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Some considerations regarding the use of the fix and chained basis indices

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Abstract. *In the authors meaning, this work submits the merits of utilizing the chain based system for building up the price indices within the time series comparatively with the utilization of the fix base system. Meantime, the authors are emphasizing the different properties, axioms or tests, which might be satisfied by an index formula.*

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The chain system is measuring the price modifications between one period and the following one by using a bilateral index formula, which considers the prices and quantities from both periods. The modifications values within a period (chain connections) are cumulated afterwards in order to get the relative prices out of the entire considered period.

Thus, if the price bilateral index is P, the chain system is generating the following model for the prices values of the first three periods:

$$1, P(p^0, p^1, q^0, q^1), P(p^0, p^1, q^0, q^1)\, P(p^1, p^2, q^1, q^2)$$

On the contrary, by using the same bilateral formula of the index P, the fix base price system is calculating only the prices values of the period *t,* relatively to the reference period 0 $P(p^0, p^t, q^0, q^t)$. The outcome shows that the fix base prices model for the periods 0,1 and 2 is:

$$1, P(p^0, p^1, q^0, q^1), P(p^0, p^2, q^0, q^2)$$

To notice that for both systems – chain and fix base – for prices using the above formulas, the value of the price of the reference period is a stable one, considered as 1. For the statistical offices the common practice consists of setting up the value of the price for the reference period at the level of 100. If proceeding likewise, then each of the numbers inserted within the above formulas must be multiplied by 100.

Because of the difficulties occurring as for obtaining the information concerning the quantities for the running period (or, equivalently, the expenses), many of the statistical offices are grounding their consumption prices index on the Laspeyres formula and the fix base system.

Therefore, it is interesting to note a number of the issues that might occur when utilizing the Laspeyres' s indices of fix base.

The main problem when utilizing the Laspeyres' s indices of fix base consists of the fact that the goods basket with fix basis for the period 0, which prices are watched during the period t may many times differ quite significantly from the basket of the period t.

Thus, if there are systematic tendencies for at least a number of prices and quantities of the index basket, then the price Laspeyres' s index of fix base $P_L(p^0, p^t, q^0, q^t)$ may differ significantly from the corresponding Paasche's index, $P_P(p^0, p^t, q^0, q^t)$. This means that, probably, these indices are not the most dequate representation of the average prices movement over the period taken into account.

The quantitative Laspeyres's index of fix base is not always available for utilization; finally, the quantities q^0 from the reference period are so much moving away from the quantities q^t of the current period, that the base must be changed for each period.

The main advantage of the "chain" system is given by the fact that, under normal conditions, the chaining will reduce the outrun between the Paasche and Laspeyres indices.

Each one of these two indices offers an asymmetric perspective on the prices modifications occurring between the two periods taken into consideration. Consequently, the utilization of the chained indices of Laspeyres or Paasche type will lead to a smaller difference between them as well as to estimations much closer to the "reality".

Based on some previous researches – Szulc (1983) and Hill (1988), Hill (1993) noticed that, when the prices fluctuate more or less, it is not appropriate to use the chain system. This phenomenon may occur in the context of seasonal usual fluctuations or of "prices wars". However, in the context of the prices modifications roughly uniform, Hill (1993) recommended the utilization of the chain indices with symmetrical weights. Fisher and Walsh indices are examples of such indices with symmetrical weights.

There are many more explanations available on the conditions under which the chaining may or may not apply. Briefly, the chaining is recommended if the prices and quantities of adjacent periods are close to an extent larger than the prices and quantities of the most remote periods in which case this strategy would lead to the reduction of the gap between the Laspeyres and Paasche indices from each "link" of the chain.

Of course, it is necessary to measure to what extent the prices and quantities of the two periods are similar. These measurements may be relative or absolute. In the case of the absolute comparisons, two vectors of the same dimension are similar if they are identical and non-similar on the contrary. In the case of the relative comparisons, two vectors are similar if they are proportional to each other and non-similar if they are not proportional.

Once the way of measuring the similarity is established, the prices and quantities of each period can be thus compared to each other enabling the drawing of a "tree" or way interconnecting all the observations, where the most similar observations are compared to each other by using a formula of bilateral index.

Hill (1995) alleged the following idea: to the extent the prices structures of two countries are more un-similar, the difference between P_L and P_P is larger; this means that $\{P_L/P_P, P_P/P_L\}$ is bigger. The issue arising out in connection with the measurement of this non-similarity of the prices structure from the two countries consists of the fact that it might happen that $P_L=P_P$ (in this case Hill's measurement would record a maximum degree of similarity), but p^0 might be very different from p^t. This is why it is necessary to proceed to a more systematic research of the similarity (or non-similarity) measurements, in order to be able to select "the best one", which might be used in the frame of the Hill's "tree" (2001), the one connecting and comparing the observations to each other.

The method of chained observations, as explained above, grounded on the similarity of the price and quantitative structures of doesn't matter which two observations, might not be practical in the context of a statistical agency since a new period added might lead to a re-ordination of the previous connections. However, the above referred "scientific" method concerning the observations chaining might be useful when is it decided whether to use the chaining or fix base indices in order to achieve the month-to-month comparisons over one year.

Some specialists argued against the utilization of the chaining principle, because this has no corresponding party in a spatial context:

„They (the chained indices) apply to inter-temporary comparisons and, contrary to the direct indices they are not applicable in the situations where there is no order or natural succession. Therefore, the idea of a chained index, for instance, has no correspondent in the frame of the inter-regional or international price comparisons since the countries cannot be arranged in a "logical" or "natural" manner (there is not a country k + 1 or k - 1 to compare with the country k) (von der Lippe (2001))".

This allegation is certainly correct but Hill's approach is leading to a "natural" set of spatial connections. Applying the same approach in the context of the time series would lead to a set of chaining between periods not necessary of the month-to-month type but which, in many situations, justify the year-to-year chaining of the data belonging to the same month.

To a certain extent, it is interesting to find out if there are indices formulas that offer the same result irrespectively if using the fix bas system or the chain system.

Comparing the succession of the chain indices defined as above with the corresponding fix base indices, it is obvious that will get the same response for all the three periods, if the formula P of the index is satisfying the following functional equation for all the price and quantitative vectors:

$$P(p^0, p^2, q^0, q^2) = P(p^0, p^1, q^0, q^1)\, P(p^1, p^2, q^1, q^2)$$

If the formula of a P index satisfies the above equation, then P satisfies the circularity test.

For the situation when the P index formula satisfies certain properties or tests, besides the circularity test mentioned above, Funke, Hacker and Voeller (1979) showed that P must have the following functional formula, initially established by Konus and Byushgens (1926)

$$P_{KB}(p^0, p^1, q^0, q^1) = \prod_{i=1}^{n} \left(\frac{p_i^1}{p_i^0} \right)^{\alpha_i}$$

where the n constants α_i satisfy the following restrictions:

$$\sum_{i=1}^{n} \alpha_i = 1 \quad \text{and } \alpha_i > 0 \ \text{ for } i = 1, \ldots, n$$

Otherwise, under very permissive conditions of regularity, the only prices index that satisfies the circularity test is the weighted geometrical average of all the ratios of individual prices, the weights being time constants.

A particular case of the family indices defined by the Funke, Hacker and Voeller equation occurs when all the weights α_i are equal. In this case, PKB came to the Jevons index (1865):

$$P_J(p^0, p^1, q^0, q^1) = \prod_{i=1}^{n} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}}$$

The issue of the indices defined by Konus, Byushgens and Jevons is that the individual prices ratios, p^1_i/p^0_i, hold weights (α_i or 1/n) which are independent as against the economical importance of the commodity i within the two periods taken into account. In other words, these weights for prices are independent as against the quantities of the commodity i consumed or against the expenses involved by the commodity i within the two periods. Consequently, these indices are not actually fit for being used by the statistical agencies at higher levels of aggregation whenever we have information connected with the expenses quotas.

The above outcomes indicate that it is not useful to require that the price index P satisfy precisely the circularity test. However, it is somehow interesting to find out indices formulas satisfying the circularity test up to a certain degree of approximation, since the utilization of a such a formula for the index would lead to values of the aggregated prices modification, which are more or less the same, irrespectively is using the chain or fix base system. Fisher (1922) discovered that, by utilizing his data set and the ideal price index Fisher P_F, the deviations from circularity have been quite small enough. This relatively high degree of correspondence between the chained and the fix base indices is characterizing other symmetrical weighted formulas as well, such as the index Walsh P_W. For most of the applications with time series from the indices theory, where the base year for the fix base indices is changed every five years (approximately), it would not count for too much the fact that the statistical agency applies a fix base index or a chain index considering the fact that a symmetrical weighted formula is used. Of course, the selection between a fix base index and a chain index degree will depend on the length of the time series taken into account and on the variation of the prices and quantities from one period to another. To the extent the prices and quantities are subject of larger fluctuations, the correspondence between the two types of index is smaller.

We can offer a theoretical explanation for the rough success of the circularity test applied to the symmetrical weighted indices formulas. Another symmetrical weighted formula is the Tornqvist P_T index. The natural logarithm of this index is defined as follows:

$$\ln P_T(p^0, p^1, q^0, q^1) = \sum_{i=1}^{n} \frac{1}{2}(s_i^0 + s_i^1)\ln\left(\frac{p_i^1}{p_i^0}\right)$$

where the expenses quotas s^t_i for the period t are defined.

Alterman, Diewert and Feenstra (1999) showed that the Tornquist PT will precisely satisfy the circularity test, provided that the logarithmic ratios of prices $\ln (p^t_i / p^{t-1}_i)$ have a linear evolution during the period t, and the expenses quotas s^t_i have a similar evolution over the time. As many economic time series for prices and quantities are roughly satisfying these conditions, the index Tornquist P_T will satisfy the circularity test with approximation as well. Generally speaking, the Tornquist index is closely approximating the Fisher and Walsh symmetrical weighted indices, so that for many economic time series (with smooth evolutions) all these three symmetrical weighted indices would satisfy the circularity test with an approximation degree high enough to allow the utilization of the fix base principle or the chain principle without counting too much.

Walsh (1901) presented the following useful alternative for the circularity test:

$$1 = P(p^0, p^1, q^0, q^1)P(p^1, p^2, q^1, q^2)...P(p^T, p^0, q^T, q^0)$$

The motivation for this test is the following: we use the bilateral index formula $P(p^0, p^1, q^0, q^1)$ in order to calculate the prices dynamics between the periods 0 and 1; we apply the same formula bought to the data of the periods 1 and 2, $P(p^1, p^2, q^1, q^2)$, in order to calculate the dynamics (change) of the prices between the periods 1 and 2, ,. . . ; we use $P(P^{T-1}, p^T , q^{T-1}, q^T)$ in order to calculate the prices dynamics between T-1 and T; we insert an artificial period T+1 that has the same price and quantity from the initial period 0 and apply $P(p^T, p^0, q^T, q^0)$ in order to calculate the prices dynamics from T to 0. Afterwards, we multiply all these indices by one another. Since we came back to the departure point, the product of these indices should ideally count as 1. Diewert (1993) called this test "the multi-period identity test". To note that if T=2 (so that there are three periods all together), then the Walsh test is reduced to the time reversion test of Fisher.

Walsh (1901) showed the way the circularity test can be used in order to evaluate how "good" any bilateral index formula is. To this purpose, he invented the artificial prices and quantities for five periods and added a sixth period holding the data of the first period. Then he evaluated the right side of the last equation above for various $P(p^0, p^1, q^0, q^1)$, and established how far away of the unit the outcomes were. His "best" results obtained products close to 1.

The framework is used also for evaluating the efficiency of the chain indices as comparatively with their direct homologues. Thus, if the right side of the last equation above proves to differ from unit, it is said that the chained indices are afflicted by the "chain derive". Sometimes, in case a formula is afflicted by the chain derive, it is recommended too apply to fix base indices instead of the chained ones.

However, if this advice is observed, it will always lead to the choice of fix base indices, provided that the index formula satisfies the circularity identity test, $P(p^0, p^0, q^0, q^0)=1$. Otherwise, it is not recommended to use Walsh circularity test in order to establish whether fix base indices or chained indices are to be used. It is correct to apply to the Walsh circularity test for its genuine purpose, namely as an approximate method to establish how "good" is a certain index formula. In order to decide whether to chain the indices or to use fix base indices, pay attention to the extent the compared observations are similar and chose the method, which would connect to the best the most similar observations.

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