_j : \Re^n \to \Re$, $j = 1,...,m$ be the functions belonging to the class C^2 from $\Re^n$.

We want to find the optimal solution of the following problem:

$$\begin{cases} [Opt]\ f(x_1, x_2, ..., x_n) \\ s.t. \quad \mathrm{g}(x_1, x_2, ..., x_n) = g_0 \\ \quad \mathrm{g_j}(x_1, x_2, ..., x_n) \le 0 \\ or \\ \quad \mathrm{g_j}(x_1, x_2, ..., x_n) \ge 0 \\ x_i \ge 0,\ \mathrm{i} = 1,2,...,\mathrm{n},\ \mathrm{j} = 1,2,..,\mathrm{m} \end{cases}$$

The problem can be solved using the *KUHN-TUCKER* method, which generalises the method of Lagrange multipliers.

Let λ be the multiplier associated to the equality constraint and λ_j be the Kuhn-Tucker multipliers of the inequality constraints in the above problem.

We assume that the multipliers are all nonnegative.

Then, if the objective function is maximized, the *maximization problem* must be written as follows**:**

$$\begin{cases} [Max] f(x_1, x_2, ..., x_n) \\ \quad g_0 - g(x_1, x_2, ..., x_n) = 0 \\ \quad g_j(x_1, x_2, ..., x_n) \ge 0 \\ x_i \ge 0, i = 1, 2, ..., n, j = 1, 2, .., m \end{cases}$$

Instead, the *minimization problem* has the form:

$$\begin{cases} [Min] f(x_1, x_2, ..., x_n) \\ \quad g_0 - g(x_1, x_2, ..., x_n) = 0 \\ \quad g_j(x_1, x_2, ..., x_n) \le 0 \\ x_i \ge 0, i = 1, 2, ..., n, j = 1, 2,..., m \end{cases}$$

Solving the problem

There is also another possibility for solving the above general problem (and this is the approach used in our paper): the sign constraints can be considered as simply inequality constraints and can be included into the Lagrange function.

Hence, for the above *maximization problem*, with this new formulation, the Lagrange function is:

$$L(\cdot) = f(x_1, x_2, ..., x_n) + \lambda[g_0 - g(x_1, x_2, ..., x_n)] + \sum_{j=1}^{m} \lambda_j g_j(x_1, x_2, ..., x_n) + \sum_{i=1}^{n} \alpha_i x_i$$

(with all multipliers satisfying $\alpha_i \geq 0$).

The first order conditions are the following:

$$\begin{cases} \dfrac{\partial L}{\partial x_i} = 0, \quad \mathrm{i} = 1,2,...,\mathrm{n} \\ \dfrac{\partial L}{\partial \lambda} = 0 \\ \dfrac{\partial L}{\partial \lambda_j} \geq 0, \quad \lambda_{\mathrm{j}} \geq 0 \ \text{ and } \ \lambda_{\mathrm{j}} \dfrac{\partial L}{\partial \lambda_j} = 0, \ \ \mathrm{j} = 1,2,...,\mathrm{m} \\ \dfrac{\partial L}{\partial \alpha_i} \geq 0, \quad \alpha_i \geq 0 \ \text{ and } \ \alpha_i \dfrac{\partial L}{\partial \alpha_i} = 0, \quad \mathrm{i} = 1,2,...,\mathrm{n} \end{cases}$$

If the problem to be solved is a minimization problem, in the above Lagrange function the last term will have the opposite sign (i.e. $-\sum_{i=1}^{n} \alpha_i \times x_i$), and the first order conditions are similar, except the sense of the inequalities from the last two groups conditions. Note that the first order conditions are only necessary conditions and not sufficient (second order conditions).